SOLUTIONS & ANSWERS FOR JEE MAINS-2021
24th February Shift 1
[PHYSICS, CHEMISTRY & MATHEMATICS]

PART – A – PHYSICS

SECTION A

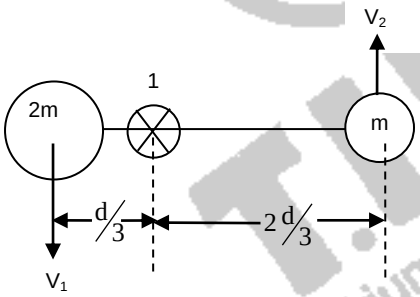
Q.1 Two stars of masses m and 2m at a distance d rotate about their common centre of mass in free space. The period of revolution is :

Options

1. $2\pi\sqrt{\frac{d^3}{3Gm}}$
2. $\frac{1}{2\pi}\sqrt{\frac{d^3}{3Gm}}$
3. $2\pi\sqrt{\frac{3Gm}{d^3}}$
4. $\frac{1}{2\pi}\sqrt{\frac{3Gm}{d^3}}$

Ans: 1

Sol:



$$F = \frac{G \times 2m^2}{d^2} = \frac{m2d}{3}\omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{3Gm}{d^3}}$$

$$\therefore T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{d^3}{3Gm}}$$

Q.2 Two equal capacitors are first connected in series and then in parallel. The ratio of the equivalent capacities in the two cases will be :

Options

1. 2 : 1
2. 1 : 4
3. 4 : 1
4. 1 : 2

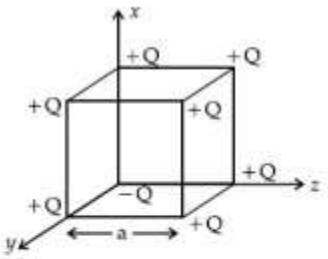
Ans: 2

Sol: In series $\Rightarrow C_1 = \frac{C}{2}$

In parallel $\Rightarrow C_2 = C + C = 2C$

$\therefore \frac{C_1}{C_2} = \frac{1}{4}$

Q.3 A cube of side 'a' has point charges +Q located at each of its vertices except at the origin where the charge is −Q. The electric field at the centre of cube is :



Options

1. $\frac{2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}\left(\hat{x}+\hat{y}+\hat{z}\right)$
2. $\frac{-2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}\left(\hat{x}+\hat{y}+\hat{z}\right)$
3. $\frac{-Q}{3\sqrt{3}\pi\varepsilon_0 a^2}\left(\hat{x}+\hat{y}+\hat{z}\right)$
4. $\frac{Q}{3\sqrt{3}\pi\varepsilon_0 a^2}\left(\hat{x}+\hat{y}+\hat{z}\right)$

Ans: 2

Sol: The net electric field at the centre will be due to only two charges

→ One at origin

→ One at diagonally opposite to it and its direction will be towards the origin.

$E_{centre} = \frac{2Q}{3\pi\varepsilon_0 a^2}\cos\theta \Rightarrow \frac{-2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}\hat{x}$ for $\vec{E}_x$

As $\cos\theta = \frac{1}{\sqrt{3}}$ for a cube (angle between diagonal with x axis)

$\vec{E}_{centre} = \frac{-2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

Q.4 The focal length f is related to the radius of curvature r of the spherical convex mirror by :

Options

1. $f = r$
2. $f = +\frac{1}{2}r$
3. $f = -\frac{1}{2}r$
4. $f = -r$

Ans: 2

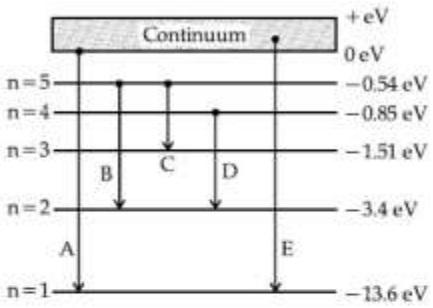
Sol: For a spherical mirror

r = 2f

$$\Rightarrow f = \frac{r}{2} = \frac{1}{2}r$$

Q.5 In the given figure, the energy levels of hydrogen atom have been shown along with some transitions marked A, B, C, D and E.

The transitions A, B and C respectively represent :



Options 1.

The ionization potential of hydrogen, second member of Balmer series and third member of Paschen series.

2.

The series limit of Lyman series, third member of Balmer series and second member of Paschen series.

3.

The first member of the Lyman series, third member of Balmer series and second member of Paschen series.

4.

The series limit of Lyman series, second member of Balmer series and second member of Paschen series.

Ans: 2

Sol: The series limit of Lyman series, third member of Balmer series and second member of Paschen series

Q.6 If Y, K and η are the values of Young's modulus, bulk modulus and modulus of rigidity of any material respectively. Choose the correct relation for these parameters.

Options

1. $K = \frac{Y\eta}{9\eta - 3Y}\ N/m^2$
2. $Y = \frac{9K\eta}{2\eta + 3K}\ N/m^2$
3. $Y = \frac{9K\eta}{3K - \eta}\ N/m^2$
4. $\eta = \frac{3YK}{9K + Y}\ N/m^2$

Ans: 1

Sol: $K = \frac{Y\eta}{9\eta - 3Y}\ N/m^2$

Q.7 Match List I with List II.

	List I		List II
(a)	Isothermal	(i)	Pressure constant
(b)	Isochoric	(ii)	Temperature constant
(c)	Adiabatic	(iii)	Volume constant
(d)	Isobaric	(iv)	Heat content is constant

Choose the correct answer from the options given below :

Options

1. (a) → (i), (b) → (iii), (c) → (ii), (d) → (iv)
2. (a) → (ii), (b) → (iii), (c) → (iv), (d) → (i)
3. (a) → (ii), (b) → (iv), (c) → (iii), (d) → (i)
4. (a) → (iii), (b) → (ii), (c) → (i), (d) → (iv)

Ans: 2

Sol: Option (2)

Q.8 A current through a wire depends on time as

$$i = \alpha_0 t + \beta t^2$$

where $\alpha_0 = 20$ A/s and $\beta = 8$ As^{-2}. Find the charge crossed through a section of the wire in 15 s.

Options

1. 2100 C
2. 11250 C
3. 260 C
4. 2250 C

Ans: 2

Sol: $q = \int_0^{15} Idt = \frac{\alpha_0 t^2}{2} + \frac{\beta t^3}{3}$

Put t = 15 s,

$q = \frac{\alpha_0 \times 15^2}{2} + \frac{\beta \times 15^3}{3}$

Put $\alpha_0 = 20$ and $\beta = 8$

$\therefore$ q = 11250 C

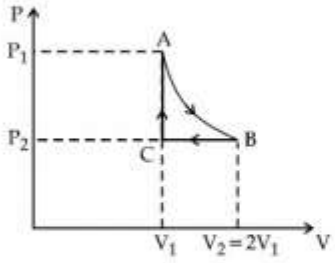
Q.9 n mole of a perfect gas undergoes a cyclic process ABCA (see figure) consisting of the following processes.

$A \to B$: Isothermal expansion at temperature T so that the volume is doubled from V_1 to $V_2 = 2V_1$ and pressure changes from P_1 to P_2.

$B \to C$: Isobaric compression at pressure P_2 to initial volume V_1.

$C \to A$: Isochoric change leading to change of pressure from P_2 to P_1.

Total workdone in the complete cycle ABCA is :



Options

1. $nRT\left(\ln 2 - \frac{1}{2}\right)$
2. $nRT\ln 2$
3. $nRT\left(\ln 2 + \frac{1}{2}\right)$
4. 0

Ans: 1

Sol: $W_{AB} = nRT\,\ell n\left(\frac{V_2}{V_1}\right) = nRT\,\ell n2$

$W_{BC} = P_2(V_f - V_i) = -P_2V_1$

But $P_2 = \frac{P_1}{2}$

$\therefore W_{BC} = \frac{-P_1V_1}{2} = \frac{-nRT}{2}$

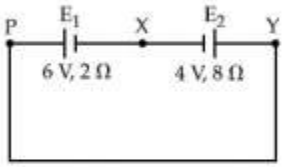
$W_{CA} = 0$ (isochoric)

$\therefore W = W_{AB} + W_{BC} + W_{CA}$

$= nRT\,\ell n2 - \frac{nRT}{2}$

$= nRT\left(\ell n2 - \frac{1}{2}\right)$

Q.10 A cell E_1 of emf 6 V and internal resistance 2 Ω is connected with another cell E_2 of emf 4 V and internal resistance 8 Ω (as shown in the figure). The potential difference across points X and Y is :



Options

1. 10.0 V
2. 2.0 V
3. 3.6 V
4. 5.6 V

Ans: 4

Sol: Current in the circuit $I = \frac{2}{10} = 0.2$

Potential across 4V cell, $V = \varepsilon + ir$

$V = 4+0.2 \times 8 = 5.6$ V

Q.11 Moment of inertia (M.I) of four bodies, having same mass and radius, are reported as :

I_1 = M.I. of thin circular ring about its diameter,

I_2 = M.I. of circular disc about an axis perpendicular to disc and going through the centre,

I_3 = M.I. of solid cylinder about its axis and

I_4 = M.I. of solid sphere about its diameter.

Then :

Options

1. $I_1 + I_3 < I_2 + I_4$
2. $I_1 + I_2 = I_3 + \frac{5}{2} I_4$
3. $I_1 = I_2 = I_3 > I_4$
4. $I_1 = I_2 = I_3 < I_4$

Ans: 3

Sol: $I_1 = \frac{Mr^2}{2}$

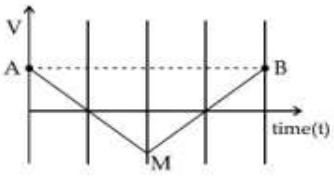
$I_2 = \frac{Mr^2}{2}$

$I_3 = \frac{Mr^2}{2}$

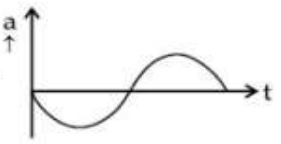
$I_4 = \frac{2}{5}Mr^2$

$\therefore I_1 = I_2 = I_3 > I_4$

Q.12 If the velocity-time graph has the shape AMB, what would be the shape of the corresponding acceleration-time graph ?



Options

1.


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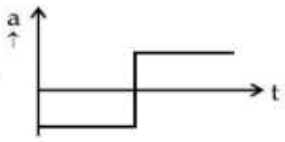

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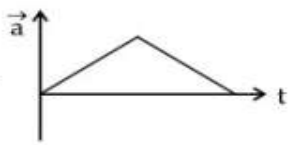
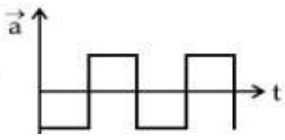


4.


