SOLUTIONS & ANSWERS FOR JEE MAINS-2021
24th February Shift 2
[PHYSICS, CHEMISTRY & MATHEMATICS]

PART – A – PHYSICS

SECTION A

Q.1 A soft ferromagnetic material is placed in an external magnetic field. The magnetic domains :

Options

1. decrease in size and changes orientation.
2. increase in size but no change in orientation.
3. may increase or decrease in size and change its orientation.
4. have no relation with external magnetic field.

Ans: 3

Sol: Option (3)

Q.2 An X-ray tube is operated at 1.24 million volt. The shortest wavelength of the produced photon will be :

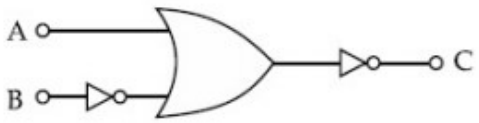
Options

1. 10^{-2} nm
2. 10^{-3} nm
3. 10^{-4} nm
4. 10^{-1} nm

Ans: 2

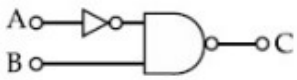
Sol: $\lambda = \frac{12400}{V} = \frac{12400}{1.24 \times 10^6} = 10^{-2}$ nm

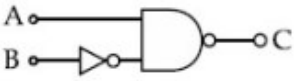
Q.3

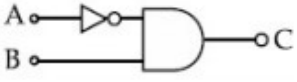


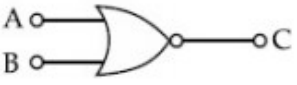
The logic circuit shown above is equivalent to :

Options

1. 

2. 

3. 

4. 

Ans: **3**

Sol: $C = \overline{A + B}$

$= \overline{A}.\overline{B} = \overline{A}.B$

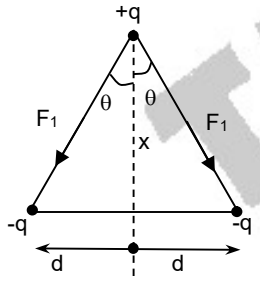
Q.4 Two electrons each are fixed at a distance '2d'. A third charge proton placed at the midpoint is displaced slightly by a distance x ($x << d$) perpendicular to the line joining the two fixed charges. Proton will execute simple harmonic motion having angular frequency : (m = mass of charged particle)

Options

1. $\left(\frac{2\pi\varepsilon_0 md^3}{q^2}\right)^{\frac{1}{2}}$
2. $\left(\frac{q^2}{2\pi\varepsilon_0 md^3}\right)^{\frac{1}{2}}$
3. $\left(\frac{\pi\varepsilon_0 md^3}{2q^2}\right)^{\frac{1}{2}}$
4. $\left(\frac{2q^2}{\pi\varepsilon_0 md^3}\right)^{\frac{1}{2}}$

Ans: **2**

Sol:



$F = -2(F_1 \cos\theta)$

$$\frac{-2q^2}{4\pi\varepsilon_0(d^2+y^2)} \times \frac{y}{\sqrt{d^2+y^2}} = -\left(\frac{q^2}{2\pi\varepsilon_0 d^3}\right)y$$

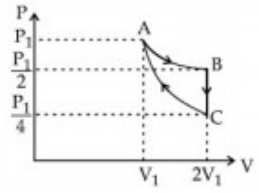
Comparing with F = -ky, here k = $\left(\frac{q^2}{2\pi\varepsilon_0 d^3}\right)$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$t = \frac{1}{2}\sqrt{\frac{q^2}{2\pi\varepsilon_0 d^3 \times m}}$$

Angular frequency, $\omega = 2\pi f = 2\pi \times \frac{1}{2\pi}\sqrt{\frac{q^2}{2\pi\varepsilon_0 d^3 \times m}} = \left(\frac{q^2}{2\pi\varepsilon_0 m\, d^3}\right)^{1/2}$

Q.5 If one mole of an ideal gas at (P_1, V_1) is allowed to expand reversibly and isothermally (A to B) its pressure is reduced to one-half of the original pressure (see figure). This is followed by a constant volume cooling till its pressure is reduced to one-fourth of the initial value (B → C). Then it is restored to its initial state by a reversible adiabatic compression (C to A). The net workdone by the gas is equal to :



Options

1. 0
2. $-\frac{RT}{2(\gamma-1)}$
3. $RT \ln 2$
4. $RT\left(\ln 2-\frac{1}{2(\gamma-1)}\right)$

Ans: 4

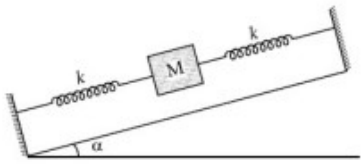
Sol: $W_{AB}=P_1V_1\,\ell n\left(\frac{2V_1}{V_1}\right)=P_1V_1\,\ell n\,2$

$W_{BC}=0$

$$W_{CA}=\frac{\left(\frac{P_1\times 2V_1}{4}-P_1V_1\right)}{\gamma}=\frac{-P_1V_1}{2(\gamma-1)}$$

$$W_{net}=P_1V_1\left[\ell n2-\frac{1}{2(\gamma-1)}\right]=RT_1\left[\ell n2-\frac{1}{2(\gamma-1)}\right]$$

Q.6 In the given figure, a body of mass M is held between two massless springs, on a smooth inclined plane. The free ends of the springs are attached to firm supports. If each spring has spring constant k, the frequency of oscillation of given body is :



Options

1. $\frac{1}{2\pi}\sqrt{\frac{2k}{Mg\sin\alpha}}$
2. $\frac{1}{2\pi}\sqrt{\frac{k}{2M}}$
3. $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$
4. $\frac{1}{2\pi}\sqrt{\frac{k}{Mg\sin\alpha}}$

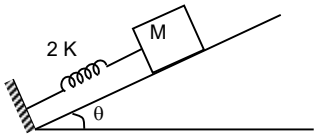
Ans: **3**

Sol: The springs are in parallel

$\therefore K_{eq} = K + K = 2K$

$$\omega = \sqrt{\frac{2K}{M}}$$

$$\therefore f = \frac{1}{2\pi}\sqrt{\frac{2K}{M}}$$



Q.7 The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of 'L' is 1.0 m from meter scale having a minimum division of 1 mm and time of one complete oscillation is 1.95 s measured from stopwatch of 0.01 s resolution. The percentage error in the determination of 'g' will be :

Options

1. 1.03%
2. 1.33%
3. 1.30%
4. 1.13%

Ans: **4**

Sol: $g \propto \frac{\ell}{T^2}$

$\therefore$ Percentage error in g is given by

$$\frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + 2.\frac{\Delta T}{T} = \frac{10^{-3}}{1} + \frac{2 \times 0.01}{1.95} = 0.0113$$

$$\frac{\Delta g}{g} \times 100 = 0.0113 \times 100 = 1.13\%$$

Q.8 A body weighs 49 N on a spring balance at the north pole. What will be its weight recorded on the same weighing machine, if it is shifted to the equator ?

[Use $g = \frac{GM}{R^2} = 9.8\ ms^{-2}$ and radius of earth, R = 6400 km.]

Options

1. 48.83 N
2. 49.83 N
3. 49 N
4. 49.17 N

Ans: **1**

Sol: $mg_p = 49$

$$m = \frac{49}{g_p} = \frac{49}{9.8} = 5\ kg$$

$$g'_{eq} = g - R\omega^2 = 9.8 - (6400 \times 10^3)\ (7.27)\ (10^{-5})$$

$$\omega = \frac{2\pi}{T} = \frac{2 \times 3.14}{24 \times 3600} = 7.27 \times 10^{-5}$$

$$\therefore g'_{eq} = 9.8 - 0.034 = 9.766$$

$W = mg'_{eq} = 5 \times 9.766 = 48.83$ N

Q.9 Zener breakdown occurs in a $p-n$ junction having p and n both :

Options

1. heavily doped and have wide depletion layer.
2. lightly doped and have narrow depletion layer.
3. lightly doped and have wide depletion layer.
4. heavily doped and have narrow depletion layer.

Ans: **4**

Sol: Option (4)

Q.10 On the basis of kinetic theory of gases, the gas exerts pressure because its molecules :

Options

1. continuously stick to the walls of container.
2. suffer change in momentum when impinge on the walls of container.
3. are attracted by the walls of container.
4. continuously lose their energy till it reaches wall.

Ans: **2**

Sol: Option (2)

Q.11 The de Broglie wavelength of a proton and α-particle are equal. The ratio of their velocities is :

Options

1. $4:1$
2. $1:4$
3. $4:3$
4. $4:2$

Ans: **1**

Sol: $\lambda_p = \lambda_\alpha$

$$\frac{h}{m_p V_p} = \frac{h}{m_\alpha V_\alpha}$$

$$\frac{1}{m_p V_p} - \frac{1}{4m_p V_\alpha}$$

$$\frac{1}{V_p} = \frac{1}{4V_\alpha}$$

$$\frac{V_\alpha}{V_p} = \frac{1}{4} \Rightarrow \frac{V_p}{V_\alpha} = 4:1$$

Q.12 Which of the following equations represents a travelling wave ?

Options

1. $y = Ae^{-x^2}(vt+\theta)$
2. $y = A\sin(15x-2t)$
3. $y = Ae^{x}\cos(\omega t-\theta)$
4. $y = A\sin x\ \cos\omega t$

Ans: **2**

Sol: Option (2)

Q.13 Match List - I with List - II.

