SOLUTIONS & ANSWERS FOR JEE MAINS-2021
25th February Shift 2
[PHYSICS, CHEMISTRY & MATHEMATICS]

PART – A – PHYSICS

SECTION A

Q.1 Match List I with List II.

	List I		List II
(a)	Rectifier	(i)	Used either for stepping up or stepping down the a.c. voltage
(b)	Stabilizer	(ii)	Used to convert a.c. voltage into d.c. voltage
(c)	Transformer	(iii)	Used to remove any ripple in the rectified output voltage
(d)	Filter	(iv)	Used for constant output voltage even when the input voltage or load current change

Choose the correct answer from the options given below :

Options

1. (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)
2. (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)
3. (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)
4. (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

Ans: 3

Sol: Option (3)

Q.2 An electron with kinetic energy K_1 enters between parallel plates of a capacitor at an angle 'α' with the plates. It leaves the plates at angle 'β' with kinetic energy K_2. Then the ratio of kinetic energies $K_1 : K_2$ will be :

Options

1. $\frac{\sin^2\beta}{\cos^2\alpha}$
2. $\frac{\cos^2\beta}{\cos^2\alpha}$
3. $\frac{\cos\beta}{\cos\alpha}$
4. $\frac{\cos\beta}{\sin\alpha}$

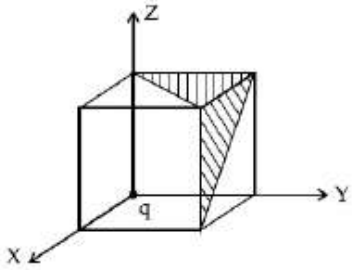
Ans: 2

Sol: Velocity perpendicular to electric field will remain the same

$$V_1 \cos\alpha = V_2 \cos\beta$$

$$\frac{K_1}{K_2} = \frac{\frac{1}{2}mV_1^2}{\frac{1}{2}mV_2^2} = \frac{\cos^2\beta}{\cos^2\alpha}$$

Q.3 A charge 'q' is placed at one corner of a cube as shown in figure. The flux of electrostatic field $\vec{E}$ through the shaded area is :

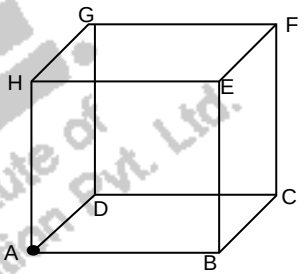


Options

1. $\frac{q}{48\varepsilon_0}$
2. $\frac{q}{8\varepsilon_0}$
3. $\frac{q}{24\varepsilon_0}$
4. $\frac{q}{4\varepsilon_0}$

Ans: 3

Sol: Flux through the cube = $\frac{q}{8\varepsilon_0}$



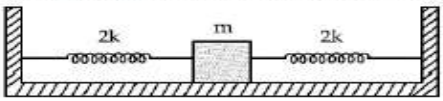
Electric field grazes the surface rest three surfaces has same orientation w.r.t change 'q'.

$$\phi_{EFGH} = \phi_{DCFG} = \phi_{EBCF} = \frac{q}{24\varepsilon_0}$$

$$\phi_{shaded} = \phi_{ECB} + \phi_{GEF} = \phi_{EFGH} = \frac{q}{24\varepsilon_0}$$

(As they are in same orientation w.r.t charge q)

Q.4 Two identical springs of spring constant '2k' are attached to a block of mass m and to fixed support (see figure). When the mass is displaced from equilibrium position on either side, it executes simple harmonic motion. The time period of oscillations of this system is :



Options

1. $2\pi\sqrt{\frac{m}{2k}}$
2. $\pi\sqrt{\frac{m}{k}}$
3. $\pi\sqrt{\frac{m}{2k}}$
4. $2\pi\sqrt{\frac{m}{k}}$

Ans: 2

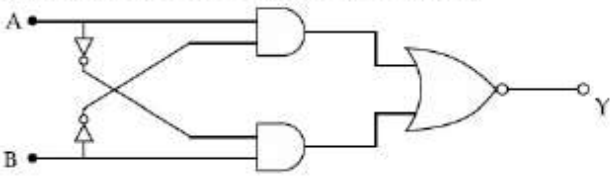
Sol: In parallel

$K_{eq} = K_1 + K_2 = 4\,K$

$$T = 2\pi\sqrt{\frac{m}{4K}}$$

$$T = \pi\sqrt{\frac{m}{K}}$$

Q.5 The truth table for the following logic circuit is :



Options

1.

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

2.

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

3.

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

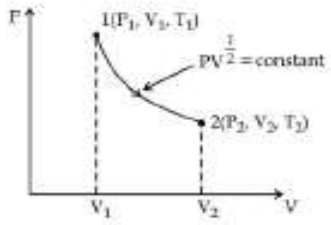
4.

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

Ans: 3

Sol: Option (3)

Q.6 Thermodynamic process is shown below on a P-V diagram for one mole of an ideal gas. If $V_2 = 2V_1$ then the ratio of temperature T_2/T_1 is :



Options

1. $\sqrt{2}$
2. $\frac{1}{2}$
3. $\frac{1}{\sqrt{2}}$
4. 2

Ans: 1

Sol: $PV^{1/2} = C$

$$\frac{nRT}{V} \times V^{1/2} = C$$

$$T \propto \sqrt{V}$$

$$\frac{T_2}{T_1} = \sqrt{\frac{V_2}{V_1}} = \sqrt{\frac{2V_1}{V_1}} = \sqrt{2}$$

Q.7 An electron of mass m_e and a proton of mass $m_p = 1836\, m_e$ are moving with the same speed. The ratio of their de Broglie wavelength $\frac{\lambda_{electron}}{\lambda_{proton}}$ will be :

Options

1. 1
2. 918
3. 1836
4. $\frac{1}{1836}$

Ans: 3

Sol: $\lambda = \frac{h}{mV} \Rightarrow \lambda \frac{1}{m}$

$$\frac{\lambda_e}{\lambda_p} = \frac{m_p}{m_e} = 1836$$

Q.8 For extrinsic semiconductors; when doping level is increased;

Options

1. Fermi-level of both p-type and n-type semiconductors will go upward for $T > T_F$ K and downward for $T < T_F$ K, where T_F is Fermi temperature.
2. Fermi-level of p and n-type semiconductors will not be affected.
3. Fermi-level of p-type semiconductors will go downward and Fermi-level of n-type semiconductor will go upward.
4. Fermi-level of p-type semiconductor will go upward and Fermi-level of n-type semiconductors will go downward.

Ans: 3

Sol: Option (3)

Q.9 The stopping potential for electrons emitted from a photosensitive surface illuminated by light of wavelength 491 nm is 0.710 V. When the incident wavelength is changed to a new value, the stopping potential is 1.43 V. The new wavelength is :

Options

1. 309 nm
2. 382 nm
3. 400 nm
4. 329 nm

Ans: 2

Sol: $e\ V_{s_1} = KE_1 = \frac{h_c}{\lambda_1} - \phi$

$$\phi = \frac{1240}{491} - 0.71$$

$$\phi = 2.52 - 0.71 = 1.81$$

$$eV_{s_2} = KE_2 = \frac{h_c}{\lambda_2} - \phi$$

$$\frac{h_c}{\lambda_2} = 1.43 + 1.81 = 3.24$$

$$\lambda_2 = \frac{1240}{3.24} = 382.72$$

$$\approx 382\,nm$$

Q.10 If e is the electronic charge, c is the speed of light in free space and h is Planck's constant, the quantity $\frac{1}{4\pi\varepsilon_0}\frac{|e|^2}{hc}$ has dimensions of :

Options

1. $[M^0 L^0 T^0]$
2. $[L\ C^{-1}]$
3. $[M\ L\ T^{-1}]$
4. $[M\ L\ T^0]$

Ans: 1

Sol: $\frac{Fr^2}{h_c} = \frac{Fr^2}{E\lambda} = M^\circ L^\circ T^\circ$

Q.11 In a ferromagnetic material, below the curie temperature, a domain is defined as :

Options

1. a macroscopic region with randomly oriented magnetic dipoles.
2. a macroscopic region with saturation magnetization.
3. a macroscopic region with zero magnetization.
4. a macroscopic region with consecutive magnetic dipoles oriented in opposite direction.

Ans: 2

Sol: Option (2)

Q.12 Consider the diffraction pattern obtained from the sunlight incident on a pinhole of diameter 0.1 μm. If the diameter of the pinhole is slightly increased, it will affect the diffraction pattern such that :

Options

1. its size decreases, but intensity increases
2. its size increases, but intensity decreases
3. its size increases, and intensity increases
4. its size decreases, and intensity decreases

Ans: 1

Sol: $\sin\theta = \dfrac{1.22\lambda}{D}$

If D is increased $\Rightarrow$ sin θ decreased

$\therefore$ size of circular fringe will decrease.

Intensity will increase.