Ans: 4

Sol: If the total time T is divided into four part

$0 \to T/4$ (deceleration constant)

$\frac{T}{4} \to \frac{T}{2}$ (constant acceleration)

$\frac{T}{2} \to \frac{3T}{4}$ (const deceleration)

$\frac{3T}{4} \to T$ (constant acceleration)

Q.13 The workdone by a gas molecule in an isolated system is given by, $W = \alpha\beta^2 e^{-\frac{x^2}{\alpha kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature. α and β are constants. Then the dimensions of β will be :

Options

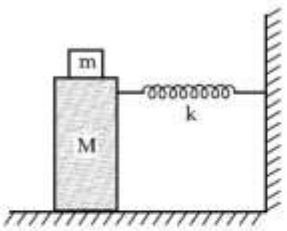
1. $[M^0 L T^0]$
2. $[M L^2 T^{-2}]$
3. $[M L T^{-2}]$
4. $[M^2 L T^2]$

Ans: 3

Sol: Dimension of $\alpha = \left[\frac{L^2}{ML^2T^{-2}}\right] = M^{-1}T^2$

Dimension of $\beta = \left[\frac{M^{1/2}LT^{-1}}{M^{-1/2}T^{1}}\right] = MLT^{-2}$

Q.14 In the given figure, a mass M is attached to a horizontal spring which is fixed on one side to a rigid support. The spring constant of the spring is k. The mass oscillates on a frictionless surface with time period T and amplitude A. When the mass is in equilibrium position, as shown in the figure, another mass m is gently fixed upon it. The new amplitude of oscillation will be :



Options

1. $A\sqrt{\frac{M+m}{M}}$
2. $A\sqrt{\frac{M-m}{M}}$
3. $A\sqrt{\frac{M}{M-m}}$
4. $A\sqrt{\frac{M}{M+m}}$

Ans: 4

Sol: Let initial velocity of M before change = V at mean position and velocity of the system (m + M) after change = V'

$$V' = \frac{M}{(m+M)}V$$

$$\frac{1}{2}KA^{1^2} = \frac{1}{2}(m+M)V^{1^2} = \frac{M^2V^2(M+m)}{2(M+m)^2}$$

$$\therefore \frac{1}{2}KA^2 = \frac{1}{2}MV^2$$

$$A^{1^2} = \frac{M}{M+m}A^2$$

$$A^1 = A\sqrt{\frac{M}{M+m}}$$

Q.15 Consider two satellites S_1 and S_2 with periods of revolution 1 hr. and 8 hr. respectively revolving around a planet in circular orbits. The ratio of angular velocity of satellite S_1 to the angular velocity of satellite S_2 is :

Options

1. 1 : 4
2. 8 : 1
3. 2 : 1
4. 1 : 8

Ans: 2

Sol:

$$\omega_1 = \frac{2\pi}{T_1} = 2\pi$$

$$\omega_2 = \frac{2\pi}{T_2} = \frac{2\pi}{8}$$

$$\frac{\omega_1}{\omega_2} = 8$$

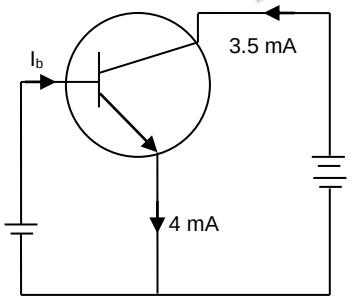
Q.16 If an emitter current is changed by 4 mA, the collector current changes by 3.5 mA. The value of β will be :

Options

1. 0.5
2. 7
3. 3.5
4. 0.875

Ans: 2

Sol:



$$I_e = I_c + I_b$$

$$4 = 3.5 + I_b$$

$$I_b = 0.5\text{mA}$$

$$\beta = \frac{I_c}{I_b} = \frac{3.5}{0.5} = 7$$

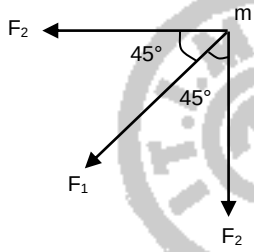
Q.17 Four identical particles of equal masses 1 kg made to move along the circumference of a circle of radius 1 m under the action of their own mutual gravitational attraction. The speed of each particle will be :

Options

1. $\sqrt{\frac{G}{2}\left(2\sqrt{2}-1\right)}$
2. $\frac{\sqrt{\left(1+2\sqrt{2}\right)G}}{2}$
3. $\sqrt{G\left(1+2\sqrt{2}\right)}$
4. $\sqrt{\frac{G}{2}\left(1+2\sqrt{2}\right)}$

Ans: 2

Sol:



$$F_{net} = F_1 + \sqrt{2}F_2 = \frac{mV^2}{R}$$

$$F_1 = \frac{Gmm}{4R^2} \text{ and } F_2 = \frac{Gmm}{2R^2}$$

$$\frac{mV^2}{R} = \frac{Gmm}{4R^2} + \frac{\sqrt{2}}{2}\frac{Gmm}{R^2}$$

$$V = \sqrt{\frac{\left(2\sqrt{2}+1\right)}{4}\frac{Gm}{R}}$$

$$V = \frac{\sqrt{\left(1+2\sqrt{2}\right)G}}{2}$$

Q.18 Each side of a box made of metal sheet in cubic shape is 'a' at room temperature 'T', the coefficient of linear expansion of the metal sheet is 'α'. The metal sheet is heated uniformly, by a small temperature ΔT, so that its new temperature is $T + \Delta T$. Calculate the increase in the volume of the metal box.

Options

1. $3a^3\alpha\Delta T$
2. $\frac{4}{3}\pi a^3\alpha\Delta T$
3. $4\pi a^3\alpha\Delta T$
4. $4a^3\alpha\Delta T$

Ans: 1

Sol: Coefficient of volume Expansion = 3α

$\Delta V = V_0(3\alpha)\Delta t$

$= a^3 \times 3\alpha \times \Delta T = 3a^3\,\alpha\Delta T$

Q.19 Given below are two statements :

Statement I : Two photons having equal linear momenta have equal wavelengths.

Statement II : If the wavelength of photon is decreased, then the momentum and energy of a photon will also decrease.

In the light of the above statements, choose the correct answer from the options given below.

Options

1. Both Statement I and Statement II are false
2. Statement I is true but Statement II is false
3. Statement I is false but Statement II is true
4. Both Statement I and Statement II are true

Ans: 2

Sol: $E = \frac{hc}{\lambda}$

$P = \frac{h}{\lambda}$

Given $P_1 = P_2 \Rightarrow \lambda_1 = \lambda_2$ statement I is true E and P are inversely proportional to λ. If λ decreases, E and P will increase

∴ II is false.

Q.20 In a Young's double slit experiment, the width of the one of the slit is three times the other slit. The amplitude of the light coming from a slit is proportional to the slit-width. Find the ratio of the maximum to the minimum intensity in the interference pattern.

Options

1. 4 : 1
2. 1 : 4
3. 2 : 1
4. 3 : 1

Ans: 1

Sol: $\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2}$

$\left[\frac{\sqrt{9I_0}+\sqrt{I_0}}{\sqrt{9I_0}-\sqrt{I_0}}\right]^2 = 4:1$

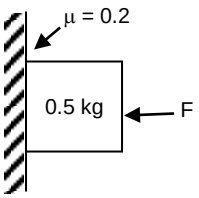
SECTION B

Q.1 The coefficient of static friction between a wooden block of mass 0.5 kg and a vertical rough wall is 0.2. The magnitude of horizontal force that should be applied on the block to keep it adhere to the wall will be _____ N.

[$g = 10\ ms^{-2}$]

Ans: 25.00

Sol:



$$\mu F = mg$$
$$F = \frac{mg}{\mu} = \frac{0.5 \times 10}{0.2} = 25N$$

Q.2 A ball with a speed of 9 m/s collides with another identical ball at rest. After the collision, the direction of each ball makes an angle of 30° with the original direction. The ratio of velocities of the balls after collision is $x : y$, where x is _____ .

Ans: 5.19

Sol: By momentum conservation along perpendicular direction

$mx \sin 30 = my \sin 30$

$\Rightarrow x : y = 1 : 1$

$\Rightarrow x = y$

$m \times 9 = m \times x \cos 30° + m \times y \cos 30°$

$$9 = 2x \frac{\sqrt{3}}{2}$$

$$x = \frac{9}{\sqrt{3}} = 5.19$$

Q.3 An unpolarized light beam is incident on the polarizer of a polarization experiment and the intensity of light beam emerging from the analyzer is measured as 100 Lumens. Now, if the analyzer is rotated around the horizontal axis (direction of light) by 30° in clockwise direction, the intensity of emerging light will be _____ Lumens.

Ans: 75.00

Sol: $I = I_0 \cos^2 \theta$

$= 100 \times \cos^2 30$

$$= 100 \times \frac{3}{4} = 75$$

Q.4 A hydraulic press can lift 100 kg when a mass 'm' is placed on the smaller piston. It can lift _____ kg when the diameter of the larger piston is increased by 4 times and that of the smaller piston is decreased by 4 times keeping the same mass 'm' on the smaller piston.

Ans: 25600.00

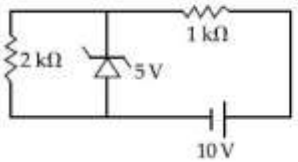
Sol:

$P_A = P_B$

$$\frac{100g}{A} = \frac{mg}{a} \Rightarrow \frac{xg}{16A} = \frac{mg}{(a/16)} = \frac{16mg}{a}$$

Dividing, $\frac{1600}{x} = \frac{1}{16} \Rightarrow x = 25600kg$

Q.5 In connection with the circuit drawn below, the value of current flowing through 2 kΩ resistor is _____ $\times 10^{-4}$ A.



Ans: 25.00

Sol: Voltage across 2 kΩ $\Rightarrow V = 5V$

Current in the load $\Rightarrow i = \frac{V}{R}$

$$= \frac{5}{2 \times 1000} = 2.5 \times 10^{-3} A = 25 \times 10^{-4} A$$

Q.6 A common transistor radio set requires 12 V (D.C.) for its operation. The D.C. source is constructed by using a transformer and a rectifier circuit, which are operated at 220 V (A.C.) on standard domestic A.C. supply. The number of turns of secondary coil are 24, then the number of turns of primary are _____.

Ans: 440.00

Sol: $\frac{N_p}{N_s} = \frac{V_p}{V_s}$

$\frac{N_p}{24} = \frac{220}{12}$

$\therefore N_p = 440$

Q.7 An audio signal $v_m = 20 \sin 2\pi(1500t)$ amplitude modulates a carrier $v_c = 80 \sin 2\pi(100{,}000t)$.

The value of percent modulation is _____.

Ans: 25.00

Sol: $A_m = 20$ $A_c = 80$

$\mu = \frac{A_m}{A_c}$

$\frac{20}{80} = 0.25$

$\therefore$ % modulation = 25%

Q.8 A resonance circuit having inductance and resistance 2×10^{-4} H and 6.28 Ω respectively oscillates at 10 MHz frequency. The value of quality factor of this resonator is _____. [$\pi = 3.14$]

Ans: 2000.00

Sol: Given $L = 2 \times 10^{-4}$ H

$R = 6.28\ \Omega$

$f = 10$ MHz

$\therefore Q = \frac{L\omega}{R}$

$$= \frac{20\pi \times 2 \times 10^{-4} \times 10^6}{6.28} = 2000$$

Q.9 An inclined plane is bent in such a way that the vertical cross-section is given by $y = \frac{x^2}{4}$ where y is in vertical and x in horizontal direction. If the upper surface of this curved plane is rough with coefficient of friction $\mu = 0.5$, the maximum height in cm at which a stationary block will not slip downward is _____ cm.