List - I		List - II	
(a)	Source of microwave frequency	(i)	Radioactive decay of nucleus
(b)	Source of infrared frequency	(ii)	Magnetron
(c)	Source of Gamma Rays	(iii)	Inner shell electrons
(d)	Source of X-rays	(iv)	Vibration of atoms and molecules
		(v)	LASER
		(vi)	RC circuit

Choose the correct answer from the options given below :

Options

1. (a)-(vi), (b)-(v), (c)-(i), (d)-(iv)
2. (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)
3. (a)-(vi), (b)-(iv), (c)-(i), (d)-(v)
4. (a)-(ii), (b)-(iv), (c)-(vi), (d)-(iii)

Ans: **2**

Sol: Option (2)

Q.14 If the source of light used in a Young's double slit experiment is changed from red to violet :

Options

1. the intensity of minima will increase.
2. the fringes will become brighter.
3. consecutive fringe lines will come closer.
4. the central bright fringe will become a dark fringe.

Ans: **3**

Sol: $\beta = \frac{D}{d}\lambda$

When source of light is changed from red to violet, wavelength decreases, so fringes will come closer.

Q.15 According to Bohr atom model, in which of the following transitions will the frequency be maximum ?

Options

1. $n=3$ to $n=2$
2. $n=4$ to $n=3$
3. $n=5$ to $n=4$
4. $n=2$ to $n=1$

Ans: **4**

Sol: Frequency will be maximum where wavelength of emitted radiation is minimum.

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Q.16 Given below are two statements :

Statement I : PN junction diodes can be used to function as transistor, simply by connecting two diodes, back to back, which acts as the base terminal.

Statement II : In the study of transistor, the amplification factor β indicates ratio of the collector current to the base current.

In the light of the above statements, choose the correct answer from the options given below.

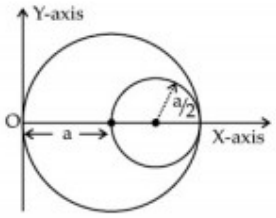
Options

1. Both Statement I and Statement II are false
2. Statement I is true but Statement II is false
3. Statement I is false but Statement II is true
4. Both Statement I and Statement II are true

Ans: **3**

Sol: Option (3)

Q.17 A circular hole of radius $\left(\frac{a}{2}\right)$ is cut out of a circular disc of radius 'a' as shown in figure. The centroid of the remaining circular portion with respect to point 'O' will be :



Options

1. $\frac{5}{6}a$
2. $\frac{2}{3}a$
3. $\frac{10}{11}a$
4. $\frac{1}{6}a$

Ans: **1**

Sol: $$X = \frac{M(a) + \left(-\frac{M}{4}\right)\left(\frac{3}{2}a\right)}{M - \frac{M}{4}} = \frac{5}{6}a$$

Q.18 A particle is projected with velocity v_0 along x-axis. A damping force is acting on the particle which is proportional to the square of the distance from the origin i.e. $ma = -\alpha x^2$. The distance at which the particle stops :

Options

1. $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{2}}$
2. $\left(\frac{2v_0^2}{3\alpha}\right)^{\frac{1}{2}}$
3. $\left(\frac{2v_0}{3\alpha}\right)^{\frac{1}{3}}$
4. $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{3}}$

Ans: **4**

Sol: $ma = -\alpha x^2$

$m.\frac{dv_0}{dt} = -\alpha x^2$

$m.\frac{dv_0}{dx}.\frac{dx}{dt} = -\alpha x^2$

$m.v_0.\frac{dv_0}{dx} = -\alpha x^2$

$mv_0.dv_0 = -\alpha x^2.dx$

Integrating,

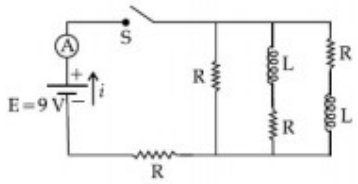
$$m.\int v_0.dv_0 = -\alpha\int x^2.dx$$

$$m.\frac{v_0^2}{2} = -\alpha\frac{x^3}{3}$$

$$x^3 = \frac{3}{2\alpha}.mv_0^2$$

$$x = \left(\frac{3V_0^2}{2\alpha}\right)^{1/3}$$

Q.19 Figure shows a circuit that contains four identical resistors with resistance R = 2.0 Ω, two identical inductors with inductance L = 2.0 mH and an ideal battery with *emf* E = 9 V. The current '*i*' just after the switch 'S' is closed will be :

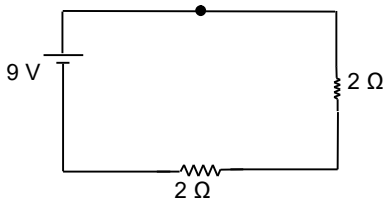


Options

1. 3.0 A
2. 9 A
3. 2.25 A
4. 3.37 A

Ans: **3**

Sol: At t = 0,



$$i = \frac{V}{R_{eff}} = \frac{9}{2+2} = 2.25\ A$$

Q.20 When a particle executes SHM, the nature of graphical representation of velocity as a function of displacement is :

Options

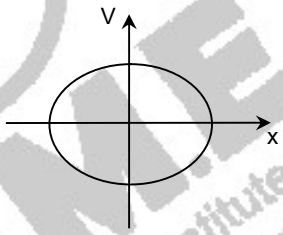
1. elliptical
2. circular
3. straight line
4. parabolic

Ans: **1**

Sol: $V = \omega\sqrt{A^2 - x^2}$

$$\frac{V^2}{\omega^2} = A^2 - x^2$$

$$\frac{V^2}{A^2\omega^2} + \frac{x^2}{A^2} = 1 \Rightarrow \text{ellipse}$$



SECTION B

Q.1 Two solids A and B of mass 1 kg and 2 kg respectively are moving with equal linear momentum. The ratio of their kinetic energies $(K.E.)_A : (K.E.)_B$ will be $\frac{A}{1}$, so the value of A will be _____.

Ans: **2.00**

Sol: $$\frac{KE_A}{KE_B} = \frac{\frac{P_A^2}{2m_A}}{\frac{P_B^2}{2m_B}} = \frac{m_B}{m_A} = 2:1$$

A = 2.00

Q.2 A cylindrical wire of radius 0.5 mm and conductivity 5×10^7 S/m is subjected to an electric field of 10 mV/m. The expected value of current in the wire will be $x^3\pi$ mA. The value of x is _____.

Ans: 5.00

Sol: $I = \frac{V}{R} = \frac{V}{\frac{\rho\ell}{A}} = \frac{VA}{\rho\ell} = \frac{V}{\ell}\cdot\frac{A}{\rho} = \frac{V}{\ell}.A\sigma$

$= E.A.\sigma$

$= 10 \times 10^{-3} \times 5 \times 10^{7} \times \pi\,(0.5 \times 10^{-3})^2$

$= 1.25 \times \pi \times 10 \times 10^{-3} \times 10^{7} \times 10^{-6}$

$= 1.25\,\pi \times 10^{-1}$

$= 0.125\,\pi\,.A$

$= 0.125 \times 10^{3}\pi$ mA

$= 125\,\pi$ mA

$= 5^3\,\pi$ mA

$\therefore x = 5.$

Q.3 A signal of 0.1 kW is transmitted in a cable. The attenuation of cable is -5 dB per km and cable length is 20 km. The power received at receiver is 10^{-x} W. The value of x is _____.

$\left[\text{Gain in dB} = 10\log_{10}\left(\frac{P_0}{P_i}\right)\right]$

Ans: 8.00

Sol: $P_i = 0.1$ KW $= 100$ W

Attenuation, - 5 dB per kilometer

$\therefore$ for 20 km $\Rightarrow$ - 100 dB

$\therefore -100 = 10\ \log_{10}\frac{P_0}{P_i} = 10\ \log\ \frac{P_0}{100}$

$\Rightarrow \log_{10}\frac{P_0}{100} = -10$

$\Rightarrow \frac{P_0}{100} = 10^{-10}$

$\Rightarrow P_0 = 10^{-10} \times 10^{2} = 10^{-8}$

$\Rightarrow x = 8$

Q.4 An electromagnetic wave of frequency 3 GHz enters a dielectric medium of relative electric permittivity 2.25 from vacuum. The wavelength of this wave in that medium will be _____ $\times 10^{-2}$ cm.

Ans: 667.00

Sol: $f = 3$ GHz $\quad \varepsilon_r = 2.28$

$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \quad \Rightarrow V = \frac{C}{\sqrt{\mu_r\varepsilon_r}}$

Usually $\mu_r \approx 1$

$\Rightarrow V = \frac{3\times10^8}{\sqrt{2.25}} = \frac{3\times10^8}{1.5} = 2\times10^8\ \text{ms}^{-1}$

But $V = f\lambda \ \Rightarrow \lambda = \frac{V}{f} = \frac{2\times10^8}{3\times10^9}$

$\therefore \lambda = 0.667\times10^{-1}\text{m}$

$= 6.67\times10^{-2}\text{m}$

$= 667\times10^{-2}$ cm

$= 667$

Q.5 The root mean square speed of molecules of a given mass of a gas at 27°C and 1 atmosphere pressure is 200 ms^{-1}. The root mean square speed of molecules of the gas at 127°C and 2 atmosphere pressure is $\frac{x}{\sqrt{3}}$ ms^{-1}. The value of x will be _____.