Q.13 $Y = A\sin(\omega t + \phi_0)$ is the time-displacement equation of a SHM. At t = 0 the displacement of the particle is $Y = \frac{A}{2}$ and it is moving along negative x-direction. Then the initial phase angle ϕ_0 will be :

Options

1. $\dfrac{2\pi}{3}$
2. $\dfrac{\pi}{6}$
3. $\dfrac{\pi}{3}$
4. $\dfrac{5\pi}{6}$

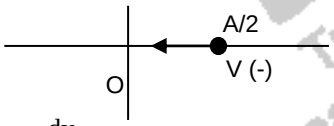
Ans: 4

Sol: $y = A\sin(\omega t + \phi_0)$

t = 0, $y = \dfrac{A}{2}$

$\dfrac{1}{2} = \sin\phi_0$

$\phi_0 = \pi/6,\ 5\pi/6$



$V = \dfrac{dy}{dt} = A\omega\cos(\omega t + \phi_0)$

$t = 0,\ V = A\omega\cos\phi_0$

$\phi_0 = \pi/6$ for V(+)

$\phi_0 = 5\pi/6$ for V (-)

$\phi_0 = 5\pi/6$

Q.14 The wavelength of the photon emitted by a hydrogen atom when an electron makes a transition from n = 2 to n = 1 state is :

Options

1. 121.8 nm
2. 913.3 nm
3. 194.8 nm
4. 490.7 nm

Ans: 1

Sol: $\frac{1}{\lambda} R\left[\frac{1}{n_{f^2}} - \frac{1}{n_{i^2}}\right]$

$\frac{1}{\lambda} R\left[\frac{1}{1} - \frac{1}{4}\right] \Rightarrow \frac{1}{\lambda} = \frac{3R}{4}$

$\Rightarrow \lambda = \frac{4}{3R} = 121.8 \text{ nm}$

Q.15 A stone is dropped from the top of a building. When it crosses a point 5 m below the top, another stone starts to fall from a point 25 m below the top. Both stones reach the bottom of building simultaneously. The height of the building is :

Options
1. 45 m
2. 50 m
3. 25 m
4. 35 m

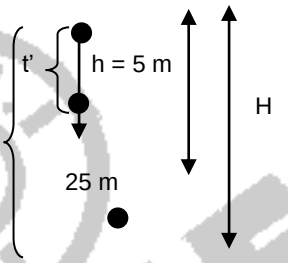
Ans: 1

Sol: For first particle

$H = \frac{1}{2} g t^2$

$5 = \frac{1}{2} g t'^2$

$t' = 5s$



For second particle

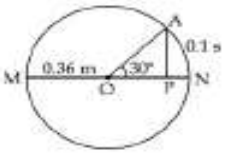
$H - 25 = \frac{1}{2} g (t-1)^2$

$\frac{1}{2} g t^2 - 25 = \frac{1}{2} g t^2 + \frac{g}{2} - gt$

t = 3 second

$H = \frac{1}{2} \times 10 \times 9 = 45 \text{ m}$

Q.16 The point A moves with a uniform speed along the circumference of a circle of radius 0.36 m and covers 30° in 0.1 s. The perpendicular projection 'P' from 'A' on the diameter MN represents the simple harmonic motion of 'P'. The restoration force per unit mass when P touches M will be :



Options
1. 0.49 N
2. 9.87 N
3. 100 N
4. 50 N

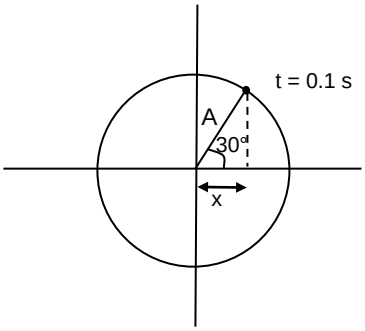
Ans: 2

Sol: $\omega = \frac{\theta}{t} = \frac{\lambda/6}{0.1} = \frac{5\pi}{3}$

For SHM

$\frac{F}{m} = -\omega^2 x$

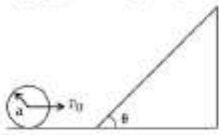
When P touches m



x = A

So $\frac{F}{m} = \left(\frac{5\pi}{3}\right)^2 \times 0.36$ = 9.87 N

Q.17 A sphere of radius 'a' and mass 'm' rolls along a horizontal plane with constant speed v_0. It encounters an inclined plane at angle θ and climbs upward. Assuming that it rolls without slipping, how far up the sphere will travel ?

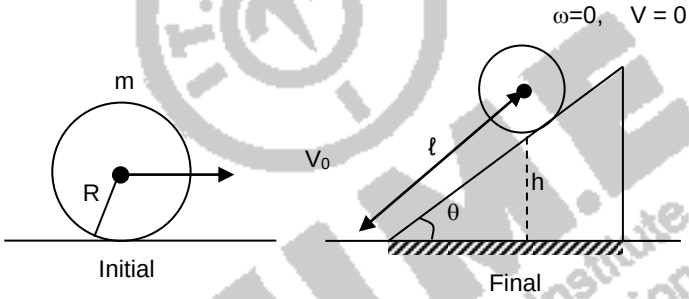


Options

1. $\frac{2}{5}\frac{v_0^2}{g\sin\theta}$
2. $\frac{v_0^2}{2g\sin\theta}$
3. $\frac{v_0^2}{5g\sin\theta}$
4. $\frac{10v_0^2}{7g\sin\theta}$

Ans:

Sol:



$K_i + U_i = K_f + U_f$

$$\frac{1}{2}mV_0^2\left(1+\frac{2}{5}\right) = mgh$$

$$h = \frac{7}{10}\frac{V_0^2}{g}$$

$$\ell = \frac{h}{\sin\theta} = \frac{7}{10}\frac{V_0^2}{g\sin\theta}$$

Q.18 Given below are two statements :

Statement I : In a diatomic molecule, the rotational energy at a given temperature obeys Maxwell's distribution.

Statement II : In a diatomic molecule, the rotational energy at a given temperature equals the translational kinetic energy for each molecule.

In the light of the above statements, choose the correct answer from the options given below :

Options

1. Statement I is true but Statement II is false.
2. Statement I is false but Statement II is true.
3. Both Statement I and Statement II are true.
4. Both Statement I and Statement II are false.

Ans: 1

Sol: Option (1)

Q.19 If a message signal of frequency 'f_m' is amplitude modulated with a carrier signal of frequency 'f_c' and radiated through an antenna, the wavelength of the corresponding signal in air is :

Options

1. $\frac{c}{f_c}$
2. $\frac{c}{f_m}$
3. $\frac{c}{f_c + f_m}$
4. $\frac{c}{f_c - f_m}$

Ans: 1

Sol: Equation of amplitude modulated wave $y = (A_c + A_m \sin \omega_m t) \sin \omega_c t$
Here angular frequency of modulated signal = ω_c
Thus frequency of modulated signal = f_c

$\therefore$ wavelength = $\frac{c}{f_c}$

Q.20 An LCR circuit contains resistance of 110 Ω and a supply of 220 V at 300 rad/s angular frequency. If only capacitance is removed from the circuit, current lags behind the voltage by 45°. If on the other hand, only inductor is removed the current leads by 45° with the applied voltage. The rms current flowing in the circuit will be :

Options

1. 1 A
2. 2 A
3. 1.5 A
4. 2.5 A

Ans: 2

Sol: When L and C are connected with R is series circuit will come in resonance. So current in the circuit will be

$$I_{rms} = \frac{V_{rms}}{R}$$

$$= \frac{220}{110} = 2A$$

SECTION B

Q.1 The percentage increase in the speed of transverse waves produced in a stretched string if the tension is increased by 4%, will be _____%.

Given 2
Answer :

Ans: 2.00

Sol: $V = \sqrt{\frac{T}{\mu}}$

$$\frac{dV}{V} = \frac{1}{2}\frac{dT}{T}$$

% change in speed = $\frac{1}{2}\frac{dT}{T} \times 100$

$$= \frac{1}{2} \times 4 = 2\%$$

Q.2 A reversible heat engine converts one-fourth of the heat input into work. When the temperature of the sink is reduced by 52 K, its efficiency is doubled. The temperature in Kelvin of the source will be _____.

Given –
Answer :

Ans: 208.00

Sol: $\eta = \frac{W}{Q_{in}} = \frac{1}{4} = 1 - \frac{T_L}{T_H}$

$\frac{T_L}{T_H} = \frac{3}{4}$

When sink temperature is decreased by 52 K

$\eta' = 2\eta = \frac{1}{2}$

$\eta' = \frac{1}{2} = 1 - \frac{(T_L - 52)}{T_H}$

$\frac{1}{2} = 1 - \frac{T_L}{T_H} + \frac{52}{T_H} \Rightarrow \frac{1}{4} = \frac{52}{T_H}$

$T_H = 208$ K

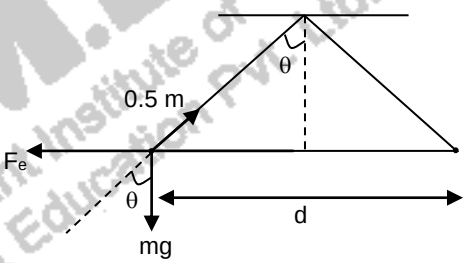
Q.3 Two small spheres each of mass 10 mg are suspended from a point by threads 0.5 m long. They are equally charged and repel each other to a distance of 0.20 m. The charge on each of the sphere is $\frac{a}{21} \times 10^{-8}$C. The value of 'a' will be _____.

