# SOLUTIONS & ANSWERS FOR JEE MAINS-2021
# 26th February Shift 2

## [PHYSICS, CHEMISTRY & MATHEMATICS]

## PART – A – PHYSICS

### SECTION A

**Q.1** The trajectory of a projectile in a vertical plane is $y = \alpha x - \beta x^2$, where $\alpha$ and $\beta$ are constants and $x$ & $y$ are respectively the horizontal and vertical distances of the projectile from the point of projection. The angle of projection $\theta$ and the maximum height attained H are respectively given by :

**Options**

1. $\tan^{-1}\alpha, \ \frac{\alpha^2}{4\beta}$
2. $\tan^{-1}\left(\frac{\beta}{\alpha}\right), \ \frac{\alpha^2}{\beta}$
3. $\tan^{-1}\beta, \ \frac{\alpha^2}{2\beta}$
4. $\tan^{-1}\alpha, \ \frac{4\alpha^2}{\beta}$

**Ans: 1**

**Sol:** $y = \alpha x - \beta x^2$

$$y = \alpha x\left(1 - \frac{x}{\alpha/\beta}\right) - - - - - (1)$$

We have

$$y = x\tan\theta\left(1 - \frac{x}{R}\right) - - - - (2)$$

Comparing (1) and (2)

$\tan\theta = \alpha \Rightarrow \theta = \tan^{-1}\alpha$

$$\frac{R}{H} = \frac{4}{\tan\theta}$$

$$H = \frac{R\tan\theta}{4} \Rightarrow H = \frac{\alpha^2}{4\beta}$$

**Q.2** Given below are two statements : one is labeled as **Assertion A** and the other is labeled as **Reason R**.

**Assertion A :** For a simple microscope, the angular size of the object equals the angular size of the image.

**Reason R :** Magnification is achieved as the small object can be kept much closer to the eye than 25 cm and hence it subtends a large angle.

In the light of the above statements, choose the most appropriate answer from the options given below :

**Options**
1. A is false but R is true
2. Both A and R are true and R is the correct explanation of A
3. A is true but R is false
4. Both A and R are true but R is NOT the correct explanation of A

**Ans:** **2**

**Sol:** Basic concept

**Q.3** Given below are two statements :

Statement I : A second's pendulum has a time period of 1 second.

Statement II : It takes precisely one second to move between the two extreme positions.

In the light of the above statements, choose the correct answer from the options given below :

**Options**
1. Both Statement I and Statement II are true
2. Statement I is true but Statement II is false
3. Statement I is false but Statement II is true
4. Both Statement I and Statement II are false

**Ans:** **3**

**Sol:** Basic concept

**Q.4** A wire of 1 Ω has a length of 1 m. It is stretched till its length increases by 25%. The percentage change in resistance to the nearest integer is :

**Options**
1. 76%
2. 56%
3. 12.5%
4. 25%

**Ans:** **2**

**Sol:**
$$\ell' = \ell + \frac{\ell}{4} = \frac{5\ell}{4}$$
$$A'\ell = A\ell$$
$$A' = \frac{A\ell}{5\ell/4} = \frac{4A}{5}$$

$$R' = \rho \frac{\left(5\,\ell/4\right)}{4\,A/5} = \frac{25}{16}\frac{\rho\ell}{A}$$

% change in R = $\frac{R' - R}{R} \times 100\%$

$$= \left(\frac{25}{16} - 1\right) \times 100 = \frac{9}{16} \times 100 = 56.25\%$$

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**Q.5** If 'C' and 'V' represent capacity and voltage respectively then what are the dimensions of λ where C/V = λ ?

**Options**

1. $[M^{-2} L^{-4} I^3 T^7]$
2. $[M^{-1} L^{-3} I^{-2} T^{-7}]$
3. $[M^{-3} L^{-4} I^3 T^7]$
4. $[M^{-2} L^{-3} I^2 T^6]$

**Ans:** 1

**Sol:** $\frac{c}{V} = \frac{Q}{V \cdot V} = \frac{Q}{V^2} = \frac{Q}{\left(W/Q\right)^2} = \frac{Q^3}{W^2}$

$$= \frac{IT}{\left[Work/Q\right]^2} = \frac{(IT)^3}{\left[ML^2T^{-2}\right]^2} = \frac{I^3T^3}{M^2L^4T^{-4}}$$

$$= M^{-2}L^{-4}T^7I^3$$

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**Q.6** The internal energy (U), pressure (P) and volume (V) of an ideal gas are related as U = 3PV + 4. The gas is :

**Options**

1. monoatomic only.
2. diatomic only.
3. polyatomic only.
4. either monoatomic or diatomic.

**Ans:** 3

**Sol:** $du = \frac{f}{2} nRdT$

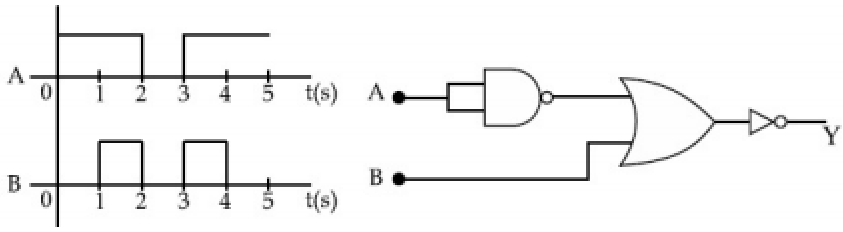
U = 3 PV + 4

U = 3 nRT + 4

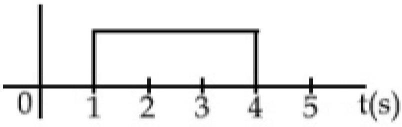
dU = 3nRdT

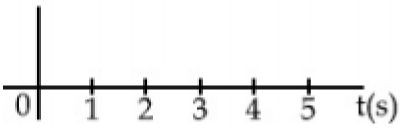
$\frac{f}{2} = 3 \Rightarrow f = 6 \Rightarrow$ Triatomic

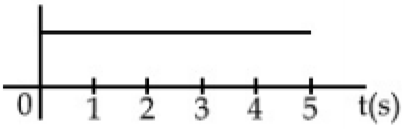
**Q.7** Draw the output signal Y in the given combination of gates.

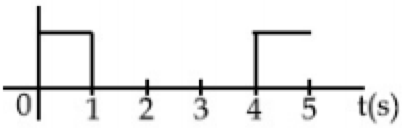



**Options**

1. 


2. 


3. 


4. 


**Ans: 4**

**Sol**: Basic concept

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**Q.8** The incident ray, reflected ray and the outward drawn normal are denoted by the unit vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively. Then choose the correct relation for these vectors.

**Options**

1. $\vec{b} = \vec{a} - \vec{c}$
2. $\vec{b} = \vec{a} + 2\vec{c}$
3. $\vec{b} = \vec{a} - 2\left(\vec{a} \cdot \vec{c}\right)\vec{c}$
4. $\vec{b} = 2\vec{a} + \vec{c}$

**Ans: 3**

**Sol:** Basic concept

**Q.9** A tuning fork A of unknown frequency produces 5 beats/s with a fork of known frequency 340 Hz. When fork A is filed, the beat frequency decreases to 2 beats/s. What is the frequency of fork A ?

**Options**
1. 342 Hz
2. 345 Hz
3. 335 Hz
4. 338 Hz

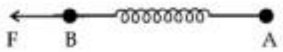
**Ans:** 3

**Sol:** Beat frequency $|f_2 - f_1| = 5$

i.e., $f_2$ = 335 Hz or $f_2$ = 445 Hz

As the second tuning fork is filed, $f_2$ increases and according to given question beat decreases. Hence the frequency of second tuning fork is 335

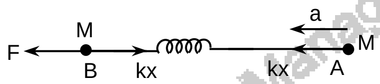
**Q.10** Two masses A and B, each of mass M are fixed together by a massless spring. A force acts on the mass B as shown in figure. If the mass A starts moving away from mass B with acceleration 'a', then the acceleration of mass B will be :




**Options**
1. $\frac{F + Ma}{M}$
2. $\frac{MF}{F + Ma}$
3. $\frac{F - Ma}{M}$
4. $\frac{Ma - F}{M}$

**Ans:** 3

**Sol:**




$$a_A = \frac{kx}{M}$$

$$a_B = \frac{F - kx}{M}$$

$$a_B = \frac{F - M_a}{M}$$

**Q.11** Given below are two statements :

Statement I : An electric dipole is placed at the centre of a hollow sphere. The flux of electric field through the sphere is zero but the electric field is not zero anywhere in the sphere.