Ans: 400.00

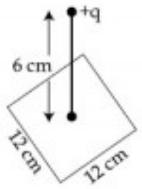
Sol: $V_{rms} = \sqrt{\frac{3RT}{M}} \propto \sqrt{T}$ $T_1 = 300$ k , $T_2 = 400$ k

$$\frac{V_{rms}1}{V_{rms}2} = \sqrt{\frac{300}{400}}$$

$V_{rms}\,2 = V_{rms}\,1 \times 2/\sqrt{3} = 200 \times 2/\sqrt{3} = 400/\sqrt{3}$

$\therefore$ x = 400.

Q.6 A point charge of $+12\ \mu C$ is at a distance 6 cm vertically above the centre of a square of side 12 cm as shown in figure. The magnitude of the electric flux through the square will be _____ $\times 10^3$ Nm^2/C.



Ans: 225.90

Sol: $\phi_{net} = \frac{q}{\varepsilon_0}$

$$\phi_{sq} = \frac{\phi_{net}}{6} = \frac{q}{6\varepsilon_0} = \frac{12\times10^{-6}}{6\varepsilon_0}$$

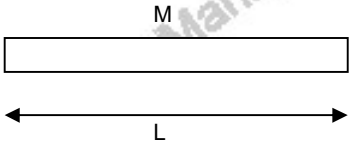
$= \frac{2}{8.85} \times 10^6 = 0.2259 \times 10^6 = 0.2259 \times 10^3 \times 10^3$

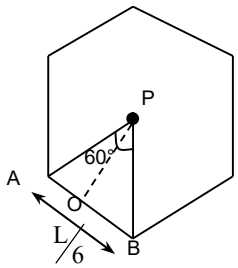
$= 225.9 \times 10^3\ Nm^2 / C.$

Q.7 A uniform thin bar of mass 6 kg and length 2.4 meter is bent to make an equilateral hexagon. The moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of hexagon is _____ $\times 10^{-1}$ kg m^2.

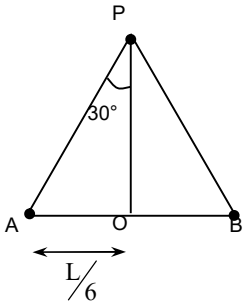
Ans: 8.00

Sol:





MI of AB about P= $\frac{M_{AB}\left(L/6\right)^2}{12}+M_{AB}(PO)^2$



OP = $\frac{L}{12\tan 30}=\frac{\sqrt{3}L}{12}$

$\therefore$ MI of AB about P= $\frac{M_{AB}\left(L/6\right)^2}{12}+M_{AB}(PO)^2$

$=\frac{1\left(L^2\right)}{36\times 12}+\frac{3L^2}{144}$

Total MI = 6 $(I_{AB})_P$

$= 6\left[\frac{L^2}{36\times 12}+\frac{3L^2}{144}\right]$

$=\frac{L^2}{7^2}+\frac{3L^2}{24}=\frac{L^2}{7^2}+\frac{L^2}{8}$

$=\frac{L^2+9L^2}{72}=\frac{10L^2}{72}$

$=\frac{10\times 2.4\times 2.4}{72}$

= 0.8

= 8×10^{-1} kg m^2

Q.8 Two cars are approaching each other at an equal speed of 7.2 km/hr. When they see each other, both blow horns having frequency of 676 Hz. The beat frequency heard by each driver will be _____ Hz. [Velocity of sound in air is 340 m/s.]

Ans: 8.00

Sol: $f'=f_0\left(\frac{V-V_0}{V-V_s}\right)=676\left(\frac{340+2}{340-2}\right)=684$

$\therefore$ Beat frequency = 684 – 676

= 8 Hz

Q.9 A series LCR circuit is designed to resonate at an angular frequency $\omega_0=10^5$ rad/s. The circuit draws 16 W power from 120 V source at resonance. The value of resistance 'R' in the circuit is _____Ω.

Ans: 900.00

Sol: $R=\frac{V_{rms}^2}{P}=\frac{(120)^2}{16}=900\,\Omega$

Q.10 A uniform metallic wire is elongated by 0.04 m when subjected to a linear force F. The elongation, if its length and diameter is doubled and subjected to the same force will be _____ cm.

Ans: 2.00

Sol: $$Y = \frac{F/A}{\Delta\ell/\ell} = \frac{F/4A}{\Delta\ell'/2\ell}$$

$$\Rightarrow \Delta\ell' = \frac{\Delta\ell}{2} = 0.02\,\text{m} = 2\ \text{cm}$$

PART – B – CHEMISTRY

SECTION A

Q.1 In polymer Buna-S : 'S' stands for :

Options
1. Strength
2. Sulphonation
3. Styrene
4. Sulphur

Ans: 3

Sol: In Buna-S, 'S' stands for styrene

Q.2 Match List - I with List - II.

	List - I (Salt)		List - II (Flame colour wavelength)
(a)	LiCl	(i)	455.5 nm
(b)	NaCl	(ii)	670.8 nm
(c)	RbCl	(iii)	780.0 nm
(d)	CsCl	(iv)	589.2 nm

Choose the correct answer from the options given below :

Options
1. (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)
2. (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)
3. (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)
4. (a)-(i), (b)-(iv), (c)-(ii), (d)-(iii)

Ans: 1

Sol: All the alkali metal & their salts impart characteristic colour to the flame

Metal	Li	Na	K	Rb	Cs
Colour	Crimson	Yellow	Pale violet	Red violet	Blue
λ / nm	670.8	589.2	766.5	780.0	455.5

Q.3 The incorrect statement among the following is :

Options

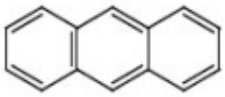
1. $VOSO_4$ is a reducing agent
2. Red colour of ruby is due to the presence of Co^{3+}
3. Cr_2O_3 is an amphoteric oxide
4. RuO_4 is an oxidizing agent

Ans: **2**

Sol: Red colour of ruby is due to the presence of Cr^{3+} ions.

Q.4 Which one of the following compounds is non-aromatic ?

Options

1. 
2. 
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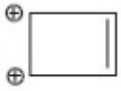
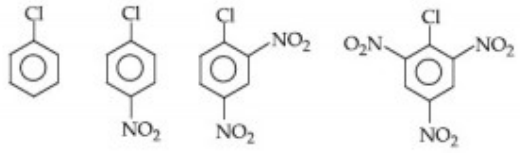
4. 

Ans: **1**

Sol: Cycloheptatriene is non-planar. It contains 6π electrons which are not in a state of delocalization. Hence option (1) i.e., cycloheptatriene is non-aromatic.

Q.5 The correct order of the following compounds showing increasing tendency towards nucleophilic substitution reaction is :



(i) (ii) (iii) (iv)

Options

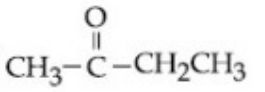
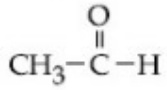
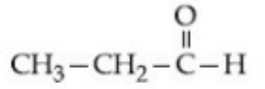
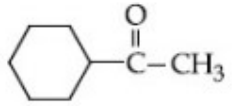
1. (iv) < (iii) < (ii) < (i)
2. (i) < (ii) < (iii) < (iv)
3. (iv) < (i) < (iii) < (ii)
4. (iv) < (i) < (ii) < (iii)

Ans: **2**

Sol: The presence of electron withdrawing groups ($-NO_2$) at ortho and para position of haloarenes increase the reactivity of nucleophilic substitution reaction.

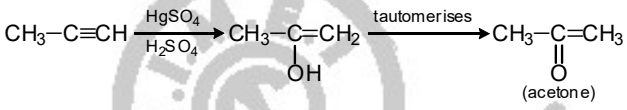
Q.6 Which one of the following carbonyl compounds cannot be prepared by addition of water on an alkyne in the presence of $HgSO_4$ and H_2SO_4 ?

Options

1. 
2. 
3. 
4. 

Ans: **3**

Sol: Reaction of alkynes with $HgSO_4$ / H_2SO_4 takes place in accordance with Markovnikoff's rule addition of water



Acetone only will be formed as the product and not propanal

Q.7 The calculated magnetic moments (spin only value) for species $[FeCl_4]^{2-}$, $[Co(C_2O_4)_3]^{3-}$ and MnO_4^{2-} respectively are :

Options

1. 4.90, 0 and 2.83 BM
2. 4.90, 0 and 1.73 BM
3. 5.82, 0 and 0 BM
4. 5.92, 4.90 and 0 BM

Ans: **2**

Sol: Magnetic moment, $\mu = \sqrt{n(n+2)}$ BM

Where n $\rightarrow$ number of unpaired electrons

In $[FeCl_4]^{2-}$ $Fe^{2+} \rightarrow [Ar]\ 3d^6$

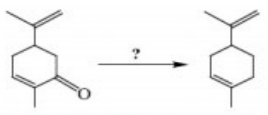
Since Cl^- is a weak ligand, pairing doesn't take place $\therefore$ n = 4

In $[Co(C_2O_4)_3]^{3-}$ $Co^{3+} \rightarrow [Ar]\ 3d^6$

Here pairing takes place since $C_2O_4^{2-}$ is a strong ligand $\therefore$ n = 0

In MnO_4^{2-}, $Mn^{6+} \rightarrow [Ar]\ 3d^1$ $\therefore$ n = 1

Q.8



Which of the following reagent is suitable for the preparation of the product in the above reaction ?