[Given $g = 10\ ms^{-2}$]

Given –
Answer :

Ans: 20.00

Sol: $mg \tan\theta = \frac{kq^2}{d^2}$



$q = \sqrt{\frac{mg \tan\theta}{25\ K}}$

$= \sqrt{\frac{10 \times 10^{-6} \times 10 \times 1}{\sqrt{24} \times (25) \times 9 \times 10^9}}$

$= \sqrt{\frac{10^{-4} \times 4}{\sqrt{24} \times 25 \times 4 \times 9 \times 10^9}}$

$= \frac{2}{3} \times \sqrt{\frac{10^{-4}}{\sqrt{24} \times 10^{11}}}$

$= \frac{a}{21} \times 10^{-8} = \frac{2}{3}\sqrt{\frac{10^{-15}}{\sqrt{24}}} = \frac{2}{3}\sqrt{\frac{10^{-6}}{\sqrt{0.49}}}$

$a = \frac{2 \times 21}{3 \times 0.7} = 20$

Q.4 If $\vec{P} \times \vec{Q} = \vec{Q} \times \vec{P}$, the angle between $\vec{P}$ and $\vec{Q}$ is θ ($0^\circ < \theta < 360^\circ$). The value of '$\theta$' will be _____°.

Given 180
Answer :

Ans: 180.00

Sol: $\overline{P} \times \overline{Q} = \overline{Q} \times \overline{P}$

$\overline{P} \times \overline{Q} = -\left(\overline{P} \times \overline{Q}\right)$

$2\left(\overline{P} \times \overline{Q}\right) = 0$

$PQ \sin\theta = 0$

$\sin\theta = 0$

$\theta = \pi = 180°$

Q.5 Two particles having masses 4 g and 16 g respectively are moving with equal kinetic energies. The ratio of the magnitudes of their linear momentum is n : 2. The value of n will be _____.

Ans: 1.00

Sol: $P = \sqrt{2mE}$

$\frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{4}{16}} = 1:2$

Given the ratio as n : 2

$\therefore$ n = 1.00

Q.6 Two identical conducting spheres with negligible volume have 2.1 nC and −0.1 nC charges, respectively. They are brought into contact and then separated by a distance of 0.5 m. The electrostatic force acting between the spheres is _____ $\times 10^{-9}$ N.

[Given : $4\pi\varepsilon_0 = \frac{1}{9 \times 10^9}$ SI unit]

Ans: 36.00

Sol: $F = \frac{Kq_1q_2}{r^2}$

$= \frac{9 \times 10^9 \times 1 \times 10^{-9}}{(0.5)^2} = 36.00 \times 10^{-9} N$

Q.7 The peak electric field produced by the radiation coming from the 8 W bulb at a distance of 10 m is $\frac{x}{10}\sqrt{\frac{\mu_0 c}{\pi}}\frac{V}{m}$. The efficiency of the bulb is 10% and it is a point source. The value of x is _____.

Ans: 0.63

Sol: $\frac{P}{4\pi r^2} \times \eta = \frac{1}{2}\varepsilon_0 E_0^2 c$

$E_0 = \sqrt{\frac{P\eta}{2\pi r^2 c \varepsilon_0}} = \sqrt{\frac{8 \times 0.1}{2 \times 100}}\sqrt{\frac{\mu_0 c}{\pi}}$

$\therefore x = 2/\sqrt{10} = 0.63$

Q.8 The wavelength of an X-ray beam is 10 Å. The mass of a fictitious particle having the same energy as that of the X-ray photons is $\frac{x}{3}h$ kg. The value of x is _____.
(h = Planck's constant)

Ans: 10.00

Sol: $\frac{hc}{\lambda} = E = mc^2$

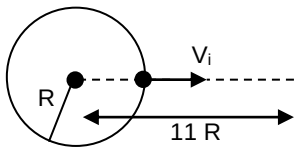
$\frac{hc}{\lambda} = \left(\frac{xh}{3}\right)c^2$

$$x = \frac{3}{c\lambda} = \frac{3}{3\times10^{8}\times10\times10^{-10}} = 10.00$$

Q.9 The initial velocity v_i required to project a body vertically upward from the surface of the earth to reach a height of 10R, where R is the radius of the earth, may be described in terms of escape velocity v_e such that $v_i = \sqrt{\frac{x}{y}} \times v_e$. The value of x will be _____.

Ans: 10.00

Sol:

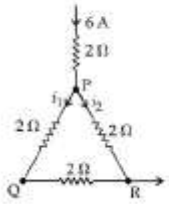


$$\frac{1}{2}mV_i^2 - \frac{GMm}{R} = \frac{-GMm}{11R}$$

$$V_i^2 = \left[2 - \frac{2}{11}\right]\frac{GM}{R}$$

$$V_i = \sqrt{\frac{20}{11}\frac{GM}{R}} = \sqrt{\frac{10}{11}}\,V_e \Rightarrow x = 10$$

Q.10 A current of 6 A enters one corner P of an equilateral triangle PQR having 3 wires of resistance 2 Ω each and leaves by the corner R. The currents i_1 in ampere is _____.



Ans: 2.00

Sol: In parallel

$$I_1R_1 = I_1R_2$$

$$\frac{I_2}{I_1} = \frac{R_1}{R_2}$$

$$I_2 = \frac{R_1}{R_1+R_2}(I_1+I_2)$$

$$= \frac{2}{2+4}\times6 = 2A$$

PART – B – CHEMISTRY

SECTION A

Q.1 Given below are two statements :

Statement I :

The identification of Ni^{2+} is carried out by dimethyl glyoxime in the presence of NH_4Cl

Statement II :

The dimethyl glyoxime is a bidentate neutral ligand.

In the light of the above statements, choose the correct answer from the options given bel

Ans: 3

Sol: Dimethyl glyoxime ion (dmg) is a bidentate anionic ligand.

Q.2 The solubility of $Ca(OH)_2$ in water is :
[Given : The solubility product of $Ca(OH)_2$ in water $= 5.5 \times 10^{-6}$]

Options
1. 1.11×10^{-6}
2. 1.11×10^{-2}
3. 1.77×10^{-6}
4. 1.77×10^{-2}

Ans: **2**

Sol: $Ca(OH)_2 \rightleftharpoons \underset{S}{Ca^{2+}} + \underset{2S}{2OH^-}$

$\therefore$ Solubility product, $K_{sp} = S \times (2S)^2 = 4S^3$

$$\therefore S = \left[\frac{K_{sp}}{4}\right]^{1/3}$$

$$= \left[\frac{5.5 \times 10^{-6}}{4}\right]^{1/3} = (1.375 \times 10^{-6})^{1/3} = 1.11 \times 10^{-2}$$

Q.3 Which among the following species has unequal bond lengths ?

Options
1. SiF_4
2. SF_4
3. BF_4^-
4. XeF_4

Ans: **2**

Sol: In SF_4, the central atom 'S' undergoes sp^3d hybridistaion. There are 4 bond pairs and one lone pair and the structure is 'see saw'. Here all the bonds are not identical.

Q.4 Given below are two statements :
Statement I :
α and β forms of sulphur can change reversibly between themselves with slow heating or slow cooling.
Statement II :
At room temperature the stable crystalline form of sulphur is monoclinic sulphur.
In the light of the above statements, choose the correct answer from the options given below :

Options
1. Statement I is true but Statement II is false.
2. Both Statement I and Statement II are false.
3. Both Statement I and Statement II are true.
4. Statement I is false but Statement II is true.

Ans: **1**

Sol: The stable form of sulphur at room temperature is rhombic sulphur (α-sulphur) which transforms to monoclinic sulphur when heated above 369 K.

Q.5 The major product of the following reaction is :

$$CH_3CH_2CH{=}CH_2 \xrightarrow[\text{Rh catalyst}]{H_2/CO}$$

Options

1. $CH_3CH_2CH_2CH_2CHO$
2. $CH_3CH_2CH_2CHO$
3. $CH_3CH_2CH{=}CH{-}CHO$
4. $CH_3CH_2C(CHO){=}CH_2$

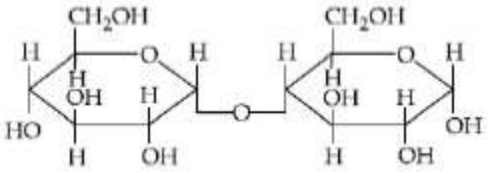
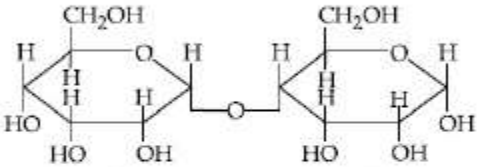
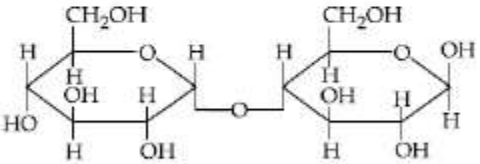
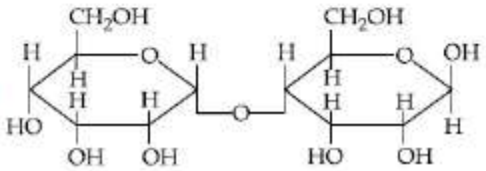
Ans: **1**

Sol: $CH_3 - CH_2 - CH = CH_2 \xrightarrow[\text{Rh catalyst}]{H_2/CO} CH_3 - CH_2 - CH_2 - CH_2 - CHO$

The above reaction is called as hydroformylation of Olefins or Oxo process.

Q.6 Which of the following is correct structure of α-anomer of maltose ?

Options

1. 
2. 
3. 
4. 

Ans: **1**

Sol: Maltose is a compound of two α-D-glucose units in which C_1 of one glucose is linked to C_4 of another glucose.

Q.7 Which of the following compound is added to the sodium extract before addition of silver nitrate for testing of halogens ?