Ans: 25.00

Sol: $\frac{dy}{dx} = \frac{1}{4} \times 2x = \frac{x}{2}$

$\frac{x}{2} = \tan\theta = \mu$

i.e.,$x = 2\mu$

$y = \frac{x^2}{4} = \frac{4\mu^2}{4} = \mu^2 = 0.5^2 = 0.25$ m = 25.00 cm

Q.10 An electromagnetic wave of frequency 5 GHz, is travelling in a medium whose relative electric permittivity and relative magnetic permeability both are 2. Its velocity in this medium is _____ $\times 10^7$ m/s.

Ans: 15.00

Sol: $\mu_r = \varepsilon_r = 2$

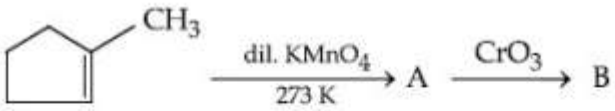
$n = \sqrt{\mu_r \varepsilon_r} = 2$

$V = \frac{c}{n} = \frac{c}{2} = \frac{3 \times 10^8}{2} = 15 \times 10^7 m/s$

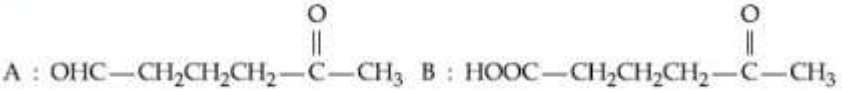
PART – B – CHEMISTRY

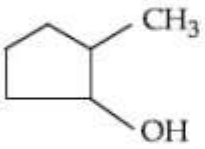
SECTION A

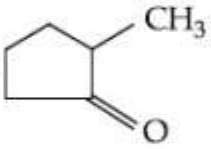
Q.1 Identify products A and B.

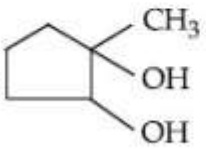


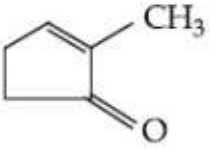
Options 1.

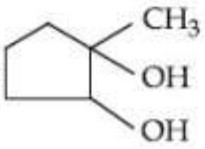


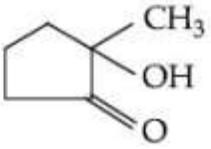
2. A: 

B: 

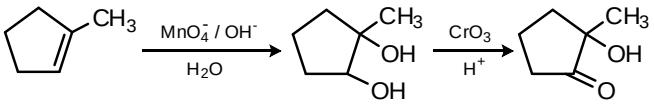
3. A: 

B: 

4. A: 

B: 

Ans: 4

Sol: 

Q.2 Out of the following, which type of interaction is responsible for the stabilisation of α-helix structure of proteins ?

Options 1. vander Waals forces

2. Ionic bonding

3. Hydrogen bonding

4. Covalent bonding

Ans: 3

Sol: α-Helical structure is stable due to hydrogen bonding between –CO– group and –NH group of peptide bond.

Q.3 Match List I with List II.

	List I (Monomer Unit)		List II (Polymer)
(a)	Caprolactum	(i)	Natural rubber
(b)	2-Chloro-1,3-butadiene	(ii)	Buna-N
(c)	Isoprene	(iii)	Nylon 6
(d)	Acrylonitrile	(iv)	Neoprene

Choose the correct answer from the options given below :

Options

1. (a) → (iii), (b) → (iv), (c) → (i), (d) → (ii)
2. (a) → (iv), (b) → (iii), (c) → (ii), (d) → (i)
3. (a) → (i), (b) → (ii), (c) → (iii), (d) → (iv)
4. (a) → (ii), (b) → (i), (c) → (iv), (d) → (iii)

Ans: 1

Sol: Isoprene – Natural rubber
Chloroprene – Neoprene
Caprolactum – Nylon 6
Butadiene-acrylonitrile – Buna-N

Q.4 Which of the following are isostructural pairs ?

A. SO_4^{2-} and CrO_4^{2-}
B. $SiCl_4$ and $TiCl_4$
C. NH_3 and NO_3^-
D. BCl_3 and $BrCl_3$

Options

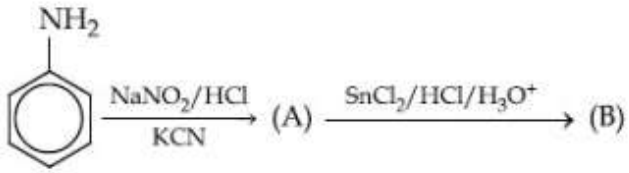
1. C and D only
2. A and C only
3. A and B only
4. B and C only

Ans: 3

Sol:

SO_4^{2-}	sp^3 hybridised with 4 bp – tetrahedral
CrO_4^{2-}	sp^3 hybridised with 4 bp – tetrahedral
$SiCl_4$	sp^3 hybridised with 4 bp – tetrahedral
$TiCl_4$	sp^3 hybridised with 4 bp – tetrahedral
NH_3	sp^3 hybridised with 3 bp and 1 lp – pyramidal
NO_3^-	sp^2 hybridised with 3 bp – trigonal planar
BCl_3	sp^2 hybridised with 3 bp – trigonal planar
$BrCl_3$	sp^3d hybridised with 3 bp and 2 lp – T-shaped

Q.5 'A' and 'B' in the following reactions are :



Options

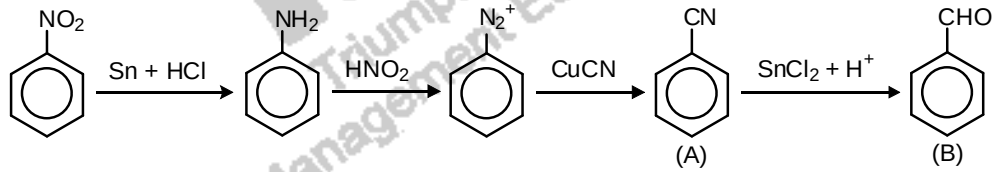
1. 

2. 

3. 

4. 

Ans: 4

Sol: 

Q.6 Which of the following ore is concentrated using group 1 cyanide salt ?

Options

1. Sphalerite
2. Siderite
3. Malachite
4. Calamine

Ans: 1

Sol: NaCN is used as depressant during the froth floatation process of sulphide ores.

Q.7 (A) $HOCl + H_2O_2 \rightarrow H_3O^+ + Cl^- + O_2$

(B) $I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H_2O + O_2$

Choose the correct option.

Options

1. H_2O_2 acts as reducing agent in equations (A) and (B).
2. H_2O_2 act as oxidizing and reducing agent respectively in equations (A) and (B).
3. H_2O_2 acts as reducing and oxidising agent respectively in equations (A) and (B).
4. H_2O_2 acts as oxidising agent in equations (A) and (B).

Ans: 1

Sol: In both cases H_2O_2 acts as reducing agent

Q.8 The electrode potential of M^{2+}/M of 3d-series elements shows positive value for :

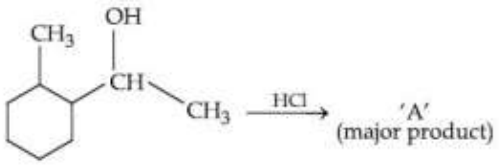
Options

1. Fe
2. Cu
3. Zn
4. Co

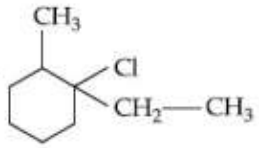
Ans: 2

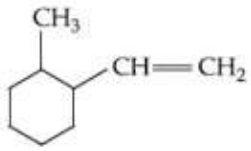
Sol: The reduction potential $E^{\circ}_{Cu^{2+}/Cu}$ is +0.34 V

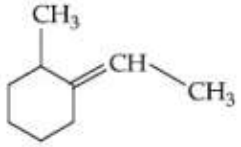
Q.9 What is the final product (major) 'A' in the given reaction ?

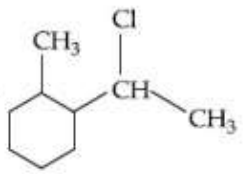


Options

1. 

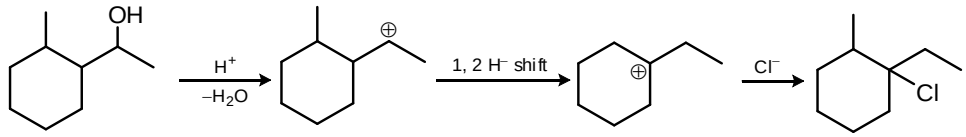
2. 

3. 

4. 

Ans: 1

Sol:



Q.10 Given below are two statements :

Statement I : Colourless cupric metaborate is reduced to cuprous metaborate in a luminous flame.

Statement II : Cuprous metaborate is obtained by heating boric anhydride and copper sulphate in a non-luminous flame.

In the light of the above statements, choose the most appropriate answer from the options given below.

Options

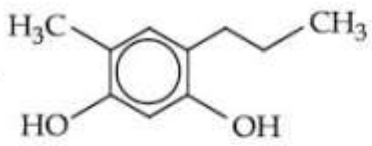
1. **Statement I** is true but **Statement II** is false
2. Both **Statement I** and **Statement II** are false
3. Both **Statement I** and **Statement II** are true
4. **Statement I** is false but **Statement II** is true

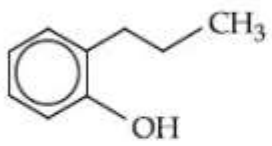
Ans: 1

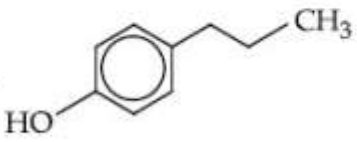
Sol: Luminous flame is reducing therefore statement I is correct. Non-luminous flame is oxidizing therefore statement II is incorrect.

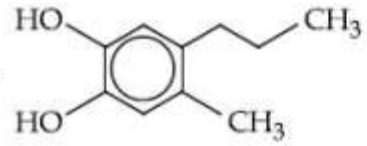
Q.11 Which of the following compound gives pink colour on reaction with phthalic anhydride in conc. H_2SO_4 followed by treatment with NaOH ?

Options

1. 

2. 

3. 

4. 

Ans: 3

Sol: P should be a phenol with para position carrying hydrogen. So that it can form phenolphthalein (or its derivative) with phthalic anhydride.