Options

1. $NaBH_4$
2. $NH_2-NH_2/C_2H_5\overset{\ominus\oplus}{O}Na$
3. Red $P + Cl_2$
4. Ni/H_2

Ans: **2**

Sol: Here only >C=O is reduced to $>CH_2$ group and the double bond is not reduced. Hence the reducing agent is hydrazine and sodium ethoxide and the reduction is called Wolff-Kishner reduction.

Q.9 Match List - I and List - II.

List - I		List - II	
(a)	$R-\overset{\overset{O}{\|\|}}{C}-Cl \rightarrow R-CHO$	(i)	$Br_2/NaOH$
(b)	$R-CH_2-COOH \rightarrow R-\underset{\underset{Cl}{\|}}{CH}-COOH$	(ii)	$H_2/Pd-BaSO_4$
(c)	$R-\overset{\overset{O}{\|\|}}{C}-NH_2 \rightarrow R-NH_2$	(iii)	$Zn(Hg)/Conc.\ HCl$
(d)	$R-\overset{\overset{O}{\|\|}}{C}-CH_3 \rightarrow R-CH_2-CH_3$	(iv)	$Cl_2/Red\ P,\ H_2O$

Choose the correct answer from the options given below :

Options

1. (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)
2. (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)
3. (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)
4. (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

Ans: **1**

Sol:

$$R-\overset{\overset{O}{\|\|}}{C}-Cl \xrightarrow[BaSO_4]{H_2/Pd} R-CHO$$ Rosenmund Reaction

$$R-CH_2-COOH \xrightarrow[H_2O]{Cl_2,\ red\ P} R-\underset{\underset{Cl}{\|}}{CH}-COOH$$ HVZ reaction

$$R-\overset{\overset{O}{\|\|}}{C}-NH_2 \xrightarrow{Br_2/NaOH} R-NH_2$$ Hoffmann's bromamide reaction

$$R-\overset{\overset{O}{\|\|}}{C}-CH_3 \xrightarrow[Conc.HCl]{Zn\ (Hg)} R-CH_2-CH_3$$ Clemmension reduction

Q.10 Most suitable salt which can be used for efficient clotting of blood will be :

Options

1. $FeCl_3$
2. $Mg(HCO_3)_2$
3. $FeSO_4$
4. $NaHCO_3$

Ans: **1**

Sol: $FeCl_3$ is used to stop bleeding because Fe^{3+} coagulates blood which is a negatively charged sol. Among the different salts given, the most preferred positively charged ion is Fe^{3+} which is in accordance with Hardy Schulz rule.

Q.11 Match List - I and List - II.

	List - I		List - II
(a)	Valium	(i)	Antifertility drug
(b)	Morphine	(ii)	Pernicious anaemia
(c)	Norethindrone	(iii)	Analgesic
(d)	Vitamin B_{12}	(iv)	Tranquilizer

Options
1. (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)
2. (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)
3. (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)
4. (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)

Ans: **1**

Sol: Valium – Tranquilizer
Morphine – Analgesic
Norethindrone – Antifertility drug
Vitamin B_{12} – Pernicious anaemia

Q.12 According to Bohr's atomic theory :

(A) Kinetic energy of electron is $\propto \frac{Z^2}{n^2}$.

(B) The product of velocity (v) of electron and principal quantum number (n), 'vn' $\propto Z^2$.

(C) Frequency of revolution of electron in an orbit is $\propto \frac{Z^3}{n^3}$.

(D) Coulombic force of attraction on the electron is $\propto \frac{Z^3}{n^4}$.

Choose the **most appropriate** answer from the options given below :

Options
1. (A) and (D) only
2. (A) only
3. (C) only
4. (A), (C) and (D) only

Ans: **2**

Sol: $KE = \frac{kze^2}{2r}$ $\therefore KE \propto \frac{z^2}{n^2}$

Velocity (v) = $\frac{2.19 \times 10^2 z}{n}$ $\therefore vn \propto z$

No. of revolutions per second = $\frac{v}{2\pi r}$

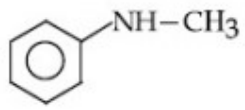
$\therefore v \propto \frac{z^2}{n^3}$

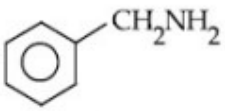
Coulombs force of attraction (F) $\propto \frac{1}{r^2}$

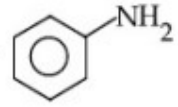
i.e., $F \propto \frac{z^2}{n^4}$

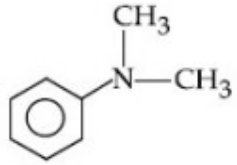
Q.13 The diazonium salt of which of the following compounds will form a coloured dye on reaction with β-Naphthol in NaOH ?

Options

1. 

2. 

3. 

4. 

Ans: **3**

Sol: Aromatic primary amines on diazotization form diazonium salt which when treated with β-naphthol in NaOH undergoes coupling reaction to give a coloured dye (azo compound)

Q.14 What is the correct order of the following elements with respect to their density ?

Options

1. Zn < Cu < Co < Fe < Cr
2. Cr < Fe < Co < Cu < Zn
3. Zn < Cr < Fe < Co < Cu
4. Cr < Zn < Co < Cu < Fe

Ans: **3**

Sol: The density in g / cm^3 of
Zn – 7.1
Cr – 7.19
Fe – 7.8
Co – 8.7
Cu – 8.9

Q.15 Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : Hydrogen is the most abundant element in the Universe, but it is not the most abundant gas in the troposphere.

Reason R : Hydrogen is the lightest element.

In the light of the above statements, choose the **correct** answer from the options given below :

Options

1. Both **A** and **R** are true but **R** is NOT the correct explanation of **A**
2. Both **A** and **R** are true and **R** is the correct explanation of **A**
3. **A** is false but **R** is true
4. **A** is true but **R** is false

Ans: **1**

Sol: Both the statements regarding hydrogen are correct. But reason (R) is not the correct explanation for assertion (A).

Q.16 Match List - I with List - II.

	List - I (Metal)		List - II (Ores)
(a)	Aluminium	(i)	Siderite
(b)	Iron	(ii)	Calamine
(c)	Copper	(iii)	Kaolinite
(d)	Zinc	(iv)	Malachite

Choose the correct answer from the options given below :

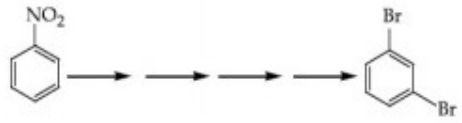
Options

1. (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)
2. (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)
3. (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)
4. (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

Ans: **3**

Sol: Aluminium – Kaonite ($Al_2Si_2O_5(OH)_4$)
Iron – Siderite ($FeCO_3$)
Copper – Malachite ($CuCO_3.Cu(OH)_2$)
Zinc – Calamine ($ZnCO_3$)

Q.17 What is the correct sequence of reagents used for converting nitrobenzene into *m*-dibromobenzene ?

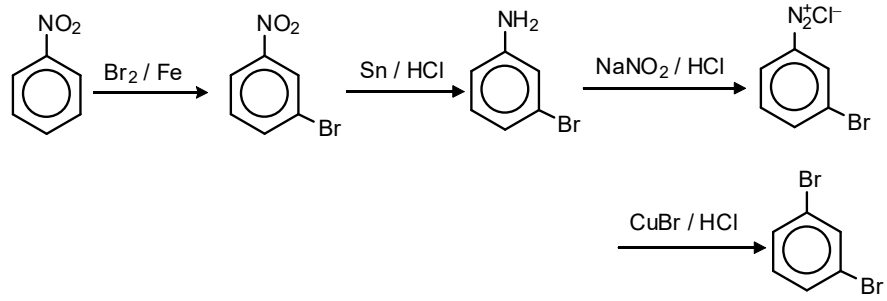


Options

1. $Br_2/Fe \rightarrow / Sn/HCl \rightarrow / NaNO_2/HCl \rightarrow / CuBr/HBr \rightarrow$
2. $Sn/HCl \rightarrow / KBr \rightarrow / Br_2 \rightarrow / H^+ \rightarrow$
3. $NaNO_2 \rightarrow / HCl \rightarrow / KBr \rightarrow / H^+ \rightarrow$
4. $Sn/HCl \rightarrow / Br_2 \rightarrow / NaNO_2 \rightarrow / NaBr \rightarrow$

Ans: **1**

Sol:



Q.18 Given below are two statements :

Statement I : The value of the parameter "Biochemical Oxygen Demand (BOD)" is important for survival of aquatic life.

Statement II : The optimum value of BOD is 6.5 ppm.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

Options

1. **Statement I is false but Statement II is true**
2. **Both Statement I and Statement II are false**
3. **Both Statement I and Statement II are true**
4. **Statement I is true but Statement II is false**

Ans: **4**

Sol: Water considered pure has BOD of less than 5 ppm whereas highly polluted water has BOD value of more than 17 ppm

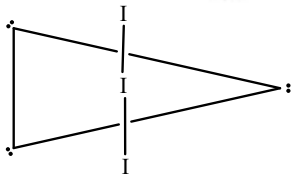
Q.19 The correct shape and I – I – I bond angles respectively in I_3^- ion are :

Options

1. T-shaped; 180° and 90°
2. Trigonal planar; 120°
3. Linear; 180°
4. Distorted trigonal planar; 135° and 90°

Ans: **3**

Sol: In I_3^-, the central atom undergoes sp^3d hybridization. There are 3 lone pairs and 2 bond pairs having linear structure.