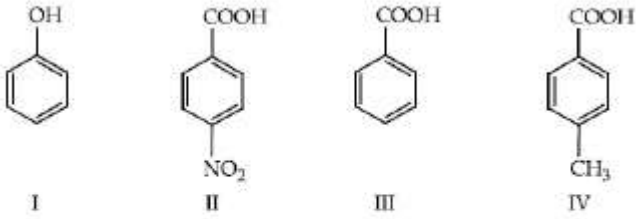
Options

1. Ammonia
2. Hydrochloric acid
3. Nitric acid
4. Sodium hydroxide

Ans: **3**

Sol: The function of adding HNO_3 before the addition of $AgNO_3$ is remove NaCN or Na_2S if formed in the reaction as HCN & H_2S

Q.8 The correct order of acid character of the following compounds is :

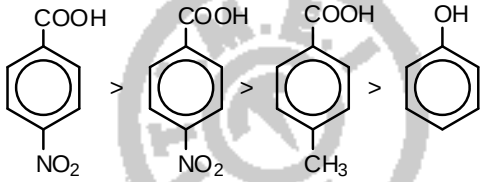


I II III IV

Options
1. III > II > I > IV
2. II > III > IV > I
3. I > II > III > IV
4. IV > III > II > I

Ans: **2**

Sol: Presence of electron withdrawing group (NO_2) in the ring of aromatic carboxylic acid increases the acidic strength whereas electron donating groups decreases their acidic strength.



(pK_a = 3.41) (pK_a = 4.19) (pK_a = 4.26) (pK_a = 10)

Q.9 Given below are two statements :
Statement I :
The pH of rain water is normally ~5.6.
Statement II :
If the pH of rain water drops below 5.6, it is called acid rain.
In the light of the above statements, choose the correct answer from the options given below :

Options
1. Both Statement I and Statement II are false.
2. Statement I is false but Statement II is true.
3. Statement I is true but Statement II is false.
4. Both Statement I and Statement II are true.

Ans: **4**

Sol: Normal rain water has pH of about 5.6 due to the dissolution of CO_2 into it. When pH of rain water falls below 5.6, it becomes acidic. Hence both the given statements are true.

Q.10 Which one of the following statements is FALSE for hydrophilic sols ?

Options
1. These sols are reversible in nature.
2. The sols cannot be easily coagulated.
3. They do not require electrolytes for stability.
4. Their viscosity is of the order of that of H_2O.

Ans: **4**

Sol: For hydrophilic sols, viscosity is much higher than that of water.

Q.11 Water does not produce CO on reacting with :

Options 1. CH_4

2. CO_2

3. C

4. C_3H_8

Ans: 2

Sol: $CO_2 + H_2O \rightarrow H_2CO_3$
$C + H_2O \rightarrow CO + H_2$
$CH_4 + H_2O \rightarrow CO + 3H_2$
$C_3H_8 + 3H_2O \rightarrow 3CO + 7H_2$

Q.12 The method used for the purification of Indium is :

Options 1. vapour phase refining

2. zone refining

3. van Arkel method

4. liquation

Ans: 2

Sol: Zone refining is a very useful refining method for semiconductors and other metals which require high purity.

Q.13 The major components of German Silver are :

Options 1. Cu, Zn and Ni

2. Cu, Zn and Ag

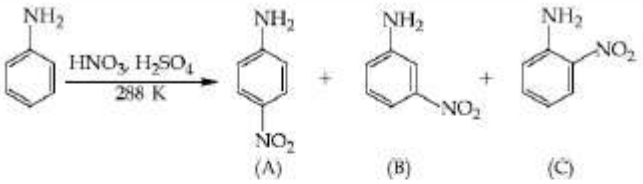
3. Zn, Ni and Ag

4. Ge, Cu and Ag

Ans: 1

Sol: German silver contains
Cu – 25-30%, Zn – 25-30% and Ni – 40-50%

Q.14



Correct statement about the given chemical reaction is :

Options 1.

$-\ddot{N}H_2$ group is *ortho* and *para* directive, so product (B) is not possible.

2.

Reaction is possible and compound (A) will be major product.

3.

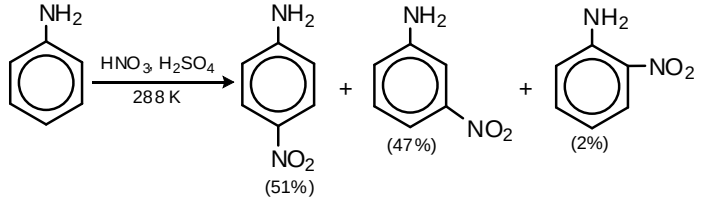
Reaction is possible and compound (B) will be the major product.

4.

The reaction will form sulphonated product instead of nitration.

Ans: 2

Sol: In strongly acidic medium, aniline is protonated to form anilinium ion, which is meta directing. Hence besides ortho and para derivatives, significant amount of meta derivative is also formed.



Q.15 The correct order of bond dissociation enthalpy of halogens is :

Options

1. $Cl_2 > Br_2 > F_2 > I_2$
2. $I_2 > Br_2 > Cl_2 > F_2$
3. $Cl_2 > F_2 > Br_2 > I_2$
4. $F_2 > Cl_2 > Br_2 > I_2$

Ans: **1**

Sol: The bond dissociation enthalpy among halogens is $Cl_2 > Br_2 > F_2 > I_2$. The reason for the low BDE for F_2 is due to its small size and large $e^- - e^-$ repulsion among the lone pairs in F_2

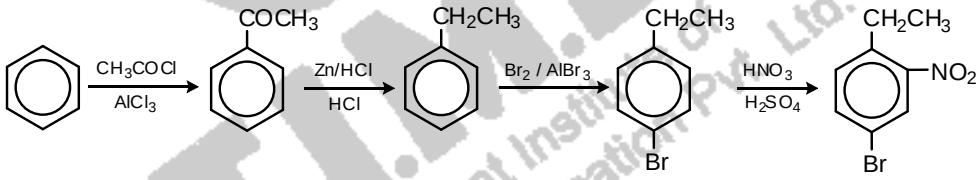
Q.16 The correct sequence of reagents used in the preparation of 4-bromo-2-nitroethyl benzene from benzene is :

Options

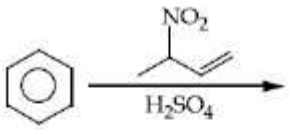
1. $Br_2/AlBr_3$, $CH_3COCl/AlCl_3$, HNO_3/H_2SO_4, Zn/HCl
2. $CH_3COCl/AlCl_3$, $Br_2/AlBr_3$, HNO_3/H_2SO_4, Zn/HCl
3. HNO_3/H_2SO_4, $Br_2/AlCl_3$, $CH_3COCl/AlCl_3$, Zn-Hg/HCl
4. $CH_3COCl/AlCl_3$, Zn-Hg/HCl, $Br_2/AlBr_3$, HNO_3/H_2SO_4

Ans: **4**

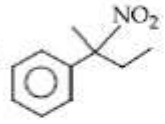
Sol:

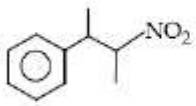


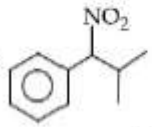
Q.17 The major product of the following reaction is :

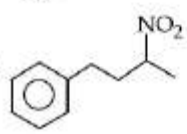


Options

1. 

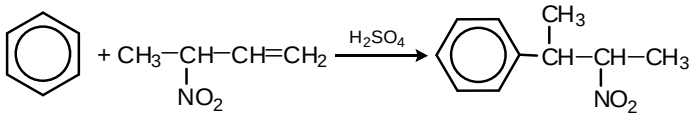
2. 

3. 

4. 

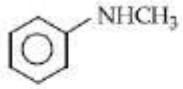
Ans: **2**

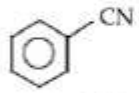
Sol:

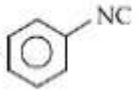


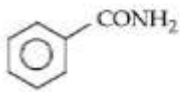
Q.18 Carbylamine test is used to detect the presence of primary amino group in an organic compound. Which of the following compound is formed when this test is performed with aniline ?

Options

1. 

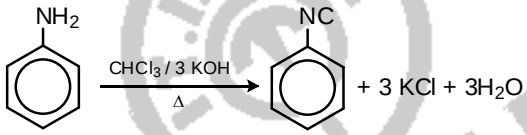
2. 

3. 

4. 

Ans: **3**

Sol:



Q.19 In which of the following order the given complex ions are arranged correctly with respect to their decreasing spin only magnetic moment ?