Q.12 Consider the elements Mg, Al, S, P and Si, the correct increasing order of their first ionization enthalpy is :

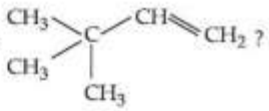
Options

1. Al < Mg < S < Si < P
2. Mg < Al < Si < P < S
3. Al < Mg < Si < S < P
4. Mg < Al < Si < S < P

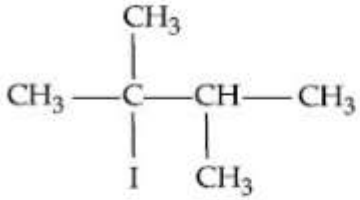
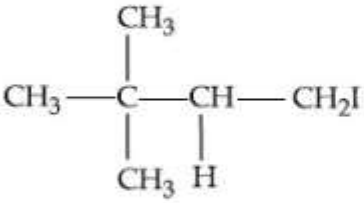
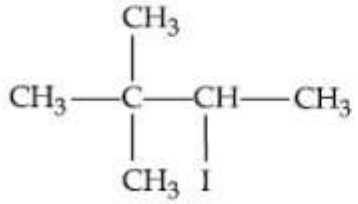
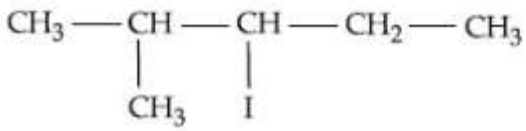
Ans: 3

Sol: The 1st ionization potential of Mg is more than Al due to the stability of completely filled 3s configuration of Mg
The 1st ionization potential of P is more than S due to the stability of completely half filled 3p configuration.

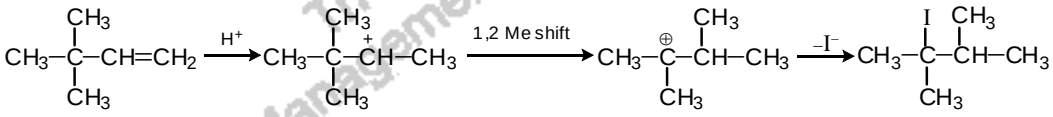
Q.13

What is the major product formed by HI on reaction with  ?

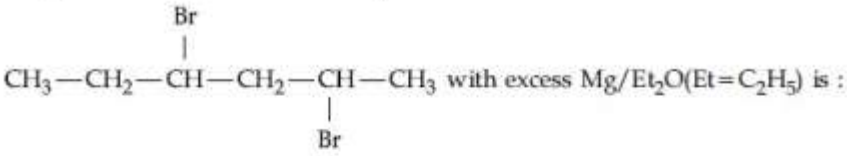
Options

1. 
2. 
3. 
4. 

Ans: 1

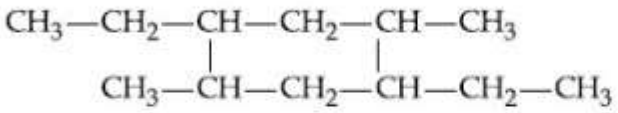
Sol: 

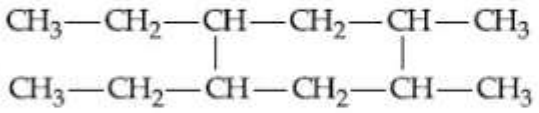
Q.14 The product formed in the first step of the reaction of

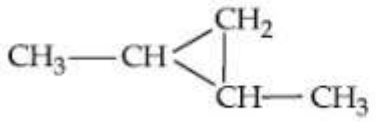


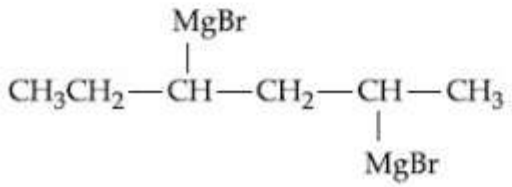
with excess $Mg/Et_2O(Et=C_2H_5)$ is :

Options

1. 

2. 

3. 

4. 

Ans: 4

Sol: Grignards reagent should be formed.

Q.15 The gas released during anaerobic degradation of vegetation may lead to :

Options

1. Ozone hole
2. Corrosion of metals
3. Global warming and cancer
4. Acid rain

Ans: 3

Sol: Anaerobic vegetative degradation releases excess of CH_4 which causes global warming and cancer.

Q.16 Al_2O_3 was leached with alkali to get X. The solution of X on passing of gas Y, forms Z. X, Y and Z respectively are :

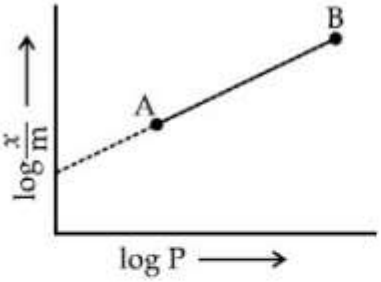
Options

1. $X=Al(OH)_3,\ Y=CO_2,\ Z=Al_2O_3$
2. $X=Na[Al(OH)_4],\ Y=CO_2,\ Z=Al_2O_3.xH_2O$
3. $X=Na[Al(OH)_4],\ Y=SO_2,\ Z=Al_2O_3$
4. $X=Al(OH)_3,\ Y=SO_2,\ Z=Al_2O_3.xH_2O$

Ans: 2

Sol: $Al_2O_3.xH_2O \xrightarrow{NaOH} \underset{(Y)}{Na[Al(OH)_4} \xrightarrow{CO_2\,(Y)} Al_2O_3.xH_2O$

Q.17 In Freundlich adsorption isotherm, slope of AB line is :



Options

1. n with (n, 0.1 to 0.5)
2. $\frac{1}{n}$ with $\left(\frac{1}{n} = 0 \text{ to } 1\right)$
3. log n with (n > 1)
4. $\log\frac{1}{n}$ with (n < 1)

Ans: 2

Sol: Freundlich isotherm = $\frac{x}{m} = k(P)^{1/n}$

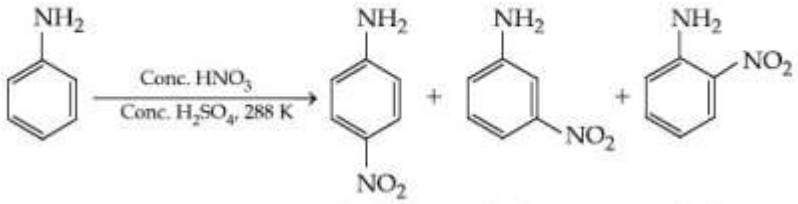
taking log on both sides

$\ln\left(\frac{x}{m}\right) = \ln k + \frac{1}{n}\ln(P)$

a plot of ln $\left(\frac{x}{m}\right)$ against ln P gives a straight

line with a slope $\frac{1}{n}\left(0 \geq \frac{1}{n} \geq 1\right)$

Q.18 In the following reaction the reason why *meta*-nitro product also formed is :



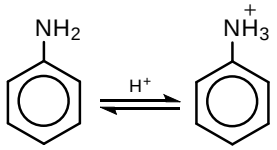
[A] 51% [B] 47% [C] 2%

Options

1. $-NO_2$ substitution always takes place at *meta*-position
2. Formation of anilinium ion
3. $-NH_2$ group is highly *meta*-directive
4. low temperature

Ans: 2

Sol: In acidic medium aniline partially get protonised to form anilinium ion



$NH_3^{\oplus}$ group is meta directing

Q.19 The major components in "Gun Metal" are :

Options

1. Cu, Zn and Ni
2. Cu, Sn and Zn
3. Cu, Ni and Fe
4. Al, Cu, Mg and Mn

Ans: 2

Sol: Gun metal is an alloy of Cu, Zn and Sn

Q.20 Which of the following reagent is used for the following reaction ?

$$CH_3CH_2CH_3 \xrightarrow{?} CH_3CH_2CHO$$

Options

1. Copper at high temperature and pressure
2. Manganese acetate
3. Potassium permanganate
4. Molybdenum oxide

Ans: 4

Sol: $CH_3 - CH_2 - CH_3 \xrightarrow{\text{Molybdenum oxide}} CH_3 - CH_2 - CHO$

SECTION B

Q.1 The reaction of sulphur in alkaline medium is given below :

$$S_{8(s)} + a\ OH^-_{(aq)} \longrightarrow b\ S^{2-}_{(aq)} + c\ S_2O_3^{2-}{}_{(aq)} + d\ H_2O_{(l)}$$

The values of 'a' is _____. (Integer answer)

Ans: 12.00

Sol: $S_8 + 12OH^- \rightarrow 4S^{2-} + 2S_2O_3^{2-} + 6H_2O$

Q.2 At 1990 K and 1 atm pressure, there are equal number of Cl_2 molecules and Cl atoms in the reaction mixture. The value of K_p for the reaction $Cl_{2(g)} = 2Cl_{(g)}$ under the above conditions is $x \times 10^{-1}$. The value of x is _____. (Rounded off to the nearest integer)

Ans: 5

Sol: Since the total pressure is 1 atm the partial pressure of $Cl_{2(g)}$ and $Cl_{(g)}$ should be 0.5 atm each

$$K_p = \frac{(P_{Cl})^2}{P_{Cl_2}} = \frac{(0.5)^2}{0.5} = 0.5 \text{ atm}$$

$K_p = 5 \times 10^{-1}$ atm

Q.3 A proton and a Li^{3+} nucleus are accelerated by the same potential. If λ_{Li} and λ_p denote the de Broglie wavelengths of Li^{3+} and proton respectively, then the value of $\frac{\lambda_{Li}}{\lambda_p}$ is $x \times 10^{-1}$.

The value of x is _____. (Rounded off to the nearest integer)

[Mass of Li^{3+} = 8.3 mass of proton]

Ans: 2.00

Sol: $$\lambda = \frac{h}{\sqrt{2mqV}}$$

$$\frac{\lambda_{Li^{3+}}}{\lambda_p} = \frac{\sqrt{2 \times m_p \times q \times V}}{\sqrt{2 \times 8.3 \times m_p \times 3 \times q \times V}}$$

$$= \frac{1}{\sqrt{24.9}} \approx \frac{1}{5} = 0.2 = 2 \times 10^{-1}$$

Q.4 The stepwise formation of $[Cu(NH_3)_4]^{2+}$ is given below :

$$Cu^{2+} + NH_3 \xrightleftharpoons{K_1} [Cu(NH_3)]^{2+}$$

$$[Cu(NH_3)]^{2+} + NH_3 \xrightleftharpoons{K_2} [Cu(NH_3)_2]^{2+}$$

$$[Cu(NH_3)_2]^{2+} + NH_3 \xrightleftharpoons{K_3} [Cu(NH_3)_3]^{2+}$$

$$[Cu(NH_3)_3]^{2+} + NH_3 \xrightleftharpoons{K_4} [Cu(NH_3)_4]^{2+}$$

The value of stability constants K_1, K_2, K_3 and K_4 are 10^4, 1.58×10^3, 5×10^2 and 10^2 respectively. The overall equilibrium constants for dissociation of $[Cu(NH_3)_4]^{2+}$ is $x \times 10^{-12}$. The value of x is _____. (Rounded off to the nearest integer)

Ans: 1

Sol: $Cu^{2+} + 4NH_3 \rightleftharpoons [Cu(NH_3)_4]^{2+}$

Overall stability constant $K = k_1 \times k_2 \times k_3 \times k_4$

$= 10^4 \times 1.5 \times 10^3 \times 5 \times 10^2 \times 10^2$

$= 7.5 \times 10^{11}$

Dissociation constant = $\frac{1}{7.5 \times 10^{11}} = 1.33 \times 10^{-12} \cong 1 \times 10^{-12}$

Q.5 When 9.45 g of $ClCH_2COOH$ is added to 500 mL of water, its freezing point drops by 0.5°C. The dissociation constant of $ClCH_2COOH$ is $x \times 10^{-3}$. The value of x is _____. (Rounded off to the nearest integer)

[$K_{f(H_2O)} = 1.86$ K kg mol^{-1}]

Ans: 36

Sol: $ClCH_2–COOH \rightleftharpoons ClCH_2COO^- + H^+$

$$\Delta T_f = i \times k_f \times \frac{W_B}{M_B} \times \frac{1000}{W_A}$$

$$i = \frac{0.5 \times 94.5 \times 500}{1.86 \times 9.45 \times 1000} = \frac{2.5}{1.86}$$

$$\alpha = \frac{i-1}{n-1} = \frac{2.5}{1.86} - 1 = \frac{0.64}{1.86} = 0.34$$

$$K_a = \frac{C\alpha^2}{1-\alpha} = \frac{0.2 \times (0.34)^2}{(1-0.34)} = 36 \times 10^{-3}$$

Q.6 The coordination number of an atom in a body-centered cubic structure is _____. [Assume that the lattice is made up of atoms.]