Q.20 The correct set from the following in which both pairs are in correct order of melting point is :

Options

1. LiF > LiCl ; NaCl > MgO
2. LiCl > LiF ; MgO > NaCl
3. LiCl > LiF ; NaCl > MgO
4. LiF > LiCl ; MgO > NaCl

Ans: **4**

Sol: The MP of LiF > LiCl as LiF is ionic and LiCl is covalent in nature.
MP of LiF – 845°C & LiCl – 605°C
The MP of MgO > NaCl, since lattice enthalpy of MgO > NaCl
MP of MgO – 2852°C & NaCl –801°C

SECTION B

Q.1 The solubility product of PbI_2 is 8.0×10^{-9}. The solubility of lead iodide in 0.1 molar solution of lead nitrate is $x \times 10^{-6}$ mol/L. The value of x is _____. (Rounded off to the nearest integer)

[Given $\sqrt{2} = 1.41$]

Ans: **141**

Sol: $PbI_2 \rightleftharpoons Pb^{2+} + 2I^-$

$K_{sp} = [Pb^{2+}][I^-]^2$

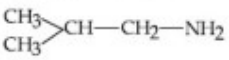
$8.0 \times 10^{-9} = 0.1 \times (2S)^2$

$$\therefore 4S^2 = \frac{8.0 \times 10^{-9}}{10^{-1}} = 8 \times 10^{-8}$$

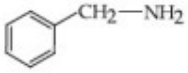
$S^2 = 2 \times 10^{-8}$

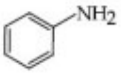
$$\therefore S = \frac{\sqrt{2} \times 10^{-8}}{10^{-1}} = 1.41 \times 10^{-4} = 141 \times 10^{-6} \text{ mol/L}$$

Q.2 The total number of amines among the following which can be synthesized by Gabriel synthesis is _____.

(A) 

(B) $CH_3CH_2NH_2$

(C) 

(D) 

Ans: **3**

Sol: Only primary aliphatic amines can be prepared by Gabriel's phthalimide synthesis

Q.3 Sucrose hydrolyses in acid solution into glucose and fructose following first order rate law with a half-life of 3.33 h at 25°C. After 9 h, the fraction of sucrose remaining is f. The value of $\log_{10}\left(\frac{1}{f}\right)$ is _____ $\times 10^{-2}$. (Rounded off to the nearest integer)

[Assume : ln 10 = 2.303, ln 2 = 0.693]

Ans: **81**

Sol: $t_{1/2} = 33.3$ h

$$n = \frac{9}{3.33} = 2.7$$

Fraction of sucrose remaining (f) = $\frac{1}{2^n}$

$\therefore \log_{10}\left(\frac{1}{f}\right) = \log_{10}(2^{2.7})$

= 2.7 log 2
= 2.7 × 0.3010
= 0.8127
= 81.27 × 10^{-2}

Q.4 C_6H_6 freezes at 5.5°C. The temperature at which a solution of 10 g of C_4H_{10} in 200 g of C_6H_6 freeze is _____ °C. (The molal freezing point depression constant of C_6H_6 is 5.12°C/m.)

Ans: **1.09**

Sol: $\Delta T_f = \frac{K_f \times W_B \times 1000}{M_B \times W_A} = \frac{5.12 \times 10 \times 1000}{58 \times 200}$

= 4.414°C

$\Delta T_f = T_f^\circ - T_f$

$\therefore T_f = T_f^\circ - \Delta T_f = 5.5 - 4.414 = 1.086$

Q.5 Among the following allotropic forms of sulphur, the number of allotropic forms, which will show paramagnetism is _____.
(A) α-sulphur (B) β-sulphur (C) S_2-form

Ans: **1**

Sol: Among the allotropic forms of sulphur, S_2 is paramagnetic in nature due to the presence of unpaired electrons in antibonding π* MO's

Q.6 Assuming ideal behaviour, the magnitude of log K for the following reaction at 25°C is $x \times 10^{-1}$. The value of x is _____. (Integer answer)

$$3HC \equiv CH_{(g)} \rightleftharpoons C_6H_{6(l)}$$

[Given : $\Delta_f G^\circ(HC \equiv CH) = -2.04 \times 10^5$ J mol^{-1} ; $\Delta_f G^\circ(C_6H_6) = -1.24 \times 10^5$ J mol^{-1} ; R = 8.314 J K^{-1} mol^{-1}]

Ans: **–855**

Sol: $\Delta_r G = \Delta_f G_{(P)} - 3\Delta_f G_{(R)}$
= $(-1.24 \times 10^5) - (-2.04 \times 10^5 \times 3)$
= $+4.88 \times 10^5$ J mol^{-1}
$\Delta G^\circ = -2.303\ RT \log K$

$\log K = \frac{\Delta G^\circ}{-2.303RT} = \frac{4.88 \times 10^5}{-2.303 \times 8.314 \times 298}$

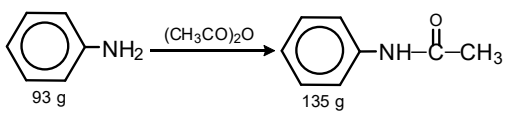
$= \frac{-4.88 \times 10^5}{5705.85} = -85.53$

= -855.3×10^{-1}

Q.7 1.86 g of aniline completely reacts to form acetanilide. 10% of the product is lost during purification. Amount of acetanilide obtained after purification (in g) is _____ $\times 10^{-2}$.

Ans: **243**

Sol:



$\therefore$ 1.86g aniline $\Rightarrow \frac{1.86 \times 135}{93} = 2.7$ g

Since 10% product is lost, amount of acetanilide formed
= 2.7 – 0.27 = 2.43 g = 243 × 10^{-2} g

Q.8 The formula of a gaseous hydrocarbon which requires 6 times of its own volume of O_2 for complete oxidation and produces 4 times its own volume of CO_2 is C_xH_y. The value of y is _____.

Ans: **8**

Sol: $C_xH_y + \left(x + \frac{y}{4}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$

According to the question given,

$x + \frac{y}{4} = 6 \ \& \ x = 4$

$\therefore \frac{y}{4} = 6 - 4 = 2$

$\therefore$ y = 8

Q.9 The magnitude of the change in oxidising power of the MnO_4^-/Mn^{2+} couple is $x \times 10^{-4}$ V, if the H^+ concentration is decreased from 1 M to 10^{-4} M at 25°C. (Assume concentration of MnO_4^- and Mn^{2+} to be same on change in H^+ concentration). The value of x is _____. (Rounded off to the nearest integer)

$\left[\text{Given : } \frac{2.303RT}{F} = 0.059\right]$

Ans: **3776**

Sol: $MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{2+} + 4H_2O$

When $[H^+]$ = 1 M

$$E_1 = E^\circ - \frac{0.059}{5}\log\frac{[Mn^{2+}]}{[MnO_4^-][H^+]^8}$$

$$= E^\circ - \frac{0.059}{5}\log\frac{[Mn^{2+}]}{[MnO_4^-]} \quad ---(1)$$

When $[H^+] = 10^{-4}$ M

$$E_2 = E^\circ - \frac{0.059}{5}\log\frac{[Mn^{2+}]}{[MnO_4^-][H^+]^8}$$

$$= E^\circ - \frac{0.059}{5}\log\left[\frac{[Mn^{2+}]}{[MnO_4^-]} \times 10^{32}\right]$$

$$= E^\circ - \frac{0.059}{5}\left[\log\frac{[Mn^{2+}]}{[MnO_4^-]} + 32\log 10\right]$$

$$= E^\circ - \frac{0.059}{5}\log\frac{[Mn^{2+}]}{[MnO_4^-]} - \frac{0.059}{5} \times 32 \quad ---(2)$$

Change in potential, $E_1 - E_2$

$$= \left[E^\circ - \frac{0.059}{5}\log\frac{[Mn^{2+}]}{[MnO_4^-]}\right] - \left[E^\circ - \frac{0.059}{5}\log\frac{[Mn^{2+}]}{[MnO_4^-]} - \frac{0.059 \times 32}{5}\right]$$

$$= \frac{0.059 \times 32}{5} = 0.3776$$

= 3776 × 10^{-4} V

Q.10 The volume occupied by 4.75 g of acetylene gas at 50°C and 740 mmHg pressure is _____ L. (Rounded off to the nearest integer)
[Given R = 0.0826 L atm K^{-1} mol^{-1}]

Ans: **5**

Sol: PV = nRT

$$V = \frac{nRR}{P} = \frac{\frac{4.75}{26} \times 0.0826 \times 323}{\frac{740}{760}}$$

$$= \frac{0.18 \times 26.68}{0.97} = 5.01\ L$$

PART – C – MATHEMATICS

SECTION A

Q.1 For the statements p and q, consider the following compound statements :
(a) $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$
(b) $((p \vee q) \wedge \sim p) \rightarrow q$
Then which of the following statements is correct ?

Options
1. (b) is a tautology but not (a).
2. (a) is a tautology but not (b).
3. (a) and (b) both are tautologies.
4. (a) and (b) both are not tautologies.