(i) $[FeF_6]^{3-}$ (ii) $[Co(NH_3)_6]^{3+}$ (iii) $[NiCl_4]^{2-}$ (iv) $[Cu(NH_3)_4]^{2+}$

Options

1. (i) > (iii) > (iv) > (ii)
2. (ii) > (i) > (iii) > (iv)
3. (ii) > (iii) > (i) > (iv)
4. (iii) > (iv) > (ii) > (i)

Ans: **1**

Sol: $[FeF_6]^{3-} \Rightarrow Fe^{3+} - [Ar]\ 3d^5\ 4s^0$

Since F^- is a weak ligand, $t_2g^3\ eg^2$ $\therefore n = 5$

$[Co(NH_3)_6]^{3+} \Rightarrow Co^{3+}\ [Ar]\ 3d^6\ 4s^0$

Since NH_3 is a strong ligand, $t_2g^6\ eg^0$ $\therefore n = 0$

$[NiCl_4]^{2-} \Rightarrow Ni^{2+} - [Ar]\ 3d^8\ 4s^0$

Since Cl^- is a weak ligand, $e^4\ t_2^4$ $\therefore n = 2$

$[Cu(NH_3)_4]^{2+} \Rightarrow Cu^{2+} - [Ar]\ 3d^9\ 4s^0$

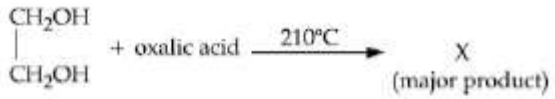
$e^4\ t_2^5$ $\therefore n = 1$

Spin only magnetic moment = $\mu = \sqrt{n(n+2)}$ BM

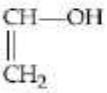
Greater the number of unpaired electrons, greater will be magnetic moment

$[FeF_6]^{3-} > [NiCl_4]^{2-} > [Cu(NH_3)_4]^{2+} > [Co(NH_3)_6]^{3+}$

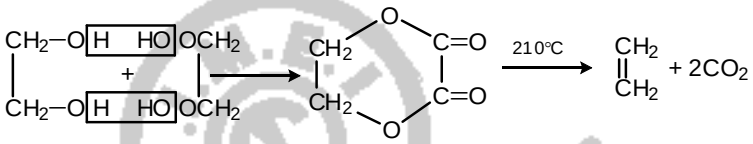
Q.20 What is 'X' in the given reaction ?



Options

1. 
2. 
3. 
4. 

Ans: **4**

Sol: 

SECTION B

Q.1 Consider titration of NaOH solution versus 1.25 M oxalic acid solution. At the end point following burette readings were obtained.

(i) 4.5 mL (ii) 4.5 mL (iii) 4.4 mL
(iv) 4.4 mL (v) 4.4 mL

If the volume of oxalic acid taken was 10.0 mL then the molarity of the NaOH solution is _____ M. (Rounded-off to the nearest integer)

Ans: **6**

Sol: At neutralization point,

$N_1V_1 = N_2V_2$

$$\therefore N_2 = \frac{N_1V_1}{V_2} = \frac{1.25 \times 2\ 10}{4.4} = 5.68\ N$$

For NaOH, N = M = 5.68 M

Q.2 Among the following, number of metal/s which can be used as electrodes in the photoelectric cell is _____. (Integer answer)

(A) Li (B) Na (C) Rb (D) Cs

Ans: **2**

Sol: Metals with low ionization enthalpies are used on photoelectric cells. ∴ Among the given, Rb & Cs are used in photoelectric cells.

Q.3 Electromagnetic radiation of wavelength 663 nm is just sufficient to ionise the atom of metal A. The ionization energy of metal A in kJ mol^{-1} is _____. (Rounded-off to the nearest integer)

[$h = 6.63 \times 10^{-34}$ Js, $c = 3.00 \times 10^8$ ms^{-1}, $N_A = 6.02 \times 10^{23}$ mol^{-1}]

Ans: **181**

Sol: $E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{663 \times 10^{-9}} = 3 \times 10^{-19}\ J$

For 1mol, $E = \frac{3 \times 10^{-19} \times 6.02 \times 10^{23}}{10^{3}} = 180.6 \cong 181$

Q.4 The number of compound/s given below which contain/s —COOH group is _____. (Integer answer)

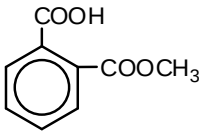
(A) Sulphanilic acid (B) Picric acid

(C) Aspirin (D) Ascorbic acid

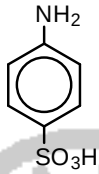
Ans: **1**

Sol: Only aspirin contains COOH group

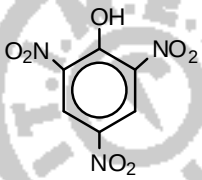
Aspirin –



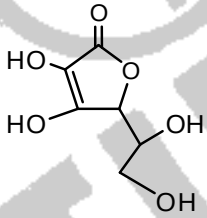
Sulphanilic acid –



Picric acid –



Ascorbic acid –



Q.5 The rate constant of a reaction increases by five times on increase in temperature from 27°C to 52°C. The value of activation energy in kJ mol^{-1} is _____. (Rounded-off to the nearest integer)

[R = 8.314 J K^{-1} mol^{-1}]

Ans: **52**

Sol: $\log \frac{k_2}{k_1} = \frac{E_a}{2.303\ R} = \left[\frac{T_2 - T_1}{T_1 T_2}\right]$

$\log 5 = \frac{E_a}{2.303 \times 8.314} = \left[\frac{325 - 300}{325 \times 300}\right]$

$E_a = \frac{0.7 \times 2.303 \times 8.314 \times 325 \times 300}{25} = 52.3\ kJ\ mol^{-1}$

Q.6 If a compound AB dissociates to the extent of 75 % in an aqueous solution, the molality of the solution which shows a 2.5 K rise in the boiling point of the solution is _____ molal. (Rounded-off to the nearest integer)

[K_b = 0.52 K kg mol^{-1}]

Ans: **3**

Sol: $\alpha = \frac{75}{100} = 0.75$

$\alpha_{diss} = \frac{i - 1}{n - 1}$

$$0.75 = \frac{i-1}{2-1}$$

$\underline{i = 1.75}$

$\Delta T_b = i.K_b.M$

$$M = \frac{\Delta T_b}{i.K_b} = \frac{2.5}{1.75 \times 0.52} = 2.75\ M$$

Q.7 The spin only magnetic moment of a divalent ion in aqueous solution (atomic number 29) is _____ BM.

Ans: **2**

Sol: M (Z = 29) $\rightarrow$ [Ar] $3d^{10}\ 4s^1$

$M^{2+} \rightarrow [Ar]\ 3d^9\ 4s^{0}$ $\quad \therefore n = 1$

Magnetic moment, $\mu = \sqrt{n(n+2)}$ BM

$= \sqrt{1(1+2)} = 1.73$ BM

Q.8 Five moles of an ideal gas at 293 K is expanded isothermally from an initial pressure of 2.1 MPa to 1.3 MPa against at constant external pressure 4.3 MPa. The heat transferred in this process is _____ kJ mol^{-1}. (Rounded-off to the nearest integer)
[Use R = 8.314 J $mol^{-1}K^{-1}$]

Ans: **16**

Sol: For an isothermal process, $\Delta T = 0$

$\therefore q - w = 0 \quad \therefore q = -w$

$w = -P_{ext}(V_2 - V_1)$

For an ideal gas, $V_1 = \frac{nRT}{p_1} \quad V_2 = \frac{nRT}{p_2}$

$$V_2 - V_1 = nRT\left[\frac{1}{p_2} - \frac{1}{p_1}\right] = \frac{5 \times 8.314 \times 293}{10^6}\left[\frac{1}{1.3} - \frac{1}{2.1}\right]$$

$$= \frac{12,810.01}{10^6} \times 0.293 = 3753.33 \times 10^{-6}\ m^3$$

$\therefore w = -4.3 \times 10^6 \times 3753.33 \times 10^{-6} = -16,139.32\ J = -16.4\ kJ$

$\therefore q = 16.14\ kJ$

Q.9 Copper reduces NO_3^- into NO and NO_2 depending upon the concentration of HNO_3 in solution. (Assuming fixed $[Cu^{2+}]$ and $P_{NO} = P_{NO_2}$) the HNO_3 concentration at which the thermodynamic tendency for reduction of NO_3^- into NO and NO_2 by copper is same is 10^x M. The value of 2x is _____. (Rounded-off to the nearest integer)
[Given, $E^\circ_{Cu^{2+}/Cu} = 0.34$ V, $E^\circ_{NO_3^-/NO} = 0.96$ V, $E^\circ_{NO_3^-/NO_2} = 0.79$ V and at 298 K, $\frac{RT}{F}(2.303) = 0.059$]

Ans: **4**

Sol: $3Cu + 8H^+ + 2NO_3^- \rightarrow 3Cu^{2+} + 2NO + 4H_2O$

$Cu + 4H^+ + 2NO_3^- \rightarrow Cu^{2+} + 2NO_2 + 2H_2O$

Let, $[HNO_3] = x \quad \therefore [H^+] = x$ and $[NO_3^-] = x$

For the same thermodynamic tendency, $E_{cell_1} = E_{cell_2}$

$$E_{NO_3^-/NO} - E_{Cu^{2+}/Cu} = E_{NO_3^-/NO_2} - E_{Cu^{2+}/Cu}$$

$$E_{NO_3^-/NO} = E_{NO_3^-/NO_2} \quad \begin{vmatrix} NO_3^- + 4H^+ + 3e^- \rightarrow NO + 2H_2O \\ NO_3^- + 2H^+ + e^- \rightarrow NO_2 + H_2O \end{vmatrix}$$

$$0.96 - \frac{0.059}{3}\log\frac{p_{NO}}{x^5} = 0.79 - \frac{0.059}{1}\log\frac{p_{NO_2}}{x^3}$$

$$0.17 = \frac{0.059}{3}\log\frac{p_{NO}}{x^5} - \frac{0.059}{1}\log\frac{p_{NO_2}}{x^3}$$

$$= \frac{0.059}{3}\left[\log\frac{p_{NO}}{x^5} - \log\frac{p_{NO_2}^3}{x^9}\right]$$

$$\frac{0.17\times 3}{0.059} = \log\frac{p_{NO}}{x^5}\times\frac{x^9}{p_{NO_2}^3}$$

$$\log x^4 = \frac{0.17\times 3}{0.059} \qquad \because p_{NO} = p_{NO_2} = 1 \text{ atm}$$

$$\log x = \frac{0.17\times 3}{0.059\times 4}$$

$$x = {}_{10}\left(\frac{0.17\times 3}{0.059\times 4}\right)$$

$$2M = \frac{0.17\times 3\times 2}{0.059\times 4} = 4.32 \cong 4$$

Q.10 The unit cell of copper corresponds to a face centered cube of edge length 3.596 Å with one copper atom at each lattice point. The calculated density of copper in kg/m^3 is _____.
[Molar mass of Cu : 63.54 g ; Avogadro Number = 6.022×10^{23}]