Ans: 8

Sol: In a BCC lattice one atom is in contact with 8 other atoms

Q.7 Number of amphoteric compounds among the following is _____.

(A) BeO (B) BaO (C) $Be(OH)_2$ (D) $Sr(OH)_2$

Ans: 2

Sol: BeO and $Be(OH)_2$ are amphoteric

Q.8 Gaseous cyclobutene isomerizes to butadiene in a first order process which has a 'k' value of $3.3 \times 10^{-4}\ s^{-1}$ at 153°C. The time in minutes it takes for the isomerization to proceed 40% to completion at this temperature is _____. (Rounded off to the nearest integer)

Ans: 26

Sol: $t_{40\%} = \frac{2.303}{3.3 \times 10^{-4}} \log \frac{100}{60}$

$= 2.303 \times 10^4 \times 0.3 \times [\log 5 - \log 3]$

$= 1532.75$ sec

$= 25.54$ min

$= 26$

Q.9 For the reaction $A_{(g)} \rightarrow B_{(g)}$, the value of the equilibrium constant at 300 K and 1 atm is equal to 100.0. The value of $\Delta_r G$ for the reaction at 300 K and 1 atm in J mol^{-1} is $-xR$, where x is _____. (Rounded off to the nearest integer)

[R = 8.31 J $mol^{-1}K^{-1}$ and ln 10 = 2.3)

Ans: 1380

Sol: $\Delta G° = -2.3\ RT \log K_{eq}$

$= -2.3 \times 300 \times R \times \log (100)$

$= -1380\ R$

$x = 1380$

Q.10 4.5 g of compound A (MW = 90) was used to make 250 mL of its aqueous solution. The molarity of the solution in M is $x \times 10^{-1}$. The value of x is _____. (Rounded off to the nearest integer)

Ans: 2

Sol: $M = \frac{4.5}{90} \times \frac{1000}{250} = 0.2 = 2 \times 10^{-1}$

PART – C – MATHEMATICS

SECTION A

Q.1 The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}$ and the plane $x+y+z=17$ is :

Options

1. 38
2. $19\sqrt{2}$
3. $\sqrt{38}$
4. $2\sqrt{19}$

Ans: **3**

Sol: $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}=\lambda$

$\Rightarrow x=\lambda+3,\ y=2\lambda+4,\ z=2\lambda+5$

Substituting in x + y + z = 17,

$5\lambda+12=17\Rightarrow\lambda=1$

$\therefore$ Point of intersection: (4, 6, 7)

$\therefore$ d = $\sqrt{(4-1)^2+(6-1)^2+(7-9)^2}=\sqrt{9+25+4}=\sqrt{38}$

Q.2 The system of linear equations

$$3x-2y-kz=10$$
$$2x-4y-2z=6$$
$$x+2y-z=5m$$

is inconsistent if :

Options

1. $k=3, m=\frac{4}{5}$
2. $k=3, m\neq\frac{4}{5}$
3. $k\neq3, m\neq\frac{4}{5}$
4. $k\neq3, m\in R$

Ans: **2**

Sol: $3x-2y-kz=10$

$2x-4y-2z=6$

$x+2y-z=5m$

$(1)+(2)\Rightarrow 4x-(k+1)z=10+5m\rightarrow(A)$

$(1)\times2\Rightarrow 6x-4y-2kz=20$

$(1)\times2-(2)\Rightarrow 4x+2(1-k)z=14\rightarrow(B)$

Consider (A) and (B)

$$\frac{4}{4}=\frac{-4}{2(1-k)}\neq\frac{10+5m}{14}$$

$$\Rightarrow 1=\frac{-2}{1-k}\Rightarrow 1-k=-2\Rightarrow k=3$$

and $10+5m\neq 14\Rightarrow 5m\neq 4\Rightarrow m\neq\frac{4}{5}$

Q.3 The function $f(x)=\frac{4x^3-3x^2}{6}-2\sin x+(2x-1)\cos x$:

Options

1. decreases in $\left[\frac{1}{2},\infty\right)$
2. increases in $\left(-\infty,\frac{1}{2}\right]$
3. decreases in $\left(-\infty,\frac{1}{2}\right]$
4. increases in $\left[\frac{1}{2},\infty\right)$

Ans: **4**

Sol: $f(x)=\frac{2}{3}x^3-\frac{1}{2}x^2-2\sin x+(2x-1)\cos x$

$$\frac{dy}{dx}=\frac{2}{3}.3x^2-\frac{1}{2}.2x-2\cos x+2\cos x+(2x-1)(-\sin x)$$

$$=2x^2-x-(2x-1)\sin x$$

$$=x(2x-1)-(2x-1)\sin x$$

$$=(2x-1)(x-\sin x)$$

We know that, As $x > 0$, $x > \sin x$

Hence f(x) is increasing when $x\geq\frac{1}{2}$

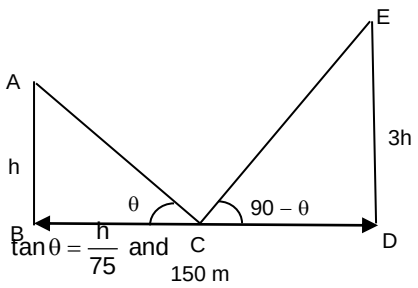
Q.4 Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is :

Options

1. $20\sqrt{3}$
2. $25\sqrt{3}$
3. 25
4. 30

Ans: **2**

Sol:



$\tan\theta = \frac{h}{75}$ and

$\tan(90-\theta) = \frac{3h}{75}$

$\tan\theta = \frac{h}{75}$, $\cot\theta = \frac{3h}{75}$

$\tan\theta.\cot\theta = \frac{3h^2}{75\times 75}$

$\Rightarrow 3h^2 = 75^2$

$\Rightarrow h^2 = \frac{75^2}{3} \Rightarrow h = \frac{75}{\sqrt{3}}$

$= 25\sqrt{3}$ m

Q.5 The value of

$-{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 + \cdots - 15.{}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \cdots + {}^{14}C_{11}$ is :

Options

1. 2^{14}
2. $2^{13} - 14$
3. $2^{16} - 1$
4. $2^{13} - 13$

Ans: **2**

Sol: Consider,

$(1-x)^{15} = 1 - {}^{15}C_1x + {}^{15}C_2x^2 - {}^{15}C_3x^3 + \ldots\ldots - {}^{15}C_{15}x^{15}$ Differentiating,

$15(1-x)^{14}(-1) = -{}^{15}C_1 + 2.{}^{15}C_2x - 3.$

${}^{15}C_3\ x^2 + \ldots\ldots\ldots - 15.{}^{15}C_{15}\ x^{14}$

$x = 1 \Rightarrow -{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 \ldots\ldots\ldots$

$-15{}^{15}C_{15} = 0 \rightarrow (1)$

We know that

${}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots\ldots\ldots$ (up to last possible term) $= 2^{n-1}$

$\Rightarrow {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \ldots\ldots\ldots + {}^{14}C_{11} + {}^{14}C_{13} = 2^{14-1}$

$\Rightarrow {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \ldots\ldots\ldots + {}^{14}C_{11} = 2^{13} - 14 \rightarrow (2)$

$(1)+(2) \Rightarrow$ The required sum $= 2^{13} - 4$

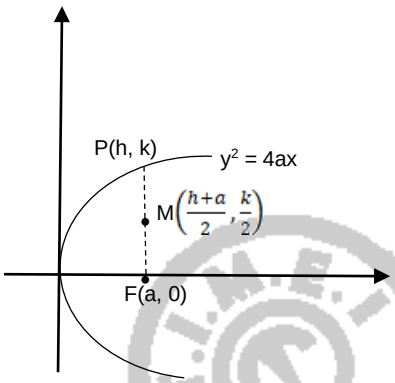
Q.6 The locus of the mid-point of the line segment joining the focus of the parabola $y^2 = 4ax$ to a moving point of the parabola, is another parabola whose directrix is :

Options

1. $x = \frac{a}{2}$
2. $x = 0$
3. $x = a$
4. $x = -\frac{a}{2}$

Ans: 2

Sol:



Let $x = \frac{h+a}{2}$ and $y = \frac{k}{2}$

$\Rightarrow h = 2x - a$ and $k = 2y$

We know that $k^2 = 4ah$

$\Rightarrow 4y^2 = 4a(2x - a)$

$\Rightarrow y^2 = 2a\left(x - \frac{a}{2}\right)$

$\Rightarrow y^2 - 4\left(\frac{a}{2}\right)\left(x - \frac{a}{2}\right)$

$\Rightarrow Y^2 = 4\left(\frac{a}{2}\right)X$

Directrix is $X = \frac{-a}{2}$

$\Rightarrow x - \frac{a}{2} = \frac{-a}{2}$

$\Rightarrow x = 0$

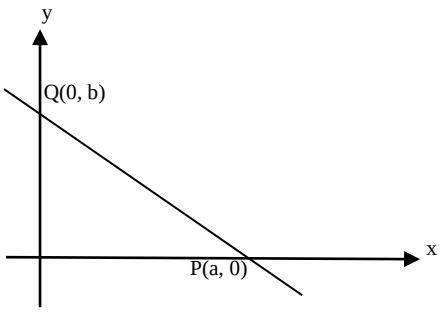
Q.7 A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C are placed at the points (1, 1), (2, 2) and (4, 4) respectively. Then which of these stones is/are on the path of the man ?