Ans: **3**

Sol: (a)

p	q	p → q	~q	~q ^ (p → q)	~p	[~p ^ (p → q] → ~p
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

(b)

p	q	p ∨ q	~p	(p ∨ q) ^ ~p	[(p ∨ q) ^ ~p] → q
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

(a) and (b) both are tautologies

Q.2 Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices (a, c), (2, b) and (a, b) be $\left(\frac{10}{3}, \frac{7}{3}\right)$. If α, β are the roots of the equation $ax^2+bx+1=0$, then the value of $\alpha^2+\beta^2-\alpha\beta$ is :

Options

1. $-\frac{69}{256}$
2. $-\frac{71}{256}$
3. $\frac{71}{256}$
4. $\frac{69}{256}$

Ans: **2**

Sol: Given a, b, c are in A.P

$\Rightarrow 2b = a + c$

Centroid of (a, c), (2, b) and (a, b) be $\left(\frac{10}{3}, \frac{7}{3}\right)$

$\left(\frac{a+2+a}{3}, \frac{c+b+b}{3}\right)=\left(\frac{10}{3}, \frac{7}{3}\right)$

$2a + 2 = 10$

$a = 4$

$2b + c = 7$

$2b + 2b - a = 7$

$4b - 4 = 7$

$b = \frac{11}{4}$

$\alpha+\beta=-\frac{b}{a}=\frac{-11}{16}$

$\alpha\beta=\frac{1}{a}=\frac{1}{4}$

$\alpha^2+\beta^2-\alpha\beta=(\alpha+\beta)^2-2\alpha\beta-\alpha\beta = (\alpha+\beta)^2-3\alpha\beta = \left(\frac{-11}{16}\right)^2-3\left(\frac{1}{4}\right) = \frac{121}{256}-\frac{3}{4}$

$= \frac{121-192}{256} = -\frac{71}{256}.$

Q.3 The negation of the statement

$\sim p \wedge (p \vee q)$ is :

Options

1. $\sim p \vee q$
2. $p \vee \sim q$
3. $p \wedge \sim q$
4. $\sim p \wedge q$

Ans: **3**

Sol: $\sim[\sim p \wedge (p \vee q)]$

$\sim[(\sim p \wedge p) \vee (\sim p \wedge q)]$
$\sim[\sim p \wedge q]$
$p \vee \sim q$.

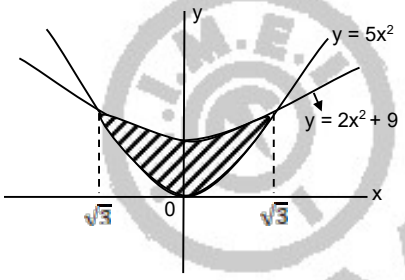
Q.4 The area of the region : $R = \{(x, y) : 5x^2 \le y \le 2x^2 + 9\}$ is :

Options

1. $11\sqrt{3}$ square units
2. $6\sqrt{3}$ square units
3. $9\sqrt{3}$ square units
4. $12\sqrt{3}$ square units

Ans: **2**

Sol: $y = 5x^2$, $y = 2x^2 + 9$
$5x^2 = 2x^2 + 9$
$3x^2 = 9$
$x^2 = 3$
$x = \pm\sqrt{3}$.



Required area = $2\int_0^{\sqrt{3}} \left[\left(2x^2 + 9\right) - 5x^2\right]dx = 2\int_0^{\sqrt{3}} \left(9 - 3x^2\right)dx = 2\left[9x - x^3\right]_0^{\sqrt{3}}$

$= 12\sqrt{3}$ Sq. units.

Q.5 For which of the following curves, the line $x + \sqrt{3}y = 2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$?

Options

1. $y^2 = \frac{1}{6\sqrt{3}}x$
2. $2x^2 - 18y^2 = 9$
3. $x^2 + y^2 = 7$
4. $x^2 + 9y^2 = 9$

Ans: **4**

Sol: Verify options

$x\left(\frac{3\sqrt{3}}{2}\right) + 9y\left(\frac{1}{2}\right) = 9$

$$x\left(\frac{\sqrt{3}}{2}\right)+\frac{3}{2}y=3$$

$$x+\sqrt{3}y=2\sqrt{3}\ .$$

Q.6 The probability that two randomly selected subsets of the set {1, 2, 3, 4, 5} have exactly two elements in their intersection, is :

Options

1. $\frac{65}{2^7}$
2. $\frac{135}{2^9}$
3. $\frac{35}{2^7}$
4. $\frac{65}{2^8}$

Ans: **2**

Sol: Required probability = $\frac{5c_2\times 3^3}{4^5}=\frac{10\times 27}{2^{10}}=\frac{2\times 5\times 27}{2^{10}}=\frac{135}{2^9}$.

Q.7 The value of the integral, $\int_1^3[x^2-2x-2]dx$, where $[x]$ denotes the greatest integer less than or equal to x, is :

Options

1. -4
2. $-\sqrt{2}-\sqrt{3}+1$
3. $-\sqrt{2}-\sqrt{3}-1$
4. -5

Ans: **3**

Sol: $\int_1^3[x^2-2x-2]dx=\int_1^3[(x-1)^2-3]dx=\int_1^3[(x-1)^2]dx+\int_1^3[-3]dx$

Let $x-1=t$

$dx=dt$

$$\int_0^2[t^2]dt-3(2)$$

$$\int_0^1[t^2]dt+\int_1^{\sqrt{2}}[t^2]dt+\int_{\sqrt{2}}^{\sqrt{3}}[t^2]dt+\int_{\sqrt{3}}^{\sqrt{4}}[t^2]dt-6$$

$$0+(\sqrt{2}-1)+2(\sqrt{3}-\sqrt{2})+3(2-\sqrt{3})-6$$

$$\sqrt{2}-1+2\sqrt{3}-2\sqrt{2}+6-3\sqrt{3}-6$$

$$-1-\sqrt{2}-\sqrt{3}\ .$$

Q.8 Let A and B be 3×3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $(A^2B^2 - B^2A^2)\,X = O$, where X is a 3×1 column matrix of unknown variables and O is a 3×1 null matrix, has :

Options

1. no solution
2. exactly two solutions
3. infinitely many solutions
4. a unique solution

Ans: **3**

Sol: Given $A^T = A$, $B^T = -B$

Let $A^2B^2 - B^2A^2 = P$

$P^T = (A^2B^2 - B^2A^2)^T = \left(A^2B^2\right)^T - \left(B^2A^2\right)^T = \left(B^2\right)^T\left(A^2\right)^T - \left(A^2\right)^T\left(B^2\right)^T = \left(B^T\right)^2\left(A^T\right)^2 - \left(A^T\right)^2\left(B^T\right)^2$

$= B^2A^2 - A^2B^2 = -(A^2B^2 - B^2A^2) = -P$

$\Rightarrow$ P is skew – symmetric matrix

$\Rightarrow |P| = 0$

Hence PX = 0 have infinite solutions.

Q.9 Let f be a twice differentiable function defined on $\mathbf{R}$ such that $f(0)=1$, $f'(0)=2$ and $f'(x)\neq0$ for all $x\in\mathbf{R}$. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$, for all $x\in\mathbf{R}$, then the value of $f(1)$ lies in the interval :

Options

1. $(9, 12)$
2. $(3, 6)$
3. $(6, 9)$
4. $(0, 3)$

Ans: **3**

Sol: $f(x)\,f''(x) - [f'(x)]^2 = 0$

$f(x)\,f''(x) = [f'(x)]^2$

$\int \frac{f''(x)}{f'(x)}dx = \int \frac{f'(x)}{f(x)}dx$

$\log|f'(x)| = \log|f(x)| + \log c$

Put x = 0

$f'(0) = cf(0)$

$2 = c$

$f'(x) = 2f(x)$

$\int \frac{f'(x)}{f(x)}dx = \int 2dx$

$\text{Log}|f(x)| = 2x + c_1$

$f(x) = e^{2x} \cdot k$

put x = 0

$f(0) = k$

$k = 1$

$f(x) = e^{2x}$

$f(1) = e^2 \Rightarrow e^2 \in (6, 9)$

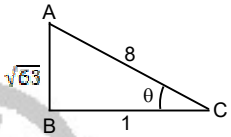
Q.10 A possible value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is :

Options

1. $2\sqrt{2}-1$
2. $\frac{1}{\sqrt{7}}$
3. $\frac{1}{2\sqrt{2}}$
4. $\sqrt{7}-1$

Ans: **2**

Sol: Let $\sin^{-1}\left(\frac{\sqrt{63}}{8}\right)=\theta$



$\sin\theta=\frac{\sqrt{63}}{8}$

$\Rightarrow \cos\theta=\frac{1}{8}$

$$\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)=\tan\left(\frac{\theta}{4}\right)=\sqrt{\frac{1-\cos\left(\theta/2\right)}{1+\cos\left(\theta/2\right)}}=\sqrt{\frac{1-\frac{\sqrt{1+\cos\theta}}{2}}{1+\sqrt{\frac{1+\cos\theta}{2}}}}=\sqrt{\frac{1-\frac{3}{4}}{1+\frac{3}{4}}}=\frac{1}{\sqrt{7}}.$$

Q.11 Let $a, b\in\mathbf{R}$. If the mirror image of the point P(a, 6, 9) with respect to the line $\frac{x-3}{7}=\frac{y-2}{5}=\frac{z-1}{-9}$ is $(20, b, -a-9)$, then $|a+b|$ is equal to :

Options

1. 90
2. 84
3. 86
4. 88

Ans: **4**

Sol: Midpoint of (a, 6, 9), (20, b, –a –9) lies on a line.