Ans: **9076**

Sol: $$d = \frac{ZM}{a^3 N_A} = \frac{4\times 63.54}{(3.59\times 10^{-10})^3\times 6.022\times 10^{23}} = \frac{4\times 63.54}{46.5\times 10^{-30}\times 6.022\times 10^{23}}$$

$$= \frac{254.16}{280.02\times 10^{-7}} = 0.9076\times 10^7 \text{ g/m}^3 = 0.9076\times 10^4 \text{ kg/m}^3 = 9076$$

PART – C – MATHEMATICS

SECTION A

Q.1 The shortest distance between the line $x - y = 1$ and the curve $x^2 = 2y$ is :

Options

1. $\frac{1}{2\sqrt{2}}$
2. $\frac{1}{2}$
3. 0
4. $\frac{1}{\sqrt{2}}$

Ans: **1**

Sol: Any point on $x^2 = 2y$ is given by $x = t \quad y = \frac{1}{2}t^2$

Slope of normal $= \frac{-1}{t}$

Shortest distance is $\perp^r$ distance from $x - y - 1 = 0$

$\Rightarrow \frac{-1}{t} = -1 \Rightarrow t = 1$

$\therefore$ point is (1, ½)

Shortest distance $= \left|\frac{1-\frac{1}{2}-1}{\sqrt{2}}\right| = \frac{1}{2\sqrt{2}}$

Q.2 Let A be a 3×3 matrix with $\det(A)=4$. Let R_i denote the i^{th} row of A. If a matrix B is obtained by performing the operation $R_2 \to 2R_2+5R_3$ on 2A, then det(B) is equal to :

Options
1. 128
2. 80
3. 16
4. 64

Ans: 4

Sol: $A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ $2A=\begin{bmatrix} 2a_{11} & 2a_{12} & 2a_{13} \\ 2a_{21} & 2a_{22} & 2a_{23} \\ 2a_{31} & 2a_{32} & 2a_{33} \end{bmatrix}$

$R_2 \to 2R_2+5R_3$ on 2A

$$\Rightarrow \begin{bmatrix} 2a_{11} & 2a_{12} & 2a_{13} \\ 4a_{21}+10a_{31} & 4a_{22}+10a_{32} & 4a_{23}+10a_{33} \\ 2a_{31} & 2a_{32} & 2a_{33} \end{bmatrix}$$

$$\Rightarrow 16\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$|B|=16|A|=16\times4=64$

Q.3 $\lim\limits_{n\to\infty}\left[\frac{1}{n}+\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+\ldots\ldots+\frac{n}{(2n-1)^2}\right]$ is equal to :

Options
1. $\frac{1}{4}$
2. $\frac{1}{2}$
3. $\frac{1}{3}$
4. 1

Ans: 2

Sol: $\lim\limits_{n\to\infty}\left[\frac{1}{n}+\frac{n}{(n+1)^2}+\ldots\ldots\ldots\ldots+\frac{n}{(2n-1)^2}\right]$

$$=\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{n}{(n+r)^2}=\int_0^1\frac{dx}{(1+x)^2}$$

$$=-\left[\frac{1}{1+x}\right]_0^1=-\left[\frac{-1}{2}-1\right]=\frac{1}{2}$$

Q.4 The following system of linear equations

$2x+3y+2z=9$
$3x+2y+2z=9$
$x-y+4z=8$

Options
1. does not have any solution
2. has a solution (α,β,γ) satisfying $\alpha+\beta^2+\gamma^3=12$
3. has infinitely many solutions
4. has a unique solution

Ans: 4

Sol: $\Delta = \begin{vmatrix} 2 & 3 & 2 \\ 3 & 2 & 2 \\ 1 & -1 & 4 \end{vmatrix} = 2(10) - 3(10) + 2(-5)$

$= -20 \neq 0$

$\Rightarrow$ has a unique solution.

Q.5 The contrapositive of the statement "If you will work, you will earn money" is :

Options
1. If you will not earn money, you will not work
2. You will earn money, if you will not work
3. To earn money, you need to work
4. If you will earn money, you will work

Ans: 1

Sol: Contrapositive of $p \Rightarrow q$ is $\sim q \Rightarrow \sim p$
So answer is
"If you will not earn money, you will not work".

Q.6 A function $f(x)$ is given by $f(x) = \frac{5^x}{5^x + 5}$, then the sum of the series

$$f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) + f\left(\frac{3}{20}\right) + \ldots\ldots + f\left(\frac{39}{20}\right)$$

is equal to :

Options
1. $\frac{39}{2}$
2. $\frac{49}{2}$
3. $\frac{29}{2}$
4. $\frac{19}{2}$

Ans: 1

Sol: $f(x) = \frac{5^x}{5^x + 5}$

$f(2-x) = \frac{5^{2-x}}{5^{2-x} + 5} = \frac{5}{5 + 5^x}$

$\therefore f(x) + f(2-x) = 1$

$\Rightarrow \left[f\left(\frac{1}{20}\right) + f\left(\frac{39}{20}\right)\right] + \left[f\left(\frac{2}{20}\right) + f\left(\frac{38}{20}\right)\right] + \ldots\ldots\ldots\ldots f\left(\frac{20}{20}\right)$

$= 19 + \frac{1}{2} = \frac{39}{2}$

Q.7

The integral $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}} dx$, $x > 0$, is equal to :

(where c is a constant of integration)

Options

1. $\log_e|x^2+5x-7|+c$
2. $\frac{1}{4}\log_e|x^2+5x-7|+c$
3. $4\log_e|x^2+5x-7|+c$
4. $\log_e\sqrt{x^2+5x-7}+c$

Ans: 3

Sol: $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}} dx$

$$= \int \frac{8x^3 + 5.4x^2}{x^4 + 5.x^3 - 7.x^2} dx$$

$$= \int \frac{4(2x+5)}{x^2+5x-7} dx = 4\log_e|x^2+5x-7|+C$$

Q.8

In a group of 400 people, 160 are smokers and non-vegetarian; 100 are smokers and vegetarian and the remaining 140 are non-smokers and vegetarian. Their chances of getting a particular chest disorder are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the chest disorder. The probability that the selected person is a smoker and non-vegetarian is :

Options

1. $\frac{28}{45}$
2. $\frac{7}{45}$
3. $\frac{14}{45}$
4. $\frac{8}{45}$

Ans: 1

Sol: A- denote the person selected has chest disorder

B_1 denote the event that the person is a smoker and non-vegetarian.

B_2 denote the event that the person is a smoker and vegetarian.

B_3 denote the event that the person is a non-smoker and vegetarian

$$P(B_1) = \frac{160}{400} = 0.4$$

$$P(B_2) = \frac{100}{400} = 0.25$$

$$P(B_3) = \frac{140}{400} = \frac{14}{40}$$

$$P\left(\frac{A}{B_1}\right) = \frac{35}{100}$$

$$P\left(\frac{A}{B_2}\right) = \frac{20}{100}$$

$$P\left(\frac{A}{B_3}\right) = \frac{10}{100}$$

Required probability is

$$P\left(\frac{B_1}{A}\right)=\frac{P(B_1)\cdot P\left(\frac{A}{B_1}\right)}{\Sigma P(B_1)P\left(\frac{A}{B_1}\right)}=\frac{\frac{16}{40}\times\frac{35}{100}}{\frac{16}{40}\times\frac{35}{100}+\frac{10}{40}\times\frac{20}{100}+\frac{14}{10}\times\frac{10}{100}}=\frac{28}{45}$$

Q.9 The minimum value of $f(x)=a^{a^x}+a^{1-a^x}$, where a, $x \in R$ and $a>0$, is equal to :

Options

1. $a+\frac{1}{a}$
2. $a+1$
3. $2\sqrt{a}$
4. $2a$

Ans: 3

Sol: we know that AM ≥ GM

$$\frac{a^{a^x}+\frac{a}{a^{a^x}}}{2}\geq\left[a^{a^x}\cdot\frac{a}{a^{a^x}}\right]^{1/2}$$

$$\Rightarrow a^{a^x}+\frac{a}{a^{a^x}}\geq 2\sqrt{a}$$

Q.10 If for the matrix, $A=\begin{bmatrix}1 & -\alpha\\ \alpha & \beta\end{bmatrix}$, $AA^T=I_2$, then the value of $\alpha^4+\beta^4$ is :

Options

1. 2
2. 3
3. 4
4. 1

Sol: $A=\begin{bmatrix}1 & -\alpha\\ \alpha & \beta\end{bmatrix}$ $\quad A^T=\begin{bmatrix}1 & \alpha\\ -\alpha & \beta\end{bmatrix}$

$$AA^T=I_2\Rightarrow\begin{bmatrix}1+\alpha^2 & \alpha-\alpha\beta\\ \alpha-\alpha\beta & \alpha^2+\beta^2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$\Rightarrow 1+\alpha^2=1$ $\qquad \alpha^2+\beta^2=1$

$\Rightarrow \alpha=0$ $\qquad \Rightarrow \beta^2=1$

$\beta=\pm 1$

$\therefore \alpha^4+\beta^4=0+1=1$

Q.11 If $I_n=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cot^n x\,dx$, then :

Options

1. $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in A.P.
2. $I_2+I_4, (I_3+I_5)^2, I_4+I_6$ are in G.P.
3. $I_2+I_4, I_3+I_5, I_4+I_6$ are in A.P.
4. $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in G.P.