Statement II : If R is the radius of a solid metallic sphere and Q be the total charge on it. The electric field at any point on the spherical surface of radius r (< R) is zero but the electric flux passing through this closed spherical surface of radius r is not zero.

In the light of the above statements, choose the correct answer from the options given below :

**Options**
1. Both Statement I and Statement II are false
2. Statement I is true but Statement II is false
3. Both Statement I and Statement II are true
4. Statement I is false but Statement II is true

**Ans:** **2**

**Sol:** Basic concept

**Q.12** An aeroplane, with its wings spread 10 m, is flying at a speed of 180 km/h in a horizontal direction. The total intensity of earth's field at that part is $2.5 \times 10^{-4}$ Wb/m$^2$ and the angle of dip is 60°. The emf induced between the tips of the plane wings will be ________.

**Options**
1. 54.125 mV
2. 62.50 mV
3. 108.25 mV
4. 88.37 mV

**Ans:** **3**

**Sol:** $B_V = B \sin 60$

emf = $B_V \ell V$

emf = $2.5 \times 10^{-4} \times \frac{\sqrt{3}}{2} \times 10 \times 50$

$= 25 \times 25 \times \sqrt{3} \times 10^{-4}$

= 108.25 mV



**Q.13** A particle executes S.H.M., the graph of velocity as a function of displacement is :

**Options**
1. a parabola.
2. an ellipse.
3. a circle.
4. a helix.

**Ans:** **2**

**Sol:**

$$V = \omega\sqrt{A^2 - x^2}$$

$$V^2 = \omega^2\left(A^2 - x^2\right)$$

$$V^2 = \omega^2 A^2 - \omega^2 x^2$$

$$\frac{x^2}{A^2}+\frac{V^2}{\omega^2A^2}=1$$

Compare with standard equation

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \Rightarrow$$ Equation of ellipse

**Q.14** A cord is wound round the circumference of wheel of radius r. The axis of the wheel is horizontal and the moment of inertia about it is I. A weight mg is attached to the cord at the end. The weight falls from rest. After falling through a distance 'h', the square of angular velocity of wheel will be :

**Options**

1. $\frac{2mgh}{I + mr^2}$
2. $\frac{2gh}{I + mr^2}$
3. $\frac{2mgh}{I + 2mr^2}$
4. $2gh$

**Ans: 1**

**Sol:** According to law of conservation of energy

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}m\left(r^2\omega^2\right) + \frac{1}{2}I\omega^2$$

$$\omega^2 = \frac{2mgh}{mr^2 + I}$$



**Q.15** A radioactive sample is undergoing α decay. At any time $t_1$, its activity is A and another time $t_2$, the activity is $\frac{A}{5}$. What is the average life time for the sample ?

**Options**

1. $\frac{\ln(t_2 + t_1)}{2}$
2. $\frac{t_2 - t_1}{\ln 5}$
3. $\frac{t_1 - t_2}{\ln 5}$
4. $\frac{\ln 5}{t_2 - t_1}$

**Ans: 2**

**Sol:** $A = A_0 e^{-\lambda t}$

$$A = A_0 e^{-\lambda t_1} -----(1)$$

$\frac{A}{5} = A_0 e^{-\lambda t_2}$ -----(2)

From (1) and (2)

$5 = \frac{e^{-\lambda t_1}}{e^{-\lambda t_2}}$

$5 = e^{\lambda(t_2 - t_1)}$

$\lambda = \frac{\ell n\,5}{t_2 - t_1}$

Average life = $\frac{1}{\lambda} = \frac{t_2 - t_1}{\ell n\,5}$

**Q.16** The recoil speed of a hydrogen atom after it emits a photon in going from n = 5 state to n = 1 state will be :

**Options**
1. 4.17 m/s
2. 4.34 m/s
3. 3.25 m/s
4. 2.19 m/s

**Ans:** 1

**Sol:** According to conservation of momentum

$mv = \frac{h}{\lambda}$

$v = \frac{h}{m\lambda}$

But $\frac{1}{\lambda} = R\left(1 - \frac{1}{25}\right)$

Or $\lambda = \frac{25}{24R}$

$\therefore v = \frac{24\,hR}{25m} = 4.17\,m/s$



**Q.17** The length of metallic wire is $l_1$ when tension in it is $T_1$. It is $l_2$ when the tension is $T_2$. The original length of the wire will be :

**Options**
1. $\frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$
2. $\frac{T_1 l_1 - T_2 l_2}{T_2 - T_1}$
3. $\frac{l_1 + l_2}{2}$
4. $\frac{T_2 l_1 + T_1 l_2}{T_1 + T_2}$

**Ans: 1**

**Sol:** $Y = \frac{F\ell}{A\Delta\ell} \Rightarrow \Delta\ell = \frac{F\ell}{AY}$

$$\ell_1 - \ell = \frac{T_1\ell}{YA}$$

$$\ell_2 - \ell = \frac{T_2\ell}{YA}$$

$$\frac{\ell_1 - \ell}{\ell_2 - \ell} = \frac{T_1}{T_2} \Rightarrow \ell_1 T_2 - \ell T_2 = T_1 \ell_2 - \ell T_1$$

$$\therefore \ell = \frac{T_1\ell_2 - T_2\ell_1}{T_1 - T_2}$$

**Q.18** A scooter accelerates from rest for time $t_1$ at constant rate $a_1$ and then retards at constant rate $a_2$ for time $t_2$ and comes to rest. The correct value of $\frac{t_1}{t_2}$ will be :

**Options**

1. $\frac{a_2}{a_1}$

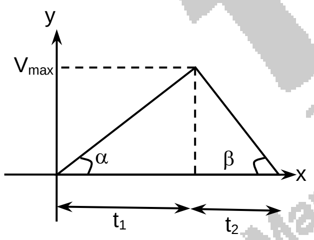
2. $\frac{a_1 + a_2}{a_1}$

3. $\frac{a_1}{a_2}$

4. $\frac{a_1 + a_2}{a_2}$

**Ans: 1**

**Sol:**




$$\tan\alpha = \frac{V_{max}}{t_1}$$

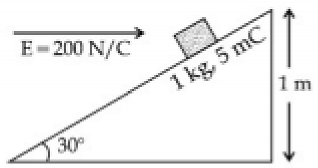
$$a_1 = \frac{V_{max}}{t_1}$$

$$\tan\beta = \frac{V_{max}}{t_2}$$

$$\frac{a_1}{a_2} = \frac{t_2}{t_1} \Rightarrow \frac{t_1}{t_2} = \frac{a_2}{a_1}$$

**Q.19** An inclined plane making an angle of 30° with the horizontal is placed in a uniform horizontal electric field $200\ \frac{N}{C}$ as shown in the figure. A body of mass 1 kg and charge 5 mC is allowed to slide down from rest at a height of 1 m. If the coefficient of friction is 0.2, find the time taken by the body to reach the bottom.

$[g = 9.8\ m/s^2;\ \sin 30° = \frac{1}{2};\ \cos 30° = \frac{\sqrt{3}}{2}]$

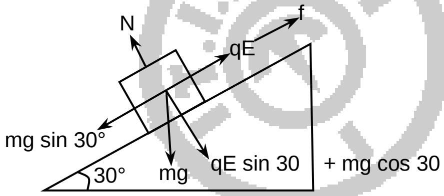



**Options**

1. 0.92 s
2. 1.3 s
3. 2.3 s
4. 0.46 s

**Ans: 2**

**Sol:**




N = q E sin 30° + mg cos 30°

$N = 5 \times 10^{-3} \times 200 \times \frac{1}{2} + 1 \times 10 \times \frac{\sqrt{3}}{2}$

N = 9.16 N

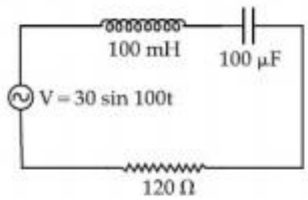
Acceleration, $a = \frac{mg \sin 30 - qE \cos 30 - \mu N}{m}$

$a = \frac{5 - 0.866 - 1.832}{1}$

a = 2.302

$t = \sqrt{\frac{2\ell}{a}} = \sqrt{\frac{2 \times 2}{2.302}} \Rightarrow t = 1.31\,s$

**Q.20** Find the peak current and resonant frequency of the following circuit (as shown in figure).




**Options**

1. 0.2 A and 50 Hz
2. 2 A and 100 Hz
3. 2 A and 50 Hz
4. 0.2 A and 100 Hz

**Ans: 1**

**Sol:** $i = \frac{V}{Z}$

$$i = \frac{V}{\sqrt{R^2 + (X_c - X_L)^2}} = \frac{30}{\sqrt{120^2 + \left(\frac{1}{100 \times 100 \times 10^{-6}} - 100 \times 100 \times 10^{-3}\right)^2}}$$

$$i = \frac{30}{\sqrt{120^2 + (100 - 10)^2}} = \frac{30}{\sqrt{120^2 + 90^2}}$$

$$i = \frac{30}{150} = 0.2A$$

$$\omega \frac{1}{\sqrt{Lc}} = \frac{1}{\sqrt{100 \times 10^{-3} \times 100 \times 10^{-6}}}$$

$$\omega = \sqrt{10^5}$$

$$\omega = 100\pi$$

f = 50 Hz

## SECTION B

**Q.1** A particle executes S.H.M. with amplitude 'a' and time period 'T'. The displacement of the particle when its speed is half of maximum speed is $\frac{\sqrt{x}a}{2}$. The value of $x$ is ________.