Midpoint = $\left(\frac{a+20}{2},\frac{b+6}{2},\frac{-a}{2}\right)$

$$\frac{\frac{a+20}{2}-3}{7}=\frac{\frac{b+6}{2}-2}{5}=\frac{-\frac{a}{2}-1}{-9}$$

$$\frac{a+14}{74}=\frac{b+2}{10}=\frac{a+2}{18}$$

$$\frac{a+14}{7}=\frac{a+2}{9}$$

$9a + 126 = 7a + 14$

$2a = -112$

$a = -56$

$$\frac{b+2}{10}=\frac{-56+14}{14}$$

$$\frac{b+2}{5}=\frac{-42}{7}$$

$$\frac{b+2}{5}=-6$$

b + 2 = –30

b = –32

(a + b) = 88

Q.12 If $n \geq 2$ is a positive integer, then the sum of the series ${}^{n+1}C_2+2\left({}^{2}C_2+{}^{3}C_2+{}^{4}C_2+....+{}^{n}C_2\right)$ is :

Options

1. $\frac{n(n-1)(2n+1)}{6}$
2. $\frac{n(2n+1)(3n+1)}{6}$
3. $\frac{n(n+1)(2n+1)}{6}$
4. $\frac{n(n+1)^2(n+2)}{12}$

Ans: **3**

Sol: By verification

take n = 3

$4C_2 + 2(2C_2 + 3C_2) = 6 + 2(1 + 3) = 14$

Put n = 3 in options 3 is satisfying

Q.13 Let $f:\mathbf{R}\to\mathbf{R}$ be defined as

$$f(x)=\begin{cases}-55x, & \text{if } x<-5\\ 2x^3-3x^2-120x, & \text{if } -5\leq x\leq 4\\ 2x^3-3x^2-36x-336, & \text{if } x>4,\end{cases}$$

Let $A=\{x\in\mathbf{R}: f \text{ is increasing}\}$. Then A is equal to :

Options

1. $(-\infty,-5)\cup(-4,\infty)$
2. $(-5,-4)\cup(4,\infty)$
3. $(-\infty,-5)\cup(4,\infty)$
4. $(-5,\infty)$

Ans: **2**

Sol: –55, x < –5

$f'(x) = 6x^2 - 6x - 120$, $-5 \leq x \leq 4$

$6x^2 - 6x - 36$, x > 4

–55, x < –5

$f'(x) = 6(x - 5)(x + 4)$, $-5 \leq x \leq 4$

6(x – 3) (x + 2), x > 4

Hence f(x) is monotonically increasing $(-5, -4) \cup (4, \infty)$

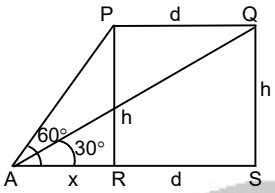
Q.14 The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20 seconds at the speed of 432 km/hour, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is :

Options

1. $3600\sqrt{3}$ m
2. $1800\sqrt{3}$ m
3. $1200\sqrt{3}$ m
4. $2400\sqrt{3}$ m

Ans: **3**

Sol:



In ΔPAR

$\tan 60° = \frac{h}{x}$

$\sqrt{3} = \frac{h}{x}$

$x = \frac{h}{\sqrt{3}}$

In ΔSAQ

$\tan 30° = \frac{h}{x+d}$

$\frac{1}{\sqrt{3}} = \frac{h}{x+d}$

$x + d = \sqrt{3}h$

$\frac{h}{\sqrt{3}} + d = \sqrt{3}h$

$d = h\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$

$d = h\left(\frac{2}{\sqrt{3}}\right)$

$\text{speed} = \frac{d}{t}$

$S = 432 \times \frac{5}{18}$ m/s

$= 120$ m/s

$d = s \times t$

$= 120 \times 20$

$d = 2400$ m

$h = \frac{\sqrt{3}d}{2}$

$= \frac{\sqrt{3} \times 2400}{2}$

$h = 1200\sqrt{3}$ m

Q.15 For the system of linear equations :

$x-2y=1,\ x-y+kz=-2,\ ky+4z=6,\ k\in \mathbf{R}$,

consider the following statements :

(A) The system has unique solution if $k\neq 2, k\neq -2$.

(B) The system has unique solution if $k=-2$.

(C) The system has unique solution if $k=2$.

(D) The system has no-solution if $k=2$.

(E) The system has infinite number of solutions if $k\neq -2$.

Which of the following statements are correct ?

Options

1. (C) and (D) only
2. (A) and (D) only
3. (A) and (E) only
4. (B) and (E) only

Ans: 2

Sol: $\Delta = \begin{vmatrix} 1 & -2 & 0 \\ 1 & -1 & k \\ 0 & k & 4 \end{vmatrix} = 1(-4-k^2) + 2(4)$

$\Delta = 4 - k^2$

$\Delta \neq 0 \rightarrow$ given equations have unique solution

$\Delta = 0 \rightarrow$ no solution

$K = 2$ (or) -2, $\Delta = 0$

$K \neq 2$ (or) -2, $\Delta \neq 0$

Q.16 If P is a point on the parabola $y=x^2+4$ which is closest to the straight line $y=4x-1$, then the co-ordinates of P are :

Options

1. $(-2, 8)$
2. $(1, 5)$
3. $(3, 13)$
4. $(2, 8)$

Ans: 4

Sol: Any point on parabola $(t, t^2 + 4)$

Distance from $(t, t^2 + 4)$ to the line $4x - y - 1 = 0$ is

$$= \frac{\left|4t-\left(t^2+4\right)-1\right|}{\sqrt{16+1}}$$

$$f(t) = \frac{\left|-t^2+4t-5\right|}{\sqrt{17}}$$

For maximum (or) minimum $f'(t) = 0$

$-2t + 4 = 0 \Rightarrow t = 2$

$\therefore$ Closest point on parabola is $(2, 8)$

Q.17 If the curve $y=ax^2+bx+c,\ x \in R$, passes through the point (1, 2) and the tangent line to this curve at origin is $y=x$, then the possible values of a, b, c are :

Options

1. $a=-1,\ b=1,\ c=1$
2. $a=1,\ b=0,\ c=1$
3. $a=\frac{1}{2},\ b=\frac{1}{2},\ c=1$
4. $a=1,\ b=1,\ c=0$

Ans: **4**

Sol: $2 = a + b + c$

$\frac{dy}{dx} = 2ax + b, \quad \left(\frac{dy}{dx}\right)_{(0,\,0)} = 1$

$\Rightarrow b = 1$

$a + 1 + c = 2$

$a + c = 1$

Verify options

Q.18 If a curve $y=f(x)$ passes through the point (1, 2) and satisfies $x\frac{dy}{dx} + y = bx^4$, then for what value of b, $\int_1^2 f(x)dx = \frac{62}{5}$?

Options

1. $\frac{31}{5}$
2. $\frac{62}{5}$
3. 10
4. 5

Ans: **3**

Sol: $\frac{dy}{dx} + \frac{y}{x} = bx^3$

I.F = $e^{\int \frac{1}{x}dx} = e^{\log_e x} = x$

Solution is

$y(x) = \int 6x^3.xdx = b\frac{x^5}{5} + C$

$y = \frac{b}{5}x^4 + \frac{C}{x}$

$\Rightarrow 2 = \frac{b}{5} + C$

$f(x) = \frac{b}{5}x^4 + \frac{C}{x}$

$\int_1^2 f(y)dy = \int_1^2 \left(\frac{b}{5}x^4 + \frac{C}{x}\right)dx = \frac{62}{5} = \left[\frac{b}{5} \times \frac{x^5}{5} + c\log x\right]_1^2 = \frac{62}{5} = \left(\frac{32}{25}b + c\log 2\right) - \frac{b}{25} = \frac{62}{5}$

$\Rightarrow \frac{310}{25} = \frac{62}{5}$

$\Rightarrow$ b = 10 and c = 0.

Q.19 Let $f(x)$ be a differentiable function defined on $[0, 2]$ such that $f'(x) = f'(2-x)$ for all $x \in (0, 2)$, $f(0) = 1$ and $f(2) = e^2$. Then the value of $\int_0^2 f(x)\,dx$ is :

Options

1. $2(1-e^2)$
2. $2(1+e^2)$
3. $1+e^2$
4. $1-e^2$

Ans: **3**

Sol:

$f'(x) = f'(2 - x)$

Integrate w.r.t. 'x'

$f(x) = -f(2 - x) + C$

Put x = 0

$f(0) = -f(2) + C$

$1 = -e^2 + C$

$C = 1 + e^2$

$\therefore f(x0 = -f(2 - x) + 1 + e^2$

$f(x) + f(2 - x) = 1 - e^2$

Let $I = \int_0^2 f(x)dx$ ----- (1)

$I = \int_0^2 f(2-x)dx$ ---- (2)

(1) + (2)

$2I = \int_0^2 [f(x) + f(2-x)]dx = \int_0^2 (1+e^2)dx$

$2I = (1+e^2)^2$

$I = 1+e^2$.

Q.20 The vector equation of the plane passing through the intersection of the planes $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k}) = 1$ and $\vec{r}\cdot(\hat{i}-2\hat{j}) = -2$, and the point (1, 0, 2) is :

Options

1. $\vec{r}\cdot(\hat{i}-7\hat{j}+3\hat{k}) = \frac{7}{3}$
2. $\vec{r}\cdot(3\hat{i}+7\hat{j}+3\hat{k}) = 7$
3. $\vec{r}\cdot(\hat{i}+7\hat{j}+3\hat{k}) = \frac{7}{3}$
4. $\vec{r}\cdot(\hat{i}+7\hat{j}+3\hat{k}) = 7$

Ans: **4**

Sol: $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$

$\left[\bar{r}.\left(\bar{i} + \bar{j} + \bar{k}\right) - 1\right] + \lambda\left[\bar{r}.\left(\bar{i} - 2\bar{j}\right) + 2\right] = 0$

$(x + y + z - 1) + (x + y + z - 1) + \lambda(x - 2y + z) = 0$

Which is passes through (1, 0, 2)

$2 + 3\lambda = 0$

$\lambda = \frac{-2}{3}$

$(x + y + z - 1) - \frac{2}{3}(x - 2y + z) = 0$

$3x + 3y + 3z - 3 - 2x + 4y - 4 = 0$

$x + 7y + 3z = 7$

$\bar{r}.\left(\bar{i} + 7\bar{j} + 3\bar{k}\right) = 7$.