Ans: 1

Sol: $I_{n+2}+I_n=\int_{\pi/4}^{\pi/2}\cot^n x\,\mathrm{cosec}^2x\,dx=\left[\frac{-(\cot x)^{n+1}}{n+1}\right]_{\pi/4}^{\pi/2}=\frac{1}{n+1}$

$\Rightarrow I_4+I_2=\frac{1}{3}, I_5+I_3=\frac{1}{4}, I_6+I_4=\frac{1}{5}$

So $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in A.P

Q.12 If the curve $x^2+2y^2=2$ intersects the line $x+y=1$ at two points P and Q, then the angle subtended by the line segment PQ at the origin is :

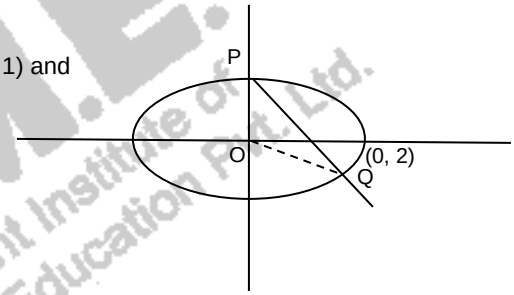
Options

1. $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{3}\right)$
2. $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{4}\right)$
3. $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{3}\right)$
4. $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{4}\right)$

Ans: 2

Sol: $\frac{x^2}{2}+\frac{y^2}{1}=1$

Solving x + y = 1 and $x^2+2y^2=2$, we get P(0, 1) and $Q\left(\frac{4}{3}, \frac{-1}{3}\right)$



Slope of OP = 1

Slope of OQ = $\frac{-1}{4}$

Required angle $=\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{4}\right)$

Q.13 A plane passes through the points A(1, 2, 3), B(2, 3, 1) and C(2, 4, 2). If O is the origin and P is (2, −1, 3), then the projection of $\overrightarrow{OP}$ on this plane is of length :

Options

1. $\sqrt{\frac{2}{7}}$
2. $\sqrt{\frac{2}{11}}$
3. $\sqrt{\frac{2}{5}}$
4. $\sqrt{\frac{2}{3}}$

Ans: 2

Sol: Plane passing through (1, 2, 3) (2, 3, 1) (2, 4, 2) is given by

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 1 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 0$$

$\Rightarrow (x-1)(3)-(y-2)+(z-3)=0$

$3x-y+z-4=0$

Given O(0, 0, 0) P(2, –1, 1)

Foot of $\perp^r$ from O to the plane is $\left(\frac{12}{11}, \frac{-4}{11}, \frac{4}{11}\right)$

Foot of $\perp^r$ from P is $\left(\frac{10}{11}, \frac{-7}{11}, \frac{7}{11}\right)$

Distance between them = $\sqrt{\frac{2}{11}}$

Q.14 $\operatorname{cosec}\left[2\cot^{-1}(5)+\cos^{-1}\left(\frac{4}{5}\right)\right]$ is equal to :

Options

1. $\frac{75}{56}$
2. $\frac{65}{33}$
3. $\frac{65}{56}$
4. $\frac{56}{33}$

Ans: 3

Sol: $\operatorname{cosec}\left[2\cot^{-1}5+\cos^{-1}4/5\right]$

$\Rightarrow \operatorname{cosec}\left[2\tan^{-1}1/5+\tan^{-1}3/4\right]$

$\Rightarrow \operatorname{cosec}\left[\tan^{-1}\frac{5}{12}+\tan^{-1}\frac{3}{4}\right]$

$= \operatorname{cosec}\left[\tan^{-1}\frac{56}{33}\right] = \operatorname{cosec}\operatorname{cosec}^{-1}\left[\frac{65}{56}\right] = \frac{65}{56}$

Q.15 Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set A × B. Then :

Options

1. $y=273x$
2. $2y=91x$
3. $2y=273x$
4. $y=91x$

Ans: 2

Sol: No. of elements in A – 3
No. of elements in B – 5
No. of elements in A × B – 15
x = No. of one-one functions from A to B = 5 × 4 × 3 = 60
y = No. of one-one functions from A to A × B = 15 × 14 × 13 = 2y = 91 x

Q.16 Let α and β be the roots of $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{3a_9}$ is :

Options

1. 1
2. 4
3. 3
4. 2

Ans: 4

Sol: α is a root of $x^2 - 6x - 2 = 0 \Rightarrow 6\alpha = \alpha^2 - 2$

$a_n = \alpha^n - \beta^n$

$$\frac{a_{10} - 2a_8}{3a_9} = \frac{\alpha^{10} - \beta^{10} - 2\left(\alpha^8 - \beta^8\right)}{3\left(\alpha^9 - \beta^9\right)} = \frac{\alpha^{10} - 2\alpha^8 - \beta^{10} + 2\beta^8}{3\left(\alpha^9 - \beta^9\right)}$$

$$= \frac{\alpha^8\left(\alpha^2 - 2\right) - \beta^8\left(\beta^2 - 2\right)}{3\alpha^9 - \beta^9} = \frac{6\left(\alpha^9 - \beta^9\right)}{3\left(\alpha^9 - \beta^9\right)} = 2$$

Q.17 If $\alpha, \beta \in R$ are such that $1 - 2i$ (here $i^2 = -1$) is a root of $z^2 + \alpha z + \beta = 0$, then $(\alpha - \beta)$ is equal to :

Options

1. 3
2. -7
3. -3
4. 7

Ans: 2

Sol: Since $\alpha, \beta \in R$ and $1 - 2i$ is one of the roots, the other root will be $1 + 2i$

Sum of roots = $1 - 2i + 1 - 2i = -\alpha$

$2 = -\alpha \Rightarrow \alpha = -2$

Product of roots = $(1 - 2i)(1 + 2i)$

$= 1 + 4 = 5 = \beta$

$\alpha - \beta = -2 - 5 = -7$

Q.18 Let A be a set of all 4-digit natural numbers whose exactly one digit is 7. Then the probability that a randomly chosen element of A leaves remainder 2 when divided by 5 is :

Options

1. $\frac{122}{297}$
2. $\frac{97}{297}$
3. $\frac{1}{5}$
4. $\frac{2}{9}$

Ans: 2

Sol: Total cases = $1 \times 9 \times 9 \times 9 = 729 +$

$3 \times 8 \times 9 \times 9 \times 9 = 1944$

2673

Favourable cases

1. No's ending with 7 = $1 \times 8 \times 9 \times 9 = 648$
2. No:'s ending with 2

1) starting with 7 = $1 \times 9 \times 9 \times 1 = 81$

2) Not starting with 7 = $8\ 9\ 2 \times 1 = 144$

Required probability $= \frac{873}{2673} = \frac{97}{297}$

Q.19 If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$, then $\sin x + \cos y$ is equal to :

Options

1. $\frac{1}{2}$
2. $\frac{\sqrt{3}}{2}$
3. $\frac{1-\sqrt{3}}{2}$
4. $\frac{1+\sqrt{3}}{2}$

Ans: 4

Sol: $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$

$$2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) - \left[2\cos^2\frac{x+y}{2} - 1\right] = \frac{3}{2}$$

$$\Rightarrow 2\cos\left(\frac{x+y}{2}\right)\left[\cos\left(\frac{x-y}{2}\right) - \cos\left(\frac{x+y}{2}\right)\right] = \frac{1}{2}$$

$$\Rightarrow 2\cos\left(\frac{x+y}{2}\right)2\sin\frac{x}{2}\sin\frac{y}{2} = \frac{1}{2}$$

$$\Rightarrow \cos\left(\frac{x+y}{2}\right)\sin\frac{x}{2}\sin\frac{y}{2} = \frac{1}{8}$$

Putting x = y = 60

Sinx + cosy = sin60 + cos60 $= \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$

Q.20 A hyperbola passes through the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its transverse and conjugate axes coincide with major and minor axes of the ellipse, respectively. If the product of their eccentricities is one, then the equation of the hyperbola is :

Options

1. $\frac{x^2}{9} - \frac{y^2}{25} = 1$
2. $\frac{x^2}{9} - \frac{y^2}{4} = 1$
3. $\frac{x^2}{9} - \frac{y^2}{16} = 1$
4. $x^2 - y^2 = 9$

Ans: 3

Sol: foci of $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is $(\pm 3, 0)$ $\quad a = 5$

$$e = \sqrt{\frac{a^2 - b^2}{a^2}} = \frac{3}{5}$$

Given product of eccentricities of ellipse and hyperbola is 1

So eccentricity of hyperbola = $\frac{5}{3}$

Since hyperbola passes through (± 3, 0)

$a^2 = 9$ $\qquad b^2 = a^2\left(e^2 - 1\right)$

$= 9\left(\frac{25}{9} - 1\right)$

$= 16$

$\Rightarrow$ Required equation is $\frac{x^2}{9} - \frac{y^2}{16} = 1$

SECTION B

Q.1 If the curves $x = y^4$ and $xy = k$ cut at right angles, then $(4k)^6$ is equal to _____.