Options

1. B only
2. C only
3. All the three
4. A only

Ans: 1

Sol:



$$\frac{\frac{1}{a}+\frac{1}{b}}{2}=\frac{1}{4}$$

$$\Rightarrow \frac{1}{a}+\frac{1}{b}=\frac{1}{2}$$

$$\Rightarrow \frac{2}{a}+\frac{2}{b}=1$$

This means that (2, 2) lies on the line $\frac{x}{a}+\frac{y}{b}=1$

Which is the equation of the line

Q.8

$$\lim_{x\to 0}\frac{\int_0^{x^2}\left(\sin\sqrt{t}\right)dt}{x^3}$$ is equal to :

Options

1. $\frac{1}{15}$
2. $\frac{2}{3}$
3. 0
4. $\frac{3}{2}$

Ans: 2

Sol: $$\lim_{x\to 0}\left[\frac{\int_0^{x^2}\sin\left(\sqrt{t}\right)dt}{x^3}\right]=?$$

Using L' Hospitals rule,

$$\lim_{x\to 0}\left[\frac{\frac{d}{dx}\left\{\int_0^{x^2}\sin\left(\sqrt{t}\right)dt\right\}}{3x^2}\right]$$

$$= \lim_{x\to 0}\left[\frac{\sin\left(\sqrt{x^2}\right).2x-0}{3x^2}\right]$$

$$\left[\because \frac{d}{dx}\left\{\int_{u(x)}^{V(x)} f(t)dt\right\} = f[v(x)]v'(x) - f[u(x)]u'(x)\right]$$

$$= \lim_{x\to 0}\left[\frac{2x\sin x}{3x^2}\right] = \lim_{x\to 0}\left(\frac{2}{3}.\frac{\sin x}{x}\right) = \frac{2}{3}$$

Q.9 An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is :

Options

1. $\frac{1}{32}$
2. $\frac{1}{2}$
3. $\frac{3}{16}$
4. $\frac{5}{16}$

Ans: **2**

Sol: Let the number of trials be 'n'.

Probability of getting odd number 2 times

$$= {}^nC_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{n-2} = {}^nC_2\left(\frac{1}{2}\right)^n$$

Probability of getting even number 3 times

$$= {}^nC_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{n-3} = {}^nC_3\left(\frac{1}{2}\right)^n$$

$$\Rightarrow {}^nC_2\left(\frac{1}{2}\right)^n = {}^nC_3\left(\frac{1}{2}\right)^n \Rightarrow {}^nC_2 = {}^nC_3 \Rightarrow n = 5$$

[$\because$ If ${}^nC_x = {}^nC_y$ then x = y or x + y = n]

The required probability

= P(X = 1) + P(X = 3) + P(X = 5)

$$= {}^5C_1\left(\frac{1}{2}\right)^5 + {}^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_5\left(\frac{1}{2}\right)^5$$

$$= \left({}^5C_1 + {}^5C_3 + {}^5C_5\right)\frac{1}{2^5} = \frac{2^4}{2^5} = \frac{1}{2}$$

$$\left[\because {}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots\ldots\ldots = {}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots\ldots = 2^{n-1}\right]$$

Q.10 If $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = a\sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c$, where c is a constant of integration, then the ordered pair (a, b) is equal to :

Options

1. $(3, 1)$
2. $(1, 3)$
3. $(-1, 3)$
4. $(1, -3)$

Ans: 2

Sol: Let $\sin x + \cos x = t \Rightarrow (\cos x - \sin x)dx = dt$

and $(\sin x + \cos x)^2 = t^2 \Rightarrow 1 + \sin 2x = t^2$

$\Rightarrow \sin 2x = t^2 - 1 \Rightarrow 8 - \sin 2x = 8 - (t^2 - 1)$

$= 9 - t^2$

$\therefore \int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx$

$= \int \frac{dt}{\sqrt{9 - t^2}} = \sin^{-1}\left(\frac{t}{3}\right) + C$

$= \sin^{-1}\left(\frac{\sin x + \cos x}{3}\right) + C \Rightarrow a = 1 \text{ and } b = 3$

Q.11 If the tangent to the curve $y = x^3$ at the point $P(t, t^3)$ meets the curve again at Q, then the ordinate of the point which divides PQ internally in the ratio 1 : 2 is :

Options

1. $2t^3$
2. $-t^3$
3. $-2t^3$
4. 0

Ans: 1

Sol: $y = x^3$

$\frac{dy}{dx} = 3x^2 = 3t^2 \quad (\because x_1 = t)$

Tangent is $y - t^3 = 3t^2(x - t) = 3t^2.x - 3t^3$

$\Rightarrow y = 3t^2x - 2t^3$ which meets the curve

$y = x^3$

Solve:

$y = 3t^2x - 2t^3 \rightarrow (1)$ and $y = x^3 \rightarrow (2)$

$\Rightarrow x^3 = 3t^2x - 2t^3 \Rightarrow x^3 - 3t^2x + 2t^3 = 0$

$\Rightarrow x^3 - t^2x - 2t^2x + 2t^3 = 0$

$\Rightarrow x(x^2 - t^2) - 2t^2(x - t) = 0$

$\Rightarrow (x - t)[x(x + t) - 2t^2] = 0$

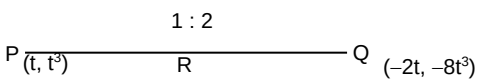
$\Rightarrow (x - t)(x^2 + tx - 2t^2) = 0$

$\Rightarrow (x - t)(x - t)(x + 2t) = 0$

$\Rightarrow x = -2t \ (\because x = t \text{ represents point P})$

$\Rightarrow y = (-2t)^3 = -8t^3$

$\therefore P(t, t^3)$ and $Q(-2t, -8t^3)$



Ordinate of R is $\frac{-8t^3+2t^3}{3} = -2t^3$

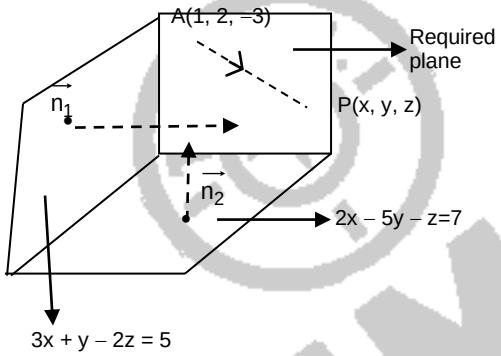
Q.12 The equation of the plane passing through the point (1, 2, −3) and perpendicular to the planes $3x+y-2z=5$ and $2x-5y-z=7$, is :

Options

1. $11x+y+17z+38=0$
2. $6x-5y+2z+10=0$
3. $6x-5y-2z-2=0$
4. $3x-10y-2z+11=0$

Ans: 1

Sol:



Clearly $\vec{n_1} = 3\hat{i}+\hat{j}-2\hat{k}$ and $\vec{n_2} = 2\hat{i}-5\hat{j}-\hat{k}$ are parallel to the required plane

$\therefore \overrightarrow{AP}$, $\vec{n_1}$ and $\vec{n_2}$ are coplanar

$$\Rightarrow \begin{vmatrix} x-1 & y-2 & z+3 \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)(-1-10)-(y-2)(-3+4) + (z+3)(-15-2)=0$$

$$\Rightarrow -11x+11-y+2-17z-51=0$$

$$\Rightarrow -11x-y-17z-38=0$$

$$\Rightarrow 11x+y+17z+38=0$$

Q.13 A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed, is :

Options

1. 1625
2. 1050
3. 575
4. 560

Ans: **1625**

Sol: $\boxed{6I, 8F} \Rightarrow \boxed{2I, 4F}$ or $\boxed{3I, 6F}$ or $\boxed{4I, 8F}$

No: of selection ${}^6C_2.{}^8C_4 + {}^6C_3.{}^8C_6 + {}^6C_4.{}^8C_8 = 1625$

Q.14 The statement among the following that is a tautology is :

Options

1. $A \wedge (A \vee B)$
2. $[A \wedge (A \rightarrow B)] \rightarrow B$
3. $A \vee (A \wedge B)$
4. $B \rightarrow [A \wedge (A \rightarrow B)]$

Ans: **2**

Sol: Consider Option (2)

A	B	A→B	A^(A→B)	{A^(A→B)}→B
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Q.15 Let p and q be two positive numbers such that $p + q = 2$ and $p^4 + q^4 = 272$. Then p and q are roots of the equation :

Options

1. $x^2 - 2x + 16 = 0$
2. $x^2 - 2x + 2 = 0$
3. $x^2 - 2x + 8 = 0$
4. $x^2 - 2x + 136 = 0$

Ans: **1**

Sol: $p + q = 2$ and $pq = k$ (assume)

$p^2 + q^2 = (p+q)^2 - 2pq = 4 - 2k$

$p^4 + q^4 = (p^2 + q^2)^2 - 2p^2q^2 = (4-2k)^2 - 2k^2$

$= 16 - 16k + 4k^2 - 2k^2$

$= 2k^2 - 16k + 16 = 272 (\text{given})$

$\Rightarrow k^2 - 8k + 8 = 136 \Rightarrow k^2 - 8k - 128 = 0$

$\Rightarrow (k-16)(k+8) = 0 \Rightarrow k = 16 \text{ or } -8$

$\therefore pq = 16 (\because p, q > 0)$ and $p + q = 2$

$\therefore$ Ans is $x^2 - 2x + 16 = 0$

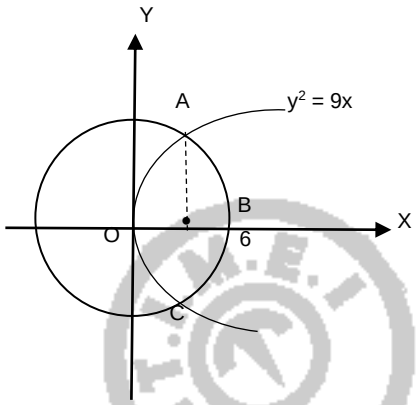
Q.16 The area (in sq. units) of the part of the circle $x^2+y^2=36$, which is outside the parabola $y^2=9x$, is :

Options

1. $12\pi + 3\sqrt{3}$
2. $24\pi + 3\sqrt{3}$
3. $24\pi - 3\sqrt{3}$
4. $12\pi - 3\sqrt{3}$

Ans: 3

Sol: $x^2+y^2=36,\ y^2=9x$



The required area =Area of the circle − Area of OABC

$=\pi(6^2)-(2\times \text{Area of OAB})$

To find A

Solve: $x^2+y^2=36$ and $y^2=9x$

$\Rightarrow x^2+9x=36 \Rightarrow x^2+9x-36=0$

$\Rightarrow (x+12)(x-3)=0 \Rightarrow x=3\ (\because x>0)$

Area of OAB = $\int_0^3 3\sqrt{x}\,dx+\int_3^6 \sqrt{36-x^2}\,dx$

$=3.\frac{2}{3}\left[x\sqrt{x}\right]_0^3+\left[\frac{x}{2}\sqrt{36-x^2}+\frac{36}{2}.\sin^{-1}\left(\frac{x}{6}\right)\right]_3^6$

$=2\left(3\sqrt{3}\right)+3(0)+\frac{36}{2}.\frac{\pi}{2}-\frac{3}{2}\sqrt{27}-18.\frac{\pi}{6}$