SECTION B

Q.1 The students $S_1, S_2, \ldots, S_{10}$ are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is _____.

Ans: **31,650**

Sol:

A	B	C
1	8	1
2	7	1
-	-	-
-	-	-
-	-	-
6	1	3

$^{10}C_1(^9C_1 + ^9C_2 + \ldots + ^9C_8) + {}^{10}C_2(^8C_1 + ^8C_2 + \ldots + ^8C_7) + {}^{10}C_3(^7C_1 + ^7C_2 + \ldots + ^7C_6)$

$= 10(510) + 45(254) + 120(126) = 31650$

Q.2 The number of the real roots of the equation $(x+1)^2 + |x-5| = \frac{27}{4}$ is _____.

Ans: **2**

Sol: $x^2 + 2x + 1 + 1\,x - 5 = \frac{27}{4}$

If $x < 5$,

$x^2 + 2x + 1 - 1x - 5 = \frac{27}{4}$

$x^2 + x = \frac{3}{4}$

$4x^2 + 4x - 3 = 0$

$4x^2 + 6x - 2x - 3 = 0$

$2x(2x + 3) - 1(2x + 3) = 0$

$x = \frac{1}{2}, -\frac{3}{2}$

if $x \geq 5$

$x^2 + 2x + 1 + 1\,x - 5 = \frac{27}{4}$

$x^2 + 3x = \frac{43}{4}$

$x = \frac{-3 \pm \sqrt{52}}{2}$ $\therefore$ Number of real roots =2

Q.3

Let $i=\sqrt{-1}$. If $\frac{\left(-1+i\sqrt{3}\right)^{21}}{(1-i)^{24}}+\frac{\left(1+i\sqrt{3}\right)^{21}}{(1+i)^{24}}=k$, and $n=[|k|]$ be the greatest integral part of $|k|$. Then $\sum_{j=0}^{n+5}(j+5)^2-\sum_{j=0}^{n+5}(j+5)$ is equal to _____.

Ans: 310

Sol: $k=\frac{[2(\frac{-1}{2}+i\frac{\sqrt{3}}{2})^{21}}{[(1-i)^2]^{12}}+\frac{[2(\frac{1}{2}+i\frac{\sqrt{3}}{2})^{21}}{[(1+i)^2]^{12}}$

$\frac{[2^{21}(cos\frac{42\pi}{3}+isin\frac{42\pi}{3})}{2^{12}}+\frac{[2^{21}(cos\frac{21\pi}{3}+isin\frac{21\pi}{3})}{2^{12}}$

$=29(\cos 14\pi+i\sin 14\pi)+29(\cos 7\pi+i\sin 7\pi)=0$

$\sum_{i=0}^{5}(j+5)^2-\sum_{i=0}^{5}(j+5)$

$=(52+62+\ldots+102)-(5+6+\ldots 10)=310$

Q.4

For integers n and r, let $\binom{n}{r}=\begin{cases}{}^{n}C_{r}, & \text{if } n\geq r\geq 0\\ 0, & \text{otherwise}\end{cases}$

The maximum value of k for which the sum

$\sum_{i=0}^{k}\binom{10}{i}\binom{15}{k-i}+\sum_{i=0}^{k+1}\binom{12}{i}\binom{13}{k+1-i}$ exists, is equal to _____.

Ans: 25

Sol: $\sum\binom{10}{i}\binom{15}{k-i}={}^{10}C_0{}^{15}C_k+{}^{10}C_1{}^{15}C_{k-1}+\ldots.+{}^{10}C_k{}^{15}C_0$

$(1+x)^{10}(1+x)^{15}=({}^{10}C_0+{}^{10}C_1x+\ldots..+\cdots)({}^{15}C_0+{}^{15}C_1x+\ldots..+)$

Comparing coefficient of x^k on both sides

$\sum\binom{10}{i}\binom{15}{k-i}={}^{25}C_k$

Similarly, $\sum\binom{12}{i}\binom{13}{k+1-i}={}^{25}C_{k+1}$

$={}^{25}C_k+{}^{25}C_{k+1}$

$={}^{26}C_{k+1}$

So, maximum possible value of k is 25

Q.5 Let a point P be such that its distance from the point (5, 0) is thrice the distance of P from the point (−5, 0). If the locus of the point P is a circle of radius r, then $4r^2$ is equal to _____.

Ans: 56.25

Sol: Let A(5, 0), B(–5, 0) and P(x, y)

Given PA=3PB

$PA^2 = 9\,PB^2$

$(x-5)^2+(y-0)^2=9[(x+5)^2+(y-0)^2]$

$8x^2+8y^2+100x+200=0$

$x^2+y^2+\frac{25}{2}x+25=0$

$r^2=\left(\frac{25}{4}\right)^2-25=\frac{225}{16}$

$4r^2 = 56.25$

Q.6 If the variance of 10 natural numbers 1, 1, 1, ..., 1, k is less than 10, then the maximum possible value of k is _____.

Ans: 11

Sol: $\sigma^2+\mu^2=\frac{\sum x_i^2}{n}$

$$\sigma^2+\frac{(9+k)^2}{100}=\frac{9+k^2}{10}$$

$$\sigma^2+\frac{(9+k)^2}{100}=\frac{9+k^2}{10}$$

$$\sigma^2=\frac{9+k^2}{10}-\frac{(9+k)^2}{100}<10$$

$$\Rightarrow k<\frac{10\sqrt{10}}{3}+1$$

Maximum integral value of k=11

Q.7 Let λ be an integer. If the shortest distance between the lines $x-\lambda=2y-1=-2z$ and $x=y+2\lambda=z-\lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is _____.

Ans: 0.4

Sol:

$$\frac{x-\lambda}{1}=\frac{y-\frac{1}{2}}{\frac{1}{2}}=\frac{z}{\frac{-1}{2}}$$

$$\vec{a}=\left(\lambda,\frac{1}{2},0\right),\vec{b}=\left(1,\frac{1}{2},\frac{-1}{2}\right)$$

$$\frac{x}{1}=\frac{y-(-2\lambda)}{\frac{1}{2}}=\frac{z-\lambda}{1}$$

$$\vec{c}=(0,-2\lambda,\lambda),\vec{d}=(1,1,1)$$

Shortest distance is given by $\frac{\frac{3}{4}+\frac{5\lambda}{2}}{\frac{\sqrt{7}}{\sqrt{2}}}=\frac{\sqrt{7}}{2\sqrt{2}}$

$$\lambda=\frac{2}{5}\Rightarrow|\lambda|=0.4$$

Q.8 If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x-2)^2+(y-3)^2=25$ at the point (5, 7) is A, then 24A is equal to _____.

Ans: 1225

Sol: Equation of normal at P is $y-7=\frac{(7-3)}{(5-2)}(x-5)$

$$y-7=\frac{4}{3}(x-5)$$

Put y = 0

$$-\frac{21}{4}=x-5\Rightarrow x=5-\frac{21}{4}=-\frac{1}{4}$$

$$\therefore B\left(\frac{-1}{4},0\right)$$

Equation of tangent at P is

$y-7=\frac{-3}{4}(x-5)$

Put y = 0

$-28=-3(x-5)$

$x=\frac{43}{3}$

Area of $\Delta PAB=\frac{1}{2}AB\times PM$

Therefore, 24A = 1225

Q.9 The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α, then 2α is _____.

Ans: 3

Sol: Let the terms be $\frac{a}{r}, a, ar$

$\frac{a}{r}.a.ar=1$

$\Rightarrow a^3=1$

$\Rightarrow a=1$

$a\left(\frac{1}{r}+1+r+r^2\right)=\frac{65}{12}$

$\frac{1+r+r^2+r^3}{r}=\frac{65}{12}----(1)$

$r \quad \| \quad \frac{1}{a} \quad \| \quad \frac{1}{ar} \quad \| \quad \frac{1}{ar^2}=\frac{65}{18}$

$\frac{1}{a}\left(r+1+\frac{1}{r}+\frac{1}{r^2}\right)-\frac{65}{18}$

$\frac{1+r+r^2+r^3}{r^2}=\frac{65}{18}---(2)$

$\frac{(1)}{(2)}\Rightarrow r=\frac{18}{12}=\frac{3}{2}$

$T_3 = ar=\frac{3}{2}=\alpha$

$\therefore 2\alpha = 3$

Q.10 If $a+\alpha=1, b+\beta=2$ and $af(x)+\alpha f\left(\frac{1}{x}\right)=bx+\frac{\beta}{x}, x\neq 0$, then the value of the expression

$\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}$ is_____.

Ans: 2

Sol: Take $a=\alpha=\frac{1}{2}$

$b=\beta=1$

$\frac{1}{2}f(x)+\frac{1}{2}f\left(\frac{1}{x}\right)=x+\frac{1}{x}$

$$\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}=2$$