Ans: 4

Sol: $x = y^4$ $\qquad xy = k$

$1 = 4y^3 \frac{dy}{dx}$ $\qquad xy' + y = 0$

$y' = \frac{-y}{x}$

Product of slopes = -1

$\frac{1}{4y^3} \cdot \frac{-y}{x} = -1$

$\frac{+1}{4y^6} = 1$ and $xy = k$

$y^6 = \frac{1}{4} \Rightarrow k = y^5$

$k^6 = y^{30}$

$= \left(\frac{1}{4}\right)^5$

$(4k)^6 = 4^6 . k^6 = 4$

Q.2 The total number of two digit numbers 'n', such that $3^n + 7^n$ is a multiple of 10, is _____.

Ans: 45

Sol: $7^x + 3^x = (5+2)^x + (5-2)^x$

Here if x is odd, the last term ${}^xC_{x-1} 5.2^{x-1}$ is divisible by 10.

But if x is even, the last term is 2^x

Then, the number is not divisible by 10

Hence, for all odd two digit natural no.'s

$7^x + 3^x$ is divisible by 10.

So required answer is 45

Q.3 If the remainder when x is divided by 4 is 3, then the remainder when $(2020 + x)^{2022}$ is divided by 8 is _____.

Ans: 1

Sol: Let x = 4k + 3

$(2020 + x)^{2022} = (2020 + 4k + 3)^{2022} = (4M + 3)^{2022} = (4M + 4 - 1)^{2022}$

$\Rightarrow$Remainder = 1

Q.4 A line is a common tangent to the circle $(x-3)^2+y^2=9$ and the parabola $y^2=4x$. If the two points of contact (a, b) and (c, d) are distinct and lie in the first quadrant, then $2(a+c)$ is equal to _____.

Ans: 9

Sol: Tangent to the parabola is of the focus

$$y = mx + \frac{1}{m}$$

$$\Rightarrow m^2x - my + 1 = 0$$

This is a tangent to circle $(x-3)^2 + y^2 = 9$

$$\Rightarrow \left|\frac{3m^2 - 0 + 1}{\sqrt{m^2 + m^4}}\right| = 3$$

$$\Rightarrow m^2 = \frac{1}{3} \Rightarrow m = \pm\frac{1}{\sqrt{3}}$$

When $m = \frac{1}{\sqrt{3}}$

Intersection with parabola is $\begin{matrix}(c,\ d)\\ (3,\ 2\sqrt{3})\end{matrix}$ and intersection with circle is $\begin{pmatrix}\frac{3}{2},\ \frac{3\sqrt{3}}{2}\\ (a,\ b)\end{pmatrix}$

$$2(a+c) = 2\left(\frac{3}{2} + 3\right) = 9$$

Q.5 A line 'l' passing through origin is perpendicular to the lines

$$l_1 : \vec{r} = (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}$$

$$l_2 : \vec{r} = (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}$$

If the co-ordinates of the point in the first octant on 'l_2' at a distance of $\sqrt{17}$ from the point of intersection of 'l' and 'l_1' are (a, b, c), then $18(a+b+c)$ is equal to _____.

Ans: 44

Sol: D.r of l_1 is 1, 2, 2
D.R of l_2 is 2, 2, 1
D.r of l which $\perp^r$ to l_1 and l_2 is –2, 3, –2

$\Rightarrow$ Equation of l is $\frac{x}{2} = \frac{y}{3} = \frac{z}{-3}$

Point of intersection of l and l_1 is P(2, –3, 2)
The general point on ℓ_2 is given by Q(2λ + 3, 2λ + 3, λ + 2)
Given PQ = $\sqrt{17}$

Solving we get $\lambda = -2, \frac{-10}{9}$

$$\lambda \neq -2 \Rightarrow \lambda = \frac{-10}{9}$$

$$\Rightarrow Q - \left(\frac{7}{9},\ \frac{7}{9},\ \frac{8}{9}\right) = (a,\ b,\ c)$$

$$\therefore 18(a+b+c) = 18\left(\frac{7}{9} + \frac{7}{9} + \frac{8}{9}\right) = 44$$

Q.6 If the curve, $y = y(x)$ represented by the solution of the differential equation $(2xy^2 - y)dx + xdy = 0$, passes through the intersection of the lines, $2x - 3y = 1$ and $3x + 2y = 8$, then $|y(1)|$ is equal to _____.

Ans: 1

Sol: $\frac{ydx - xdy}{y^2} = 2xdx$

$$d\left(\frac{x}{y}\right) = 2xdx$$

$$\frac{x}{y} = \frac{2x^2}{2} + C$$

$$\frac{x}{y} = x^2 + C \rightarrow (1)$$

Point of intersection of $2x - 3y = 1$ and $3x + 2y = 8$ is (2, 1)
Substitute in (1)
$2 = 4 + C \Rightarrow C = -2$

$$\Rightarrow \frac{x}{y} = x^2 - 2$$

When x = 1, $\frac{1}{y} = -1 \Rightarrow y = -1$

$$|y(1)| = 1$$

Q.7 The value of $\int_{-2}^{2} |3x^2 - 3x - 6|\,dx$ is _____.

Ans: 19

Sol: $3x^2 - 3x - 6 \geq 0$

$$\Rightarrow 3(x^2 - x - 2) \geq 0$$

$$3(x-2)(x+1) \geq 0$$

$$3\int_{-2}^{2} |x^2 - x - 2|\,dx = 3\left[\int_{-2}^{-1} (x^2 - x - 2)dx + \int_{-1}^{2} (-x^2 + x + 2)dx\right] = 19$$

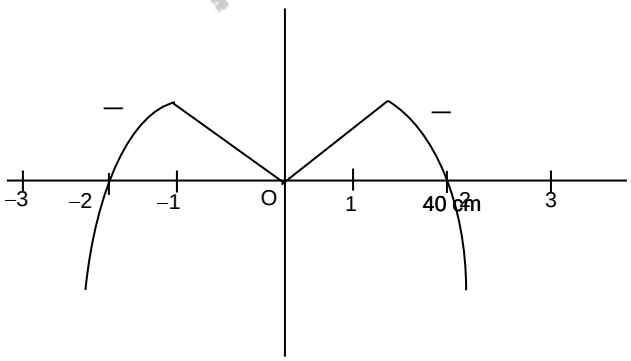
Q.8 A function f is defined on $[-3, 3]$ as

$$f(x) = \begin{cases} \min\{|x|, 2 - x^2\}, & -2 \leq x \leq 2 \\ [|x|], & 2 < |x| \leq 3 \end{cases}$$

where $[x]$ denotes the greatest integer $\leq x$. The number of points, where f is not differentiable in $(-3, 3)$ is _____.

Ans: 5

Sol: The graph of the function is as shown below



There are 5 sharp turning points. So no: of points at which the function is not differentiable is 5

Q.9 Let $\vec{a} = \hat{i} + \alpha\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - \alpha\hat{j} + \hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $8\sqrt{3}$ square units, then $\vec{a} \cdot \vec{b}$ is equal to _____.

Ans: 2

Sol:

$\bar{a} = i + \alpha j + 3k$ $\qquad$ $\bar{b} = 3i - \alpha j + k$

$|a| = \sqrt{1+\alpha^2+9}$ $\qquad$ $|b| = \sqrt{9+\alpha^2+1}$

$|a|^2 = 10 + \alpha^2$ $\qquad$ $|b|^2 = 10 + \alpha^2$

Area of parallelogram $= |\bar{a} \times \bar{b}| = 8\sqrt{3}$

$$\bar{a} \times \bar{b} = \begin{vmatrix} i & j & k \\ 1 & \alpha & 3 \\ 3 & -\alpha & 1 \end{vmatrix} = 4\alpha i + 8j - 4\alpha k$$

$|a \times b| = \sqrt{16\alpha^2 + 64 + 16\alpha^2} = \sqrt{32\alpha^2 + 64}$

$|a \times b|^2 = 32\alpha^2 + 64 = 192$

$\Rightarrow \alpha^2 = 4$

Now $|a \times b|^2 + (a \cdot b)^2 = |a|^2 |b|^2$

$192 + (a \cdot b)^2 = 196$

$(a \cdot b)^2 = 4$

$a \cdot b = 2$

Q.10 If $\lim\limits_{x \to 0} \frac{ax - (e^{4x} - 1)}{ax(e^{4x} - 1)}$ exists and is equal to b, then the value of $a - 2b$ is _____.

Ans: 5

Sol: $\lim\limits_{x \to 0} \frac{ax - \left(e^{4x} - 1\right)}{ax\left(e^{4x} - 1\right)}$

Applying L Hospitals rule

$\lim\limits_{x \to 0} \frac{a - 4e^4 x}{a\left(e^{4x} - 1\right) + ax4e^{4x}} \Rightarrow a = 4$

Applying L Hospitals rule again

$\lim\limits_{x \to 0} \frac{-16e^{4x}}{a \cdot 4e^{4x} + a \cdot 4e^{4x} + ax16a^{4x}} = b$

$\Rightarrow \frac{-16}{4a + 4a} = b$

$\frac{-16}{8 \times 4} = b \Rightarrow b = \frac{-1}{2}$

$a - 2b = 4 - 2 \times \frac{-1}{2} = 5$