$=6\sqrt{3}+9\pi-\frac{9\sqrt{3}}{2}-3\pi=6\pi+\frac{3\sqrt{3}}{2}$

$\therefore$ Required area = $36\pi-2\left(6\pi+\frac{3\sqrt{3}}{2}\right)$

$=36\pi-12\pi-3\sqrt{3}=24\pi-3\sqrt{3}$

Q.17 The population $P = P(t)$ at time 't' of a certain species follows the differential equation $\frac{dP}{dt} = 0.5P - 450$. If $P(0) = 850$, then the time at which population becomes zero is :

Options

1. $\frac{1}{2}\log_e 18$
2. $2\log_e 18$
3. $\log_e 9$
4. $\log_e 18$

Ans: 2

Sol: $\frac{dp}{dt} = 0.5p - 450 \Rightarrow \int \frac{dp}{0.5p - 450} = \int dt$

$\Rightarrow \frac{\log(0.5p - 450)}{0.5} = t + C$

$\Rightarrow \log\left(\frac{p}{2} - 450\right) = \frac{1}{2}t + 2C$

$\Rightarrow \frac{p}{2} - 450 = e^{\left(\frac{t}{2} + 2C\right)} = e^{2C}.e^{t/2} = C'e^{t/2}$

$\Rightarrow \frac{p}{2} = C; e^{t/2} + 450 \Rightarrow p = k.e^{t/2} + 900$

$p(0) = 850 \Rightarrow 850 = k + 900 \Rightarrow k = -50$

$\therefore p = -50e^{t/2} + 900 = 0 \Rightarrow e^{t/2} = 18$

$\Rightarrow \frac{t}{2} = \log_e 18$

$\Rightarrow t = 2\log_e 18$

Q.18

Let $f : R \to R$ be defined as $f(x) = 2x - 1$ and $g : R - \{1\} \to R$ be defined as $g(x) = \frac{x - \frac{1}{2}}{x - 1}$.

Then the composition function $f(g(x))$ is :

Options

1. both one-one and onto
2. onto but not one-one
3. neither one-one nor onto
4. one-one but not onto

Ans: 4

Sol: $f(x) = 2x - 1,\ g(x) = \frac{x - \frac{1}{2}}{x - 1}$

$f[g(x)] = 2\left(\frac{x - \frac{1}{2}}{x - 1}\right) - 1$

$= \left(\frac{2x - 1}{x - 1}\right) - 1 = \frac{2x - 1 - x + 1}{x - 1}$

$= \frac{x}{x-1}$

Let $fog(x_1) = fog(x_2)$

$\Rightarrow \frac{x_1}{x_1-1} = \frac{x_2}{x_2-1}$

$\Rightarrow x_1x_2 - x_1 = x_1x_2 - x_2 \Rightarrow -x_1 = -x_2 \Rightarrow x_1 = x_2$

$\therefore$ fog is one-one

$fog = y = \frac{x}{x-1} \Rightarrow xy - y = x$

$xy - x = y \Rightarrow x(y-1) = y$

$\Rightarrow x = \frac{y}{y-1}$

Clearly y = 1 does not have a pre image

Hence $fog(x)$ is not onto

Q.19 If $e^{\left(\cos^2 x+\cos^4 x+\cos^6 x+\ldots\right)\log_e 2}$ satisfies the equation $t^2-9t+8=0$, then the value of $\frac{2\sin x}{\sin x+\sqrt{3}\cos x}\left(0<x<\frac{\pi}{2}\right)$ is :

Options

1. $2\sqrt{3}$
2. $\frac{1}{2}$
3. $\sqrt{3}$
4. $\frac{3}{2}$

Ans: **2**

Sol: The given expression

$= \left[e^{\log 2}\right]^{\left(\cos^2 x+\cos^4 x+\cos^6 x+\ldots\ldots\text{to }\infty\right)}$

$= 2^{\left(\frac{\cos^2 x}{1-\cos^2 x}\right)}$ $\left[\because a+ar+ar^2+\ldots\ldots\text{ to }\infty = \frac{a}{1-r},\ -1<r<1\right]$

$= 2^{\cot^2 x}$; $t^2-9t+8=0 \Rightarrow (t-8)(t-1)=0$

$\Rightarrow t = 1$ or 8

$\Rightarrow 2^{\cot^2 x} = 2^0$ or $2^{\cot x} = 2^3 \Rightarrow \cot x = \sqrt{3}$ $\left(\because 0<x<\frac{\pi}{2}\right)$

$\Rightarrow x = 30°$

$\therefore \frac{2\sin x}{\sin x+\sqrt{3}\cos x} = \frac{2\left(1/2\right)}{\frac{1}{2}+\sqrt{3}.\frac{\sqrt{3}}{2}} = \frac{1}{2}$

Q.20 If $f: R \to R$ is a function defined by $f(x) = [x-1]\cos\left(\frac{2x-1}{2}\right)\pi$, where $[\cdot]$ denotes the greatest integer function, then f is :

Options

1. continuous only at $x = 1$
2. discontinuous at all integral values of x except at $x = 1$
3. continuous for every real x
4. discontinuous only at $x = 1$

Ans: 3

Sol: $f(x) = [x-1]\cos\left(\frac{2x-1}{2}\right)\pi$

Let us check the continuity ar x = n, $n \in Z$

$\lim\limits_{x \to n^+} f(x) = \lim\limits_{h \to 0} f(n+h)$

$= \lim\limits_{h \to 0}[(n+h)-1]\cos\left\{\frac{2(n+h)-1}{2}\right\}\pi$

$= (n-1)\cos\frac{(2n-1)\pi}{2} = 0$

$\left[\because \cos\left(\text{odd multiples of } \frac{\pi}{2}\right) = 0\right]$

$\lim\limits_{x \to n^-} f(x) = \lim\limits_{h \to 0} f(n-h)$

$= \lim\limits_{h \to 0}[(n-h)-1]\cos\left\{\frac{2(n-h)-1}{2}\right\}\pi$

$= (n-2)\times\cos\left(\frac{2n-1}{2}\right)\pi = 0$ and $f(n) = 0$

$\therefore f(x)$ is continuous for all $x \in Z$

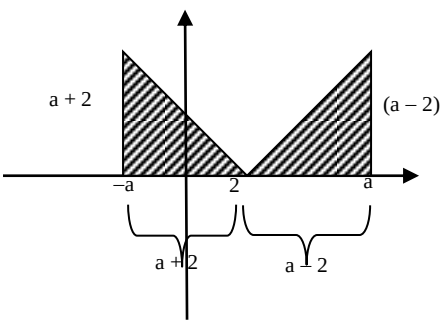
If $x \notin Z$, $[x-1]$ and $\cos\left(\frac{2x-1}{2}\right)\pi$ are continuous. Hence there is also continuous.

SECTION B

Q.1 If $\int_{-a}^{a} (|x| + |x-2|)dx = 22$, $(a > 2)$ and $[x]$ denotes the greatest integer $\le x$, then $\int_{a}^{-a} (x + [x])\,dx$ is equal to _____.

Ans: 3

Sol: $2\int_0^a x\,dx + \int_{-a}^{a} |x-2|\,dx = a^2 + \int_{-a}^{a} |x-2|\,dx$



From the graph,

$$\int_{-a}^{a} |x-2|\,dx = \frac{1}{2}(a-2)^2 + \frac{1}{2}(a+2)^2$$

$$= \frac{1}{2}\left(2a^2+8\right) = a^2+4$$

$$\therefore a^2 + \left(a^2+4\right) = 22 \Rightarrow 2a^2 = 18 \Rightarrow a^2 = 9 \Rightarrow a = 3$$

$$\int_{a}^{-a} (x+[x])dx = -\int_{-a}^{a} x\,dx - \int_{-a}^{a} [x]dx = -\int_{-a}^{a} [x]dx$$

$$= \int_{-3}^{3} [x]dx = -\left\{\int_{-3}^{-2} [x]dx + \int_{-2}^{-1} [x]dx + \int_{-1}^{0} [x]dx\right.$$

$$\left. + \int_{0}^{1} [x]dx + \int_{1}^{2} [x]dx + \int_{2}^{3} [x]dx\right\}$$

$$= -\left\{\int_{-3}^{-2} -3dx + \int_{-2}^{-1} -2dx + \int_{-1}^{0} -1dx + \int_{0}^{1} 0dx\right.$$

$$\left. + \int_{1}^{2} 1dx + \int_{2}^{3} 2dx\right\}$$

$$= \{-3(1) + -2(1) + -1(1) + 0 + 1(1) + 2(1)\}$$

$$= -\{-3\} = 3$$

Q.2 Let M be any 3×3 matrix with entries from the set {0, 1, 2}. The maximum number of such matrices, for which the sum of diagonal elements of M^TM is seven, is _____.

Ans: **540**

Sol: Let $M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \Rightarrow M^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$

$$M^T.M = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$= \begin{bmatrix} a^2+d^2+g^2 & ---- & ---- \\ ---- & b^2+e^2+h^2 & ---- \\ ---- & ---- & c^2+f^2+i^2 \end{bmatrix}$$

Sum of the diagonal elements

$= a^2+b^2+c^2+d^2+e^2+f^2+g^2+h^2+i^2 = 7$

where $a^2, b^2, \ldots\ldots i^2 \in \{0, 1, 4\}$

Case:1 (excluding 4)

1, 1, 1, 1, 1, 1, 1, 0, 0 and its arrangements

$$\Rightarrow \frac{9!}{7!\,2!} = \frac{9\times8}{2\times1} = 36$$

Case: 2 (including 4)

4, 1, 1, 1, 0, 0, 0, 0, 0 and its arrangements

$$\Rightarrow \frac{9!}{3!\times5!} = \frac{9\times8\times7\times6}{1\times2\times3} = 504$$

Ans: 504 + 36 = 540

Q.3 If the least and the largest real values of α, for which the equation $z+\alpha|z-1|+2i=0$ ($z\in C$ and $i=\sqrt{-1}$) has a solution, are p and q respectively; then $4(p^2+q^2)$ is equal to _____.

Ans: **10**

Sol: $Z+\alpha\,|\,z-1|+2i=0$

Let $Z = x + iy \Rightarrow (x + iy) + \alpha|(x - 1) + iy| + 2i = 0$

$\Rightarrow (x+iy)+\alpha\sqrt{(x-1)^2+y^2}+2i=0$

$\Rightarrow \left[x+\alpha\sqrt{(x-1)^2+y^2}\right]+i(y+2)=0+i0$

$\Rightarrow y = -2$ and

$x+\alpha\sqrt{(x-1)^2+4}=0 \Rightarrow \alpha\sqrt{(x-1)^2+4}=-x$

$\alpha^2 [x-1]^2 + 4] = x^2$

$\alpha^2 (x^2 - 2x + 5) = x^2$

$(\alpha^2 - 1)x^2 - 2\alpha^2x + 5\alpha^2 = 0$

Since $x\in R$,

$(-2\alpha^2)^2 - 4(\alpha^2 - 1)(5\alpha^2) \geq 0$

$4\alpha^4 - 20\alpha^4 + 20\alpha^2 \geq 0$

$-16\alpha^4 + 20\alpha^2 \geq 0$

$\Rightarrow 5 - 4\alpha^2 \geq 0$

$\Rightarrow 4\alpha^2 - 5 \leq 0$

$\alpha^2 \leq \frac{5}{4} \Rightarrow \frac{\sqrt{5}}{2} \leq \alpha \leq \frac{\sqrt{5}}{2}$

$\alpha_{min} = \frac{-\sqrt{5}}{2}$

$\alpha_{max} = \frac{\sqrt{5}}{2}$

$4[p^2 + q^2]$

$= 4\left(\frac{5}{4}+\frac{5}{4}\right) = 10$

Q.4 Let $P=\begin{bmatrix}3 & -1 & -2\\ 2 & 0 & \alpha\\ 3 & -5 & 0\end{bmatrix}$, where $\alpha\in R$. Suppose $Q=[q_{ij}]$ is a matrix satisfying $PQ=kI_3$ for some non-zero $k\in R$. If $q_{23}=-\frac{k}{8}$ and $|Q|=\frac{k^2}{2}$, then α^2+k^2 is equal to _____.