**Ans: 3.00**

**Sol:** $V = \omega\sqrt{A^2 - X^2}$

$$\frac{\omega A}{2} = \omega\sqrt{A^2 - X^2}$$

$$\frac{A^2}{4} = A^2 - X^2$$

X = $\pm\frac{\sqrt{3}}{2}A$

i.e., x = 3

**Q.2** Time period of a simple pendulum is T. The time taken to complete $\frac{5}{8}$ oscillations starting from mean position is $\frac{\alpha}{\beta}T$. The value of $\alpha$ is ________ .

**Ans: 7.00**

**Sol:** Distance covered in one oscillation = 4A

Distance covered = $\frac{5}{8}\times 4A = \frac{5}{2}A$

Time required to cover $\frac{5}{2}A = 2A + \frac{A}{2}$

Distance from mean position is

$\frac{T}{2}+\frac{T}{12}=7\frac{T}{12}$

$=\frac{\alpha}{\beta}T$

$\therefore\ \alpha = 7.00$

**Q.3** The volume V of a given mass of monoatomic gas changes with temperature T according to the relation $V = KT^{\frac{2}{3}}$. The workdone when temperature changes by 90 K will be $x$R. The value of $x$ is ________.
[R = universal gas constant]

**Ans: 60.00**

**Sol:** $V = KT^{2/3}$

$V = K\left(\frac{PV}{nR}\right)^{2/3}$

$P^{2/3}V^{2/3-1} = c$

$P^{2/3}V^{-1/3} = c$

$PV^{-1/2} = c$

$W = \frac{R(T_1 - T_2)}{1-\eta}$

$= \frac{R(90)}{1+\frac{1}{2}} = \frac{90R}{3/2} = 60R$

The value of x is 60.00

**Q.4** 27 similar drops of mercury are maintained at 10 V each. All these spherical drops combine into a single big drop. The potential energy of the bigger drop is ________ times that of a smaller drop.

**Ans: 243.00**

**Sol:** $\frac{4}{3}\pi R^3 = 27\times\frac{4}{3}\pi r^3$

R = 3r

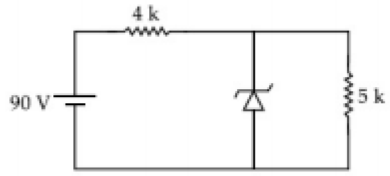
Potential energy of bigger drop

$PE = \frac{3}{5}\frac{KQ^2}{R}$

$= \frac{3}{5}\frac{K(27\,q)^2}{3r}$

$= \frac{3}{5}\frac{Kq^2}{r}\left(\frac{3^6}{3}\right) = 243\ PE_{smallerdrop}$

**Q.5** The zener diode has a $V_z = 30$ V. The current passing through the diode for the following circuit is ________ mA.



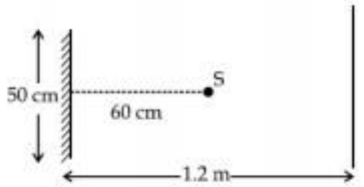

**Ans: 9.00**

**Sol:** Current through 5 KΩ = $\frac{30}{5} = 6\,mA$

Current through 4 KΩ = $\frac{60}{4\,K\Omega} = 15\,mA$

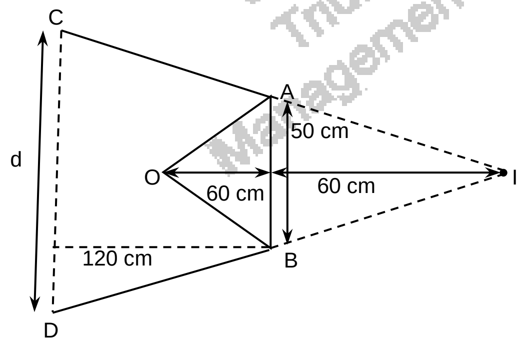
Current trough zener diode = 15 – 6 = 9 mA

**Q.6** A point source of light S, placed at a distance 60 cm infront of the centre of a plane mirror of width 50 cm, hangs vertically on a wall. A man walks infront of the mirror along a line parallel to the mirror at a distance 1.2 m from it (see in the figure). The distance between the extreme points where he can see the image of the light source in the mirror is __________ cm.




**Ans: 150.00**

**Sol:**




In ΔABI and CDI

$\frac{60}{180} = \frac{50}{d} \Rightarrow d = 150\ cm$

**Q.7** 1 mole of rigid diatomic gas performs a work of $\frac{Q}{5}$ when heat Q is supplied to it. The molar heat capacity of the gas during this transformation is $\frac{xR}{8}$. The value of $x$ is ________.
[R = universal gas constant]

**Ans:** **25.00**

**Sol:** $\Delta U = Q - W$

$$= Q - \frac{Q}{5} = \frac{4Q}{5}$$

$$\Delta T = \frac{\Delta U}{n\,C_V} = \frac{4Q}{5 \times 1 \times \frac{5R}{2}} = \frac{8Q}{25R}$$

$$C = \frac{\Delta Q}{n\Delta T} = \frac{Q}{1 \times \frac{8Q}{25R}} = \frac{25R}{8} \Rightarrow x = 25$$

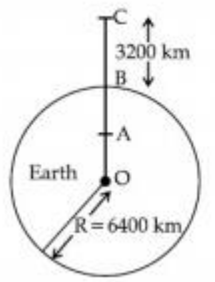
**Q.8** If the highest frequency modulating a carrier is 5 kHz, then the number of AM broadcast stations accommodated in a 90 kHz bandwidth are ________.

**Ans:** **9.00**

**Sol:** Band width = 2f = 10 kHz

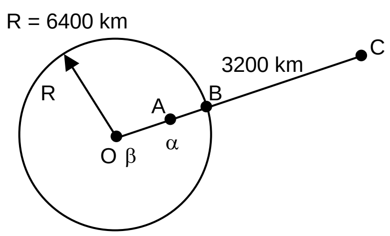
Number of AM broadcast stations = $\frac{90}{10} = 9$

**Q.9** In the reported figure of earth, the value of acceleration due to gravity is same at point A and C but it is smaller than that of its value at point B (surface of the earth). The value of OA : AB will be $x : y$. The value of $x$ is ________.




**Ans:** **4.00**

**Sol:**




$E_A = E_B$

$$\frac{GM}{(6400 + 3200)^2} = \frac{GM}{(6400)^3}\beta$$

$\beta = \frac{8}{9} \times 3200$

$\alpha = R - \beta$

$\alpha = 6400 - \frac{8}{9} \times 3200$

$= \frac{10}{9} \times 3200$

$\frac{\alpha}{\beta} = \frac{10}{8} = \frac{5}{4}$

i.e., x : y = 4 : 5 $\Rightarrow$ x = 4.00

**Q.10** Two stream of photons, possessing energies equal to twice and ten times the work function of metal are incident on the metal surface successively. The value of ratio of maximum velocities of the photoelectrons emitted in the two respective cases is $x : y$. The value of $x$ is __________.

**Ans:** **1.00**

**Sol:** $E = \phi + \frac{1}{2} m v_{max}^2$

$E_1 = 2\phi = \phi + \frac{1}{2} m v_1^2$

i.e., $2\phi = mv_1^2$

$E_2 = 10\phi = \phi + \frac{1}{2} m v_2^2$

i.e., $18\phi = mv_2^2$

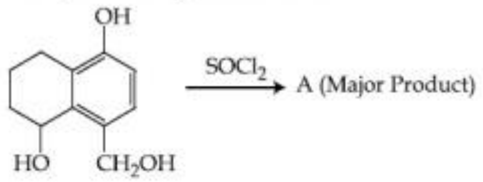
$\frac{v_1^2}{v_2^2} = \frac{1}{9} \Rightarrow \frac{v_1}{v_2} = \frac{1}{3}$

i.e., $\alpha = 1$

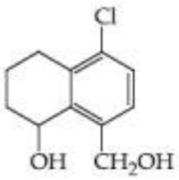
# PART – B – CHEMISTRY

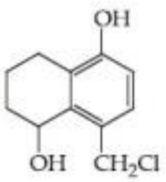
## SECTION A

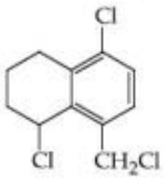
Q.1 Identify A in the given reaction.




Options

1.



2.



3.
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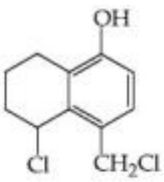
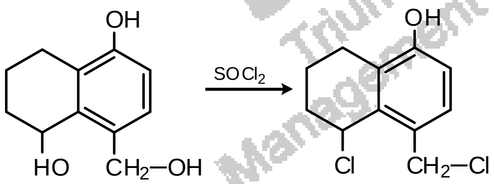



**Ans:** 4

**Sol:**



Nucleophilic substitution reaction is less preferred in aromatic compounds.

Q.2 Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.

**Assertion A** : In $TlI_3$, isomorphous to $CsI_3$, the metal is present in +1 oxidation state.

**Reason R** : Tl metal has fourteen *f* electrons in its electronic configuration.

In the light of the above statements, choose the most appropriate answer from the options given below :

Options

1. **A** is not correct but **R** is correct
2. Both **A** and **R** are correct and **R** is the correct explanation of **A**
3. **A** is correct but **R** is not correct
4. Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**

**Ans:** 4

**Sol:** Both the statements are correct, but (R) is not the correct explanation for (A).

Q.3 Match List-I with List-II.

| | List-I | | List-II |
|---|---|---|---|
| (a) | Sucrose | (i) | β-D-Galactose and β-D-Glucose |
| (b) | Lactose | (ii) | α-D-Glucose and β-D-Fructose |
| (c) | Maltose | (iii) | α-D-Glucose and α-D-Glucose |

Choose the correct answer from the options given below :

Options
1. (a) → (iii), (b) → (i), (c) → (ii)
2. (a) → (iii), (b) → (ii), (c) → (i)
3. (a) → (ii), (b) → (i), (c) → (iii)
4. (a) → (i), (b) → (iii), (c) → (ii)

**Ans:** 3

**Sol:** In sucrose, the glycosidic linkage is between $C_1$ of α-D-glucose and $C_2$ of β-D-fructose
In Lactose, the glycosidic linkage is between $C_1$ of β-D-galactose and $C_4$ of β-D-glucose
In Maltose, the glycosidic linkage is between $C_1$ of α-D-glucose and $C_4$ of α-D-glucose

Q.4 Match List-I with List-II.

| | List-I | | List-II |
|---|---|---|---|
| (a) | Siderite | (i) | Cu |
| (b) | Calamine | (ii) | Ca |
| (c) | Malachite | (iii) | Fe |
| (d) | Cryolite | (iv) | Al |
| | | (v) | Zn |

Choose the correct answer from the options given below :

Options
1. (a) → (iii), (b) → (v), (c) → (i), (d) → (iv)
2. (a) → (iii), (b) → (i), (c) → (v), (d) → (ii)
3. (a) → (i), (b) → (ii), (c) → (v), (d) → (iii)
4. (a) → (i), (b) → (ii), (c) → (iii), (d) → (iv)

**Ans:** 1

**Sol:** Siderite – $FeCO_3$
Calamine – $ZnCO_3$
Malachite – $CuCO_3.Cu(OH)_2$
Cryolite – $Na_3AlF_6$

Q.5
A. Phenyl methanamine
B. N,N-Dimethylaniline
C. N-Methyl aniline
D. Benzenamine

Choose the correct order of basic nature of the above amines.

Options
1. D > B > C > A
2. A > B > C > D
3. D > C > B > A
4. A > C > B > D

**Ans:** 2

**Sol:** The $pK_b$ values of
Phenyl methanamine – 4.70
N,N-Dimethylaniline – 8.92

N-Methylaniline – 9.3
Benzene amine – 9.38

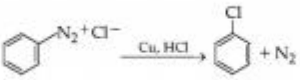
**Q.6** The nature of charge on resulting colloidal particles when $FeCl_3$ is added to excess of hot water is :

Options
1. negative
2. sometimes positive and sometimes negative
3. positive
4. neutral

**Ans:** 3

**Sol:** When $FeCl_3$ is added to hot water, a positively charged sol of hydrated ferric oxide is formed due to the adsorption of $Fe^{3+}$ ions.

**Q.7** Match List-I with List-II.

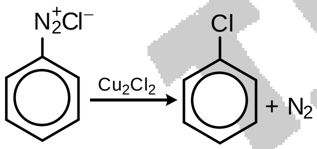
| | List-I | | List-II |
|---|---|---|---|
| (a) |  | (i) | Wurtz reaction |
| (b) |  | (ii) | Sandmeyer reaction |
| (c) | $2CH_3CH_2Cl + 2Na \xrightarrow{Ether} C_2H_5-C_2H_5 + 2NaCl$ | (iii) | Fittig reaction |
| (d) | $2C_6H_5Cl + 2Na \xrightarrow{Ether} C_6H_5-C_6H_5 + 2NaCl$ | (iv) | Gatterman reaction |

Choose the correct answer from the options given below :

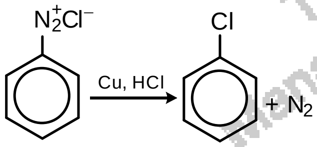
Options
1. (a) → (iii), (b) → (i), (c) → (iv), (d) → (ii)
2. (a) → (ii), (b) → (iv), (c) → (i), (d) → (iii)
3. (a) → (ii), (b) → (i), (c) → (iv), (d) → (iii)
4. (a) → (iii), (b) → (iv), (c) → (i), (d) → (ii)

**Ans:** 2

**Sol:**



– Sandmeyer's reaction



– Gattermann reaction

$2CH_3-CH_2Cl + 2Na \xrightarrow{Ether} C_2H_5-C_2H_5 + 2NaCl$ – Wurtz reaction

$2C_6H_5Cl + 2Na \xrightarrow{Ether} C_6H_5-C_6H_5 + 2NaCl$ – Fittig reaction

**Q.8** 2,4-DNP test can be used to identify :

Options
1. halogens
2. aldehyde
3. ether
4. amine

**Ans:** 2

**Sol:** 2,4-DNP test is used to identify carbonyl compounds (aldehydes & ketones)

**Q.9** In $\overset{1}{C}H_2=\overset{2}{C}=\overset{3}{C}H-\overset{4}{C}H_3$ molecule, the hybridization of carbon 1, 2, 3 and 4 respectively, are :

Options
1. $sp^3, sp, sp^3, sp^3$
2. $sp^2, sp, sp^2, sp^3$
3. $sp^2, sp^3, sp^2, sp^3$
4. $sp^2, sp^2, sp^2, sp^3$

**Ans:** **2**

**Sol:** $\overset{sp^2}{H_2C}=\overset{sp}{C}=\overset{sp^2}{CH}=\overset{sp^2}{CH_3}$

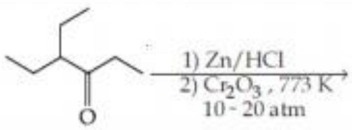
**Q.10** Which pair of oxides is acidic in nature ?

Options
1. $CaO, SiO_2$
2. $B_2O_3, CaO$
3. $B_2O_3, SiO_2$
4. $N_2O, BaO$

**Ans:** **3**

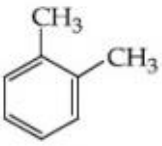
**Sol:** $B_2O_3$ – Acidic oxide
$SiO_2$ – Acidic oxide
$N_2O$ – Neutral oxide
$CaO$ – Basic oxide
$BaO$ – Basic oxide

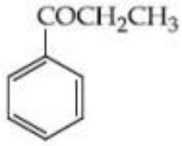
**Q.11**



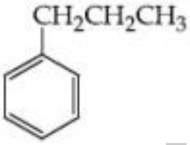

Considering the above reaction, the major product among the following is :

**Options**

1. 

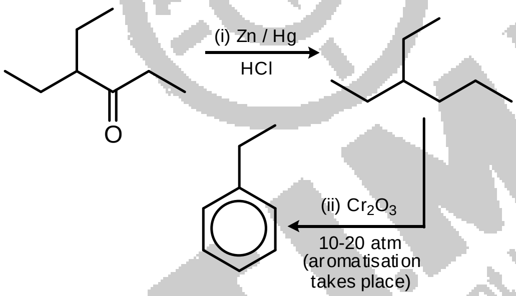

2. 


3. 


4. 


**Ans:** **3**

**Sol:**




**Q.12** Which of the following forms of hydrogen emits low energy $\beta^-$ particles ?

**Options**

1. Proton $H^+$
2. Deuterium ${}^2_1H$
3. Protium ${}^1_1H$
4. Tritium ${}^3_1H$

**Ans:** **4**

**Sol:** Tritium (${}^3_1H$) is the radioactive isotope of hydrogen which emits low energy β-particles.

**Q.13** Ceric ammonium nitrate and $CHCl_3$/alc. KOH are used for the identification of functional groups present in ________ and ________ respectively.

**Options**

1. alcohol, amine
2. alcohol, phenol
3. amine, phenol
4. amine, alcohol

**Ans:** 1

**Sol:** Ceric ammonium nitrate (CAN) is used to test for the presence of alcohol. With CAN (yellow coloured reagent), alcohols produce a colour change from yellow to red.
Primary amines with $CHCl_3$ / alc.KOH give the respective isocyanide (carbylamine)

**Q.14** The correct order of electron gain enthalpy is :

Options
1. Te > Se > S > O
2. O > S > Se > Te
3. S > Se > Te > O
4. S > O > Se > Te

**Ans:** 3

**Sol:** The negative electron gain enthalpy value in kJ $mol^{-1}$ is
S = –200
Se = –195
Te = –190
O = –141

**Q.15** Match List-I with List-II.

| | List-I | | List-II |
|---|---|---|---|
| (a) | Sodium Carbonate | (i) | Deacon |
| (b) | Titanium | (ii) | Castner-Kellner |
| (c) | Chlorine | (iii) | van-Arkel |
| (d) | Sodium hydroxide | (iv) | Solvay |

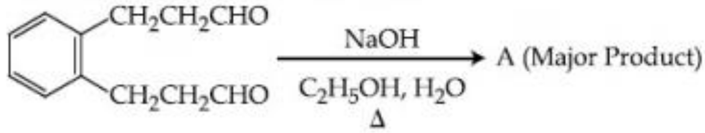
Choose the correct answer from the options given below :

Options
1. (a) → (iv), (b) → (iii), (c) → (i), (d) → (ii)
2. (a) → (iv), (b) → (i), (c) → (ii), (d) → (iii)
3. (a) → (i), (b) → (iii), (c) → (iv), (d) → (ii)
4. (a) → (iii), (b) → (ii), (c) → (i), (d) → (iv)

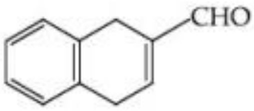
**Ans:** 1

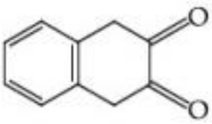
**Sol:** Sodium carbonate – Solvay process
Titanium – van-Arkel process
Chlorine – Deacon's process
Sodium hydroxide – Castner-Kellner process

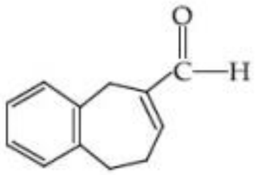
**Q.16** Identify A in the given chemical reaction.

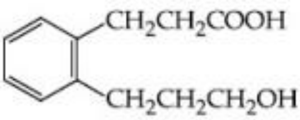



**Options**

1. 

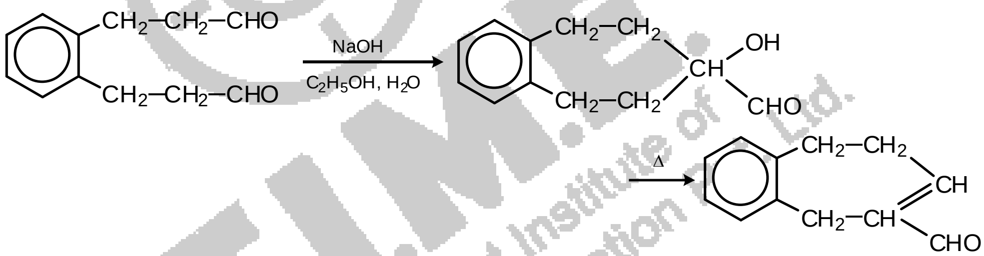

2. 


3. 

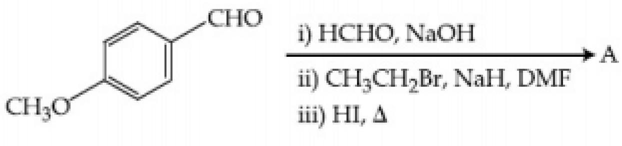

4. 


**Ans:** **3**

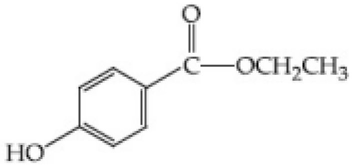
**Sol:**

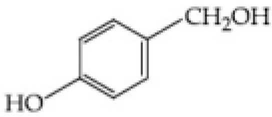


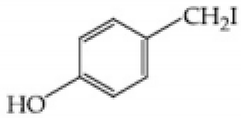
**Q.17** Identify A in the following chemical reaction.

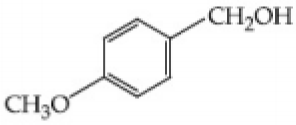



**Options**

1. 

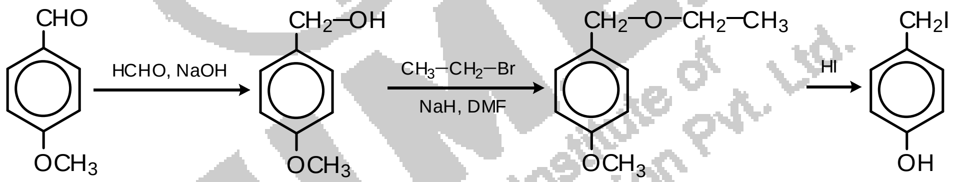

2. 


3. 


4. 


**Ans:** 3

**Sol:**




**Q.18** Match List-I with List-II.

| | List-I (Molecule) | | List-II (Bond order) |
|---|---|---|---|
| (a) | $Ne_2$ | (i) | 1 |
| (b) | $N_2$ | (ii) | 2 |
| (c) | $F_2$ | (iii) | 0 |
| (d) | $O_2$ | (iv) | 3 |

Choose the correct answer from the options given below :

**Options**

1. (a) → (ii), (b) → (i), (c) → (iv), (d) → (iii)
2. (a) → (iii), (b) → (iv), (c) → (i), (d) → (ii)
3. (a) → (i), (b) → (ii), (c) → (iii), (d) → (iv)
4. (a) → (iv), (b) → (iii), (c) → (ii), (d) → (i)

**Ans:** 2

**Sol:** For $Ne_2$, B.O = $\frac{1}{2}$ [10 – 10] = 0

For $N_2$, B.O = $\frac{1}{2}$ [10 – 4] = 3

For $F_2$, B.O = $\frac{1}{2}$ [10 – 8] = 1

For $O_2$, B.O = $\frac{1}{2}$ [10 – 6] = 2

**Q.19** Calgon is used for water treatment. Which of the following statement is NOT true about Calgon ?

**Options**
1. It is polymeric compound and is water soluble.
2. It doesnot remove $Ca^{2+}$ ion by precipitation.
3. Calgon contains the 2nd most abundant element by weight in the Earth's crust.
4. It is also known as Graham's salt.

**Ans:** 3

**Sol:** Calgon is sodium hexameta phosphate. The second most abundant element is silicon and it is not present in Calgon.

**Q.20** Seliwanoff test and Xanthoproteic test are used for the identification of _________ and _________ respectively.

**Options**
1. aldoses, ketoses
2. proteins, ketoses
3. ketoses, aldoses
4. ketoses, proteins

**Ans:** 4

**Sol:** Seliwanoff's test is a chemical test which distinguishes between aldose and ketose. Xanthoproteic reaction is a method that can be used to detect the presence of protein soluble in a solution.

## SECTION B

**Q.1** The number of stereoisomers possible for $[Co(ox)_2(Br)(NH_3)]^{2-}$ is _________.
[ox = oxalate]

**Ans:** 3

**Sol:** $[Co(ox)_2(Br)(NH_3)]^{2-}$ can exhibit cis and trans isomerism. Among these, cis isomer exhibit optical isomerism.

**Q.2** The pH of ammonium phosphate solution, if $pk_a$ of phosphoric acid and $pk_b$ of ammonium hydroxide are 5.23 and 4.75 respectively, is _________.

**Ans:** 7

**Sol:** $pH = 7 + \frac{1}{2}[pK_a - pK_b]$

$= 7 + \frac{1}{2}[5.23 - 4.75] = 7.24$

**Q.3** When 12.2 g of benzoic acid is dissolved in 100 g of water, the freezing point of solution was found to be −0.93°C ($K_f(H_2O) = 1.86$ K kg $mol^{-1}$). The number (n) of benzoic acid molecules associated (assuming 100% association) is _________.

**Ans:** 2

**Sol:** $\Delta T_f = \frac{i.K_f.W_B.1000}{M_B W_A}$

$i = \frac{0.93 \times 122 \times 1000}{1.86 \times 12.2 \times 1000} = 0.5$

$\alpha = \frac{1-i}{1-\frac{1}{n}}$

$1 = \frac{1-0.5}{1-\frac{1}{n}} = \frac{0.5}{1-\frac{1}{n}}$

$\therefore 1-\frac{1}{n} = 0.5$

$\frac{1}{n} = 0.5 = \frac{1}{2}$

$\therefore$ n = 2

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**Q.4** Emf of the following cell at 298 K in V is $x \times 10^{-2}$.

$Zn|Zn^{2+}\ (0.1\ M)||Ag^{+}\ (0.01\ M)|Ag$

The value of $x$ is ________. (Rounded off to the nearest integer)

[Given : $E^{\theta}_{Zn^{2+}/Zn} = -0.76\ V$ ; $E^{\theta}_{Ag^{+}/Ag} = +0.80\ V$ ; $\frac{2.303RT}{F} = 0.059$]

**Ans:** **147**

**Sol:** $Zn_{(s)} + 2Ag^{+}_{(aq)} \rightarrow Zn^{2+}_{(aq)} + 2Ag_{(s)}$

$E = E^{\circ}_{cell} - \frac{0.059}{2}\log\frac{[Zn^{2+}]}{[Ag^{+}]^2}$

$= \mathbf{1.56} - \frac{0.059}{2}log\frac{10^{-1}}{[10^{-2}]^2}$

$= 1.56 - 0.0295 \log 10^3$

$= 1.56 - 0.0885$

$= 1.4715 = 147.15 \times 10^{-2}$

**Q.5** The average S – F bond energy in kJ $mol^{-1}$ of $SF_6$ is ________. (Rounded off to the nearest integer)

[Given : The values of standard enthalpy of formation of $SF_6(g)$, S(g) and F(g) are - 1100, 275 and 80 kJ $mol^{-1}$ respectively.]

Given –

**Ans:** **309**

**Sol:** $SF_{6(g)} \rightarrow S_{(g)} + 6F_{(g)}$

$\Delta_r H = \Sigma \Delta_r H_{(P)} - \Sigma \Delta_r H_{(R)}$

$= [275 + 6 \times 80] - [-1100] = 1855$ kJ

$\Delta_r H = 6 \times BE_{(S-F)}$

$\therefore BE_{(S-F)} = \frac{1855}{6} = 309.16$ kJ

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**Q.6** In mildly alkaline medium, thiosulphate ion is oxidized by $MnO_4^-$ to "A". The oxidation state of sulphur in "A" is ________.

**Ans:** **6**

**Sol:** In faintly alkaline medium, thiosulphate is oxidized by $MnO_4^-$ to sulphate

$\mathbf{8}MnO_4^- + \mathbf{3}S_2O_3^{2-} + H_2O \rightarrow 8MnO_2 + 6SO_4^{2-} + 2OH^-$

The oxidation state of S in $SO_4^{2-}$ is +6

**Q.7** The $NaNO_3$ weighed out to make 50 mL of an aqueous solution containing 70.0 mg $Na^+$ per mL is ________ g. (Rounded off to the nearest integer)

[Given : Atomic weight in g $mol^{-1}$ - Na : 23 ; N : 14 ; O : 16]

Given

**Ans:** **13**

**Sol:** 1 mL $\Rightarrow$ 70 Mg $Na^+$

$\therefore$ 50 mL $\Rightarrow 70 \times 50 = 3500$ Mg $Na^+$ = 3.5 g $Na^+$

23 g $Na^+ \rightarrow$ 85 g $NaNO_3$

$\mathbf{3.5\ g\ Na^+} \rightarrow \frac{3.5 \times 85}{23} = 12.9\ g$

**Q.8** A ball weighing 10 g is moving with a velocity of 90 $ms^{-1}$. If the uncertainty in its velocity is 5%, then the uncertainty in its position is ________ $\times 10^{-33}$ m. (Rounded off to the nearest integer)
[Given : $h = 6.63 \times 10^{-34}$ Js]

**Ans:** 1

**Sol:** $\Delta V = 90 \times \frac{5}{100} = 4.5\, ms^{-1}$

$\Delta x \,.\, m\Delta V = \frac{h}{4\pi}$

$\Delta x = \frac{h}{m.\Delta V.4\pi}$

$= \frac{6.63 \times 10^{-34}}{10 \times 10^{-3} \times 4.5 \times 4 \times 3.14} = \frac{6.63 \times 10^{-34}}{0.5652}$

$= 11.73 \times 10^{-34} = 1.173 \times 10^{-33}$

**Q.9** If the activation energy of a reaction is 80.9 kJ $mol^{-1}$, the fraction of molecules at 700 K, having enough energy to react to form products is $e^{-x}$. The value of $x$ is ________. (Rounded off to the nearest integer)
[Use $R = 8.31$ J $K^{-1}$ $mol^{-1}$]

**Ans:** 14

**Sol:** $e^{-\frac{E_a}{RT}} = e^{-x}$

i.e., $\frac{E_a}{RT} = x$

$\therefore \frac{80.9 \times 10^{-3}}{8.31 \times 700} = 13.9$

**Q.10** The number of octahedral voids per lattice site in a lattice is ________. (Rounded off to the nearest integer)

**Ans:** 1

**Sol:** For 'N' number of particles per unit cell there will be 'N' octahedral voids.

# PART – C – MATHEMATICS

## SECTION A

**Q.1** Let $f(x) = \sin^{-1} x$ and $g(x) = \frac{x^2 - x - 2}{2x^2 - x - 6}$. If $g(2) = \lim_{x \to 2} g(x)$, then the domain of the function $fog$ is :

**Options**

1. $(-\infty, -2] \cup \left[-\frac{3}{2}, \infty\right)$
2. $(-\infty, -1] \cup [2, \infty)$
3. $(-\infty, -2] \cup [-1, \infty)$
4. $(-\infty, -2] \cup \left[-\frac{4}{3}, \infty\right)$

**Ans:** 4

**Sol:** $g(2) = \lim_{x \to 2} \left(\frac{x^2-x-2}{2x^2-x-6}\right) = \lim_{x \to 2} \frac{2x-1}{4x-1} = \frac{3}{7}$

$fog(2) = f\{g(2)\} = \sin^{-1}\left(\frac{x^2 - x - 2}{2x^2 - x - 6}\right) = \sin^{-1}\left(\frac{(x-2)(x+1)}{(x-2)(2x+3)}\right) = \sin^{-1}\left(\frac{x+1}{2x+3}\right)$

$-1 \leq \frac{x+1}{2x+3} \leq 1 \;\; \ldots\ldots (1)$

$$\frac{x+1}{2x+3} = \frac{\frac{1}{2}(2x+3) - \frac{3}{2} + 1}{2x+3} = \frac{1}{2} + \frac{-\frac{1}{2}}{2x+3}$$

$$\therefore\ -1 \le \frac{1}{2} + \frac{-1/2}{2x+3} \le 1$$

$$-\frac{3}{2} \le \frac{-\frac{1}{2}}{2x+3} \le \frac{1}{2}$$

$$3 \ge \frac{1}{2x+3} \ge -1$$

$$-1 \le \frac{1}{2x+3} \le 0^- \quad \text{and } 0^+ \le \frac{1}{2x+3} \le 3$$

$$\left(\text{Since } \frac{1}{2x+3} \ne 0\right)$$

$$-1 \ge 2x+3 > -\infty \quad \text{and} \quad \infty > 2x+3 \ge \frac{1}{3}$$

$$-4 \ge 2x > -\infty \quad \text{and} \quad \infty > 2x \ge -\frac{8}{3}$$

$$-2 \ge x > -\infty \quad \text{and} \quad \infty > x \ge -\frac{4}{3}$$

$$x \in (-\infty, -2] \cup \left[-\frac{4}{3}, \infty\right)$$

**Q.2** A natural number has prime factorization given by $n = 2^x 3^y 5^z$, where $y$ and $z$ are such that $y + z = 5$ and $y^{-1} + z^{-1} = \frac{5}{6}$, $y > z$. Then the number of odd divisors of n, including 1, is :

**Options**
1. 6
2. 12
3. 11
4. $6x$

**Ans:** 2

**Sol:** $n = 2^x . 3^y . 5^z$

$y + z = 5 \ldots\ldots \quad (1)$

$\frac{1}{y} + \frac{1}{z} = \frac{5}{6} \ldots\ldots (2)$

$(2) \Rightarrow \frac{y+z}{yz} = \frac{5}{6} \quad \Rightarrow \frac{5}{yz} = \frac{5}{6} \quad \Rightarrow yz = 6 \ldots\ldots (3)$

From (1) and (3), $y = 3$ and $z = 2$ (Since $y > z$)

$\therefore n = 2^x . 3^3 . 5^2$

Consider : $1, 3, 3^2, 3^3$ and $1, 5, 5^2$

(Factors of $3^3$ and $5^2$)

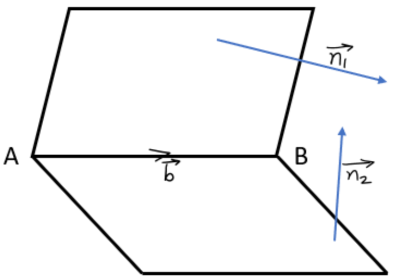
$\therefore$ number of odd factors $= 4 \times 3 = 12$

**Q.3** Let L be a line obtained from the intersection of two planes $x + 2y + z = 6$ and $y + 2z = 4$. If point $P(\alpha, \beta, \gamma)$ is the foot of perpendicular from (3, 2, 1) on L, then the value of $21(\alpha + \beta + \gamma)$ equals :

**Options**
1. 142
2. 136
3. 102
4. 68

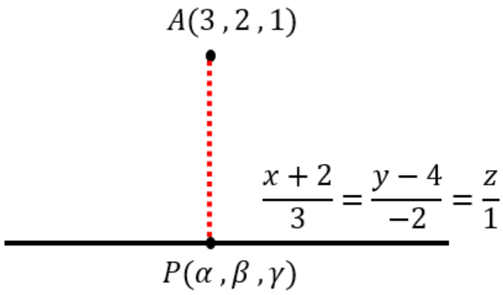
**Ans:** 3

**Sol:**




$b = n_1 \times n_2 = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 3i - 2j + k$

Equation of the line AB is $\dfrac{x+2}{3} = \dfrac{y-4}{-2} = \dfrac{z-0}{1}$




$(\alpha,\beta,\gamma)$ lies on the line

$\Rightarrow \dfrac{\alpha+2}{3} = \dfrac{\beta-4}{-2} = \dfrac{\gamma}{1} = \lambda$

$\Rightarrow \alpha = 3\lambda - 2, \beta = -2\lambda + 4$ and $\gamma = \lambda$

AP is perpendicular to the line $\Rightarrow \overrightarrow{AP}.\vec{b} = 0$

$\overrightarrow{AP} = (\alpha - 3)i + (\beta - 2)j + (\gamma - 1)k$

$\overrightarrow{AP}.\vec{b} = 3(\alpha - 3) - 2(\beta - 2) + 1(\gamma - 1) = 0$

$\Rightarrow 3(3\lambda - 5) - 2(-2\lambda + 2) + 1(\lambda - 1) = 0$

$\lambda = \dfrac{10}{7}$

$\alpha = 3\lambda - 2 = \dfrac{16}{7}, \beta = -2\lambda + 4 = \dfrac{8}{7}, \gamma = \lambda = \dfrac{10}{7}$

$21(\alpha + \beta + \gamma) = 21\left(\dfrac{16}{7} + \dfrac{8}{7} + \dfrac{10}{7}\right) = 102$



**Q.4** If $0 < a, b < 1$, and $\tan^{-1}a + \tan^{-1}b = \dfrac{\pi}{4}$, then the value of

$(a + b) - \left(\dfrac{a^2 + b^2}{2}\right) + \left(\dfrac{a^3 + b^3}{3}\right) - \left(\dfrac{a^4 + b^4}{4}\right) + \dots$ is :

**Options**

1. $e$
2. $\log_e\left(\dfrac{e}{2}\right)$
3. $\log_e 2$
4. $e^2 - 1$

**Ans:** 3

**Sol:** $\tan^{-1} a + \tan^{-1} b = \frac{\pi}{4}$

$\tan^{-1}\left(\dfrac{a+b}{1-ab}\right) = \dfrac{\pi}{4}$

$\frac{a+b}{1-ab}=1$ $\quad a+b=1-ab$

a +b + ab=1 $\quad 1 + a + b + ab = 2$

(1+a)(1+b)=2

The given sum= $\left(a-\frac{a^2}{2}+\frac{a^3}{3}-\frac{a^4}{4}+\cdots\right)+\left(b-\frac{b^2}{2}+\frac{b^3}{3}-\frac{b^4}{4}+\cdots\right)$

$=\log(1+a)+\log(1+b)=\log\{(1+a)(1+b)\}$

$\left[\log(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots\right]$ =log2

**Q.5** If vectors $\vec{a_1}=x\hat{i}-\hat{j}+\hat{k}$ and $\vec{a_2}=\hat{i}+y\hat{j}+z\hat{k}$ are collinear, then a possible unit vector parallel to the vector $x\hat{i}+y\hat{j}+z\hat{k}$ is :

**Options**

1. $\frac{1}{\sqrt{2}}\left(\hat{i}-\hat{j}\right)$
2. $\frac{1}{\sqrt{3}}\left(\hat{i}-\hat{j}+\hat{k}\right)$
3. $\frac{1}{\sqrt{2}}\left(-\hat{j}+\hat{k}\right)$
4. $\frac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}-\hat{k}\right)$

**Ans:** 2

**Sol:** $a_1=xi-j+k$ and $a_2=i+yj+zk$ are collinear.

$\frac{x}{1}=-\frac{1}{y}=\frac{1}{z}=\lambda \quad \Rightarrow x=\lambda, y=-\frac{1}{\lambda}, z=\frac{1}{\lambda}$

$xi+yj+zk=\lambda i-\frac{1}{\lambda}j+\frac{1}{\lambda}k \ (=\vec{a})$

$\hat{a}=\frac{\lambda i-\frac{1}{\lambda}j+\frac{1}{\lambda}k}{\sqrt{\lambda^2+\frac{1}{\lambda^2}+\frac{1}{\lambda^2}}}=\frac{\lambda^2 i-j+k}{\sqrt{\lambda^4+2}}$

If $\lambda=1$, $\hat{a}=\frac{i-j+k}{\sqrt{3}}$



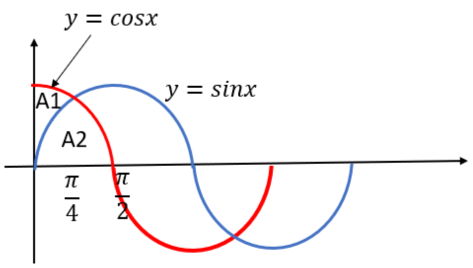
**Q.6** Let $A_1$ be the area of the region bounded by the curves $y=\sin x$, $y=\cos x$ and y-axis in the first quadrant. Also, let $A_2$ be the area of the region bounded by the curves $y=\sin x$, $y=\cos x$, x-axis and $x=\frac{\pi}{2}$ in the first quadrant. Then,

**Options**

1. $A_1:A_2=1:\sqrt{2}$ and $A_1+A_2=1$
2. $A_1:A_2=1:2$ and $A_1+A_2=1$
3. $2A_1=A_2$ and $A_1+A_2=1+\sqrt{2}$
4. $A_1=A_2$ and $A_1+A_2=\sqrt{2}$

**Ans:** 1

**Sol:**



$$A_1 = \int_0^{\pi/4} (\cos x - \sin x)dx = [\sin x + \cos x]_0^{\pi/4} = \sqrt{2} - 1$$

$$A_2 = \int_0^{\frac{\pi}{4}} \sin x\, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x\, dx = -[\cos x]_0^{\frac{\pi}{4}} + [\sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = 2 - \sqrt{2} = \sqrt{2}(\sqrt{2} - 1)$$

$A_1 : A_2 = 1 : \sqrt{2}$ and $A_1 + A_2 = 1$

**Q.7** Let $f : \mathbf{R} \to \mathbf{R}$ be defined as

$$f(x) = \begin{cases} 2\sin\left(-\frac{\pi x}{2}\right), & \text{if } x < -1 \\ |ax^2 + x + b|, & \text{if } -1 \le x \le 1 \\ \sin(\pi x), & \text{if } x > 1 \end{cases}$$

If $f(x)$ is continuous on **R**, then a + b equals :

**Options**
1. 3
2. −3
3. −1
4. 1

**Ans:** 3

**Sol:**

$$\lim_{x \to (-1)^-} f(x) = \lim_{x \to (-1)^-} \left\{2\sin\left(-\frac{\pi x}{2}\right)\right\} = 2\sin\frac{\pi}{2} = 2$$

$$\lim_{x \to (-1)^+} f(x) = \lim_{x \to (-1)^+} |(ax^2 + x + b)| = |a - 1 + b|$$

$|a - 1 + b| = 2 \ldots. (1)$

$$\lim_{x \to (1)^-} f(x) = \lim_{x \to (1)^-} |(ax^2 + x + b)| = |a + 1 + b|$$

$$\lim_{x \to (1)^+} f(x) = \lim_{x \to (1)^+} \sin\pi x = 0$$

$|a + 1 + b| = 0 \ldots. (2)$

From (1) and (2), a + b = –1

**Q.8** For $x > 0$, if $f(x) = \int_1^x \frac{\log_e t}{(1+t)} dt$, then $f(e) + f\left(\frac{1}{e}\right)$ is equal to :

**Options**
1. $\frac{1}{2}$
2. −1
3. 0
4. 1

**Ans:** 1

**Sol:**

$f(x) = \int_1^x \frac{\log t}{t+1}$

$$f(e) + f\left(\frac{1}{e}\right) = \int_1^e \frac{\log t}{t+1} dt + \int_1^{1/e} \frac{\log t}{t+1} dt$$

$$I_2 = \int_1^{1/e} \frac{\log t}{t+1} dt$$

$$\text{Let } t = \frac{1}{y} \Rightarrow dt = -\frac{1}{y^2} dy$$

$$\text{If } t = 1, y = 1 \text{ and if } t = \frac{1}{e}, y = e$$

$$I_2 = \int_1^e \frac{\log\left(\frac{1}{y}\right) \cdot \left(-\frac{1}{y^2}\right)}{1 + \frac{1}{y}} dy = \int_1^e \frac{\left(-\frac{1}{y^2}\right)(\log 1 - \log y)}{\left(\frac{y+1}{y}\right)} dy = \int_1^e \frac{1}{y^2} \log y \times \frac{y}{y+1} \,.\, dy$$

$= \int_1^e \frac{\log y}{y+1} dy = \int_1^e \log y \left(\frac{1}{y} - \frac{1}{y+1}\right) dy = \int_1^e \left(\frac{\log y}{y} - \frac{\log y}{y+1}\right) dy$

$= \int_1^e \frac{\log y}{y} dy - \int_1^e \frac{\log y}{y+1} dy = \int_1^e \frac{\log y}{y} dy - I_1$

$I_1 + I_2 = \int_1^e \frac{\log y}{y} dy$

Let $\log y = u \Rightarrow \frac{1}{y} dy = du$

$y = 1 \Rightarrow u = \log 1 = 0$ and $y = e \Rightarrow u = \log e = 1$

$\text{Ans} = I_1 + I_2 = \int_0^1 u\,.du = \left[\frac{u^2}{2}\right]_0^1 = \frac{1}{2}$

**Q.9** Let slope of the tangent line to a curve at any point $P(x, y)$ be given by $\frac{xy^2 + y}{x}$. If the curve intersects the line $x + 2y = 4$ at $x = -2$, then the value of $y$, for which the point $(3, y)$ lies on the curve, is :

**Options**
1. $\frac{18}{35}$
2. $-\frac{18}{11}$
3. $-\frac{4}{3}$
4. $-\frac{18}{19}$

**Ans:** 4

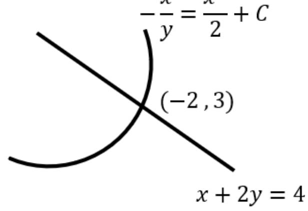
**Sol:** $\frac{dy}{dx} = \frac{xy^2+y}{x} = y^2 + \frac{y}{x}$

$y = vx \Rightarrow \frac{dy}{dx} = v + x.\frac{dv}{dx}$

$v + x\,.\frac{dv}{dx} = v^2x^2 + v$

$\frac{dv}{dx} = v^2x \quad \Rightarrow \int \frac{dv}{v^2} = \int x.\,dx - \frac{1}{v} = \frac{x^2}{2} + C$

$-\frac{x}{y} = \frac{x^2}{2} + C$




The point $(-2\,, 3)$ lies on the curve $-\frac{x}{y} = \frac{x^2}{2} + C$

$\frac{2}{3} = \frac{4}{2} + C \quad \Rightarrow C = -\frac{4}{3}$

Equation of the curve is $-\frac{x}{y} = \frac{x^2}{2} - \frac{4}{3} \ldots..(1)$

Substituting $(3\,, y)$ in (1), $-\frac{3}{y} = \frac{9}{2} - \frac{4}{3} \quad \Rightarrow y = -\frac{18}{19}$

**Q.10** Let $F_1(A, B, C) = (A \wedge {\sim}B) \vee [{\sim}C \wedge (A \vee B)] \vee {\sim}A$ and $F_2(A, B) = (A \vee B) \vee (B \to {\sim}A)$ be two logical expressions. Then :

**Options**
1. $F_1$ is a tautology but $F_2$ is not a tautology
2. $F_1$ is not a tautology but $F_2$ is a tautology
3. Both $F_1$ and $F_2$ are not tautologies
4. $F_1$ and $F_2$ both are tautologies

**Ans:** 2

**Sol:** If $A = T$ , $B = T$ and $C = T$ then $F_1 = F$

If $A = T$ , $B = T$ and $C = F$ then $F_1 = T$

Hence $F_1$ is neither tautology nor contradiction.

$F_2 = (A \vee B) \vee (B \rightarrow \sim A) = (A \vee B) \vee (\sim B \vee \sim A)$ [Since $p \Rightarrow q = \sim p \vee q$]

$= (A \vee \sim A) \vee (B \vee \sim B) = T \vee T = T$ (Tautology)

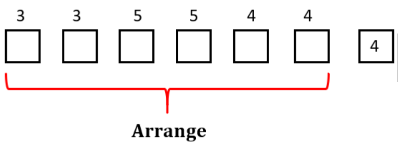
**Q.11** A seven digit number is formed using digits 3, 3, 4, 4, 4, 5, 5. The probability, that number so formed is divisible by 2, is :

**Options**

1. $\frac{6}{7}$
2. $\frac{4}{7}$
3. $\frac{1}{7}$
4. $\frac{3}{7}$

**Ans:** 4

**Sol:** Favourable:




Number of favourable case $= \frac{6!}{2!\,.\,2!\,.\,2!} = 90$

Total number of cases $= \frac{7!}{2!\,.\,3!\,.\,2!} = 210$

$P = \frac{90}{210} = \frac{3}{7}$

**Q.12** Consider the following system of equations :

$x + 2y - 3z = a$

$2x + 6y - 11z = b$

$x - 2y + 7z = c,$

where a, b and c are real constants. Then the system of equations :

**Options**

1. has a unique solution for all a, b and c
2. has a unique solution when $5a = 2b + c$
3. has no solution for all a, b and c
4. has infinite number of solutions when $5a = 2b + c$

**Ans:** 4

**Sol:** $|A| = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 6 & -11 \\ 1 & -2 & 7 \end{vmatrix} = 0$

$\Rightarrow$ No solution or infinite number of solutions.

$(2) - 2 \times (1) \Rightarrow 2y - 5z = b - 2a \ldots\ldots (A)$

$(1) - (3) \Rightarrow 4y - 10z = a - c \ldots\ldots\ldots (B)$

For infinite number of solutions,