Ans: **17**

Sol: $PQ = kI \Rightarrow Q = k.P^{-1}$

$P=\begin{bmatrix}3 & -1 & -2\\ 2 & 0 & \alpha\\ 3 & -5 & 0\end{bmatrix}$

$P^{-1}=\frac{1}{|P|}.\text{adj}P=\frac{1}{3(5\alpha)+1(-3\alpha)+20}$

$\begin{bmatrix}- & - & -\\ - & - & -(3\alpha+4)\\ - & - & -\end{bmatrix}$

$Q=kP^{-1}=\frac{k}{(12\alpha+20)}\begin{bmatrix}- & - & -\\ - & - & -(3\alpha+4)\\ - & - & -\end{bmatrix}$

$\Rightarrow q_{23}=\frac{-k(3\alpha+4)}{(12\alpha+20)}=\frac{-k}{8}\Rightarrow\frac{3\alpha+4}{12\alpha+20}=\frac{1}{8}$

$24\alpha+32=12\alpha+20\Rightarrow 12\alpha=-12\Rightarrow\alpha=-1$

$\therefore |P| = 12\alpha + 20 = 12(-1) + 20 = 8$

$|PQ| = |KI| \Rightarrow |P| . |Q| = K^3 |I|$

$8 \times \frac{k^2}{2} = k^3 \Rightarrow 4k^2 = k^3 \Rightarrow k = 4 (\because k \neq 0)$

$\alpha^2 + k^2 = 1 + 16 = 17$

Q.5 The minimum value of α for which the equation $\frac{4}{\sin x} + \frac{1}{1 - \sin x} = \alpha$ has at least one solution in $\left(0, \frac{\pi}{2}\right)$ is _____.

Ans: **9**

Sol: $\alpha = \frac{4}{\sin x} + \frac{1}{1 - \sin x}$

$\frac{d\alpha}{dx} = \frac{-4 \cos x}{\sin^2 x} + \frac{-1}{(1 - \sin x)^2} . (-\cos x)$

$= \frac{-4 \cos x}{\sin^2 x} + \frac{\cos x}{(1 - \sin x)^2} = 0$

$\Rightarrow \frac{4 \cos x}{\sin^2 x} = \frac{\cos x}{(1 - \sin x)^2} \Rightarrow \cos x = 0$ or

$\frac{4}{\sin^2 x} = \frac{1}{(1 - \sin x)^2} \Rightarrow 4(1 - t)^2 = t^2$ when t=sinx

$\Rightarrow t = 2$ or $t = \frac{2}{3}$ where $-1 \leq t \leq 1$

$\therefore t = \frac{2}{3}$ or $\sin x = \frac{2}{3}$

If $\sin x = \frac{2}{3}$, $\alpha = \frac{4}{2/3} + \frac{1}{\left(1 - \frac{2}{3}\right)} = 6 + 3 = 9$

If cosx = 0 then sinx = 1 or sinx = –1

But sinx = 1 is not possible, for which α is not defined.

Sinx = –1 is not possible since $x \in \left(0, \frac{\pi}{2}\right)$

As $x \to 0$ or $x \to 1$, $\alpha \to \infty$

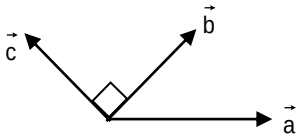
Point of local minima is at $\sin x = \frac{2}{3}$

$\therefore \alpha_{min} = 9$

Q.6 Let three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ be such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$, $\vec{a} \cdot \vec{c} = 7$ and $\vec{b}$ is perpendicular to $\vec{c}$, where $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{k}$, then the value of $2\left|\vec{a} + \vec{b} + \vec{c}\right|^2$ is _____.

Ans: **75**

Sol:



$\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar

$\vec{c}$ is $\perp^r$ to $\vec{b}$ and $\vec{c}$ is $\perp^r$ to $\vec{a} \times \vec{b}$

$$\Rightarrow \vec{c} = \lambda.\vec{b} \times \left(\vec{a} \times \vec{b}\right)$$

$$= \lambda\left[(\vec{b}.\vec{b})\vec{a} - (\vec{b}.\vec{a})\vec{b}\right] = \lambda\left[|\vec{b}|^2\,\vec{a} - (\vec{a}.\vec{b})\vec{b}\right]$$

$$= \lambda\left[5\left(-\hat{i}+\hat{j}+\hat{k}\right) - (-2+1)\left(2\hat{i}+\hat{k}\right)\right]$$

$$= \lambda\left(-5\hat{i}+5\hat{j}+5\hat{k}+2\hat{i}+\hat{k}\right) = \lambda\left(-3\hat{i}+5\hat{j}+6\hat{k}\right)$$

$$\vec{a}\cdot\vec{c} = \lambda(3+5+6) = 7 \Rightarrow \lambda = \frac{1}{2}$$

$$\therefore \vec{a}+\vec{b}+\vec{c} = -\hat{i}+\hat{j}+\hat{k}+2\hat{i}+\hat{k}+\frac{-3}{2}\hat{i}+\frac{5}{2}\hat{j}+3\hat{k}$$

$$= \frac{-1}{2}\hat{i}+\frac{7}{2}\hat{j}+5\hat{k}$$

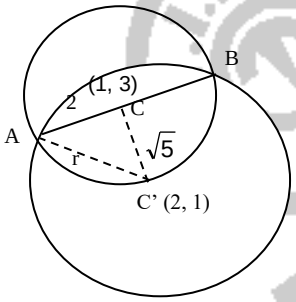
$$2|\vec{a}+\vec{b}+\vec{c}|^2 = 2\left(\frac{1}{4}+\frac{49}{4}+25\right) = 75$$

Q.7 If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle 'C', whose center is at (2, 1), then its radius is _____.

Ans: **3**

Sol: $x^2 + y^2 - 2x - 6y + 6 = 0$

Centre: (1, 3) and r = 2



$$CC' = \sqrt{(1-2)^2 + (3-1)^2} = \sqrt{1+4} = \sqrt{5}$$

$$r = \sqrt{4+5} = 3$$

Q.8 Let $A = \{n \in N : n \text{ is a 3-digit number}\}$

$B = \{9k+2 : k \in N\}$

and $C = \{9k+l : k \in N\}$ for some l $(0 < l < 9)$

If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400, then l is equal to _____.

Ans: **5**

Sol: $B = \{101, 110, 119, \ldots\ldots\ldots\ldots 992\}$

$$n(B) = \frac{992-101}{9} + 1 = 100$$

$$S_B = \frac{100}{2}[101+992] = 54650$$

$C = \{99+\ell, 108+\ell, \ldots\ldots\ldots\ldots 990+\ell\}$ 40 cm

$$n(C) = \frac{990-99}{9} + 1 = 100$$

$$S_C = \frac{100}{2}[99+\ell+990+\ell] = 50(1089+2\ell)$$

$$S_B + S_C = 54650 + 50(1089+2\ell) = 109600$$

$$\Rightarrow \ell = 5$$

Q.9 Let B_i $(i = 1, 2, 3)$ be three independent events in a sample space. The probability that only B_1 occur is α, only B_2 occurs is β and only B_3 occurs is γ. Let p be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha - 2\beta)\, p = \alpha\beta$ and $(\beta - 3\gamma)\, p = 2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0, 1)). Then $\frac{P(B_1)}{P(B_3)}$ is equal to _____.

Ans: **6**

Sol: Let $P(B_1) = x$, $P(B_2) = y$ and $P(B_3) = z$

$\alpha = x(1 - y)(1 - z)$

$\beta = y(1 - x)(1 - z)$

$\gamma = z(1 - x)(1 - y)$

$P = (1 - x)(1 - y)(1 - z)$

$$(\alpha - 2\beta)P = \alpha\beta$$

$$P = \frac{xy(1-z)^2(1-x)(1-y)}{(1-z)[x(1-y)-2y(1-x)]} = (1-x)(1-y)(1-z)$$

$$\Rightarrow \frac{xy(1-x)(1-y)(1-z)}{(x-xy-2y+2xy)} = (1-x)(1-y)(1-z)$$

$$\Rightarrow xy = xy + x - 2y \Rightarrow x = 2y \rightarrow (1)$$

$$(P - 3\gamma)P = 2\beta\gamma$$

$$\Rightarrow P = \frac{2\beta\gamma}{\beta - 3\gamma} = \frac{2y(1-x)(1-z)z(1-x)(1-y)}{y(1-x)(1-z)-3z(1-x)(1-y)}$$

$$\Rightarrow (1-x)(1-y)(1-z) = \frac{2yz(1-x)^2(1-y)(1-z)}{(1-x)[y(1-z)-3z(1-y)]}$$

$$\Rightarrow 1 = \frac{2yz}{(y - yz - 3z + 3zy)} = \frac{2yz}{y - 3z + 2yz}$$

$$\Rightarrow y = 3z \rightarrow (2)$$

$$(1)\,\&\,(2) \Rightarrow x = 2(3z) \Rightarrow x = 6z \Rightarrow \frac{x}{z} = 6$$

Q.10 $\lim\limits_{n\to\infty} \tan\left\{\sum\limits_{r=1}^{n} \tan^{-1}\left(\frac{1}{1 + r + r^2}\right)\right\}$ is equal to _____.

Ans: **1**

Sol: $\tan^{-1}\left(\frac{1}{1+r+r^2}\right) = \tan^{-1}\left[\frac{(r+1)-r}{1+(r+1)r}\right] = \tan^{-1}(r+1) - \tan^{-1} r$ where r = 1, 2, 3,…….n

1^{st} term = $\tan^{-1} 2 - \tan^{-1} 1$

2^{nd} term = $\tan^{-1} 3 - \tan^{-1} 2$

3^{rd} term = $\tan^{-1} 4 - \tan^{-1} 3$

……

……

n^{th} term = $\tan^{-1}(n+1) - \tan^{-1} n$

$S_n = \tan^{-1}(n+1) - \tan^{-1} 1$

As $n \to \infty$, $S = \tan^{-1}\infty - \tan^{-1} 1 = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

$$\therefore \tan\left[\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right]_{(n\to\infty)} = \tan\frac{\pi}{4} = 1\,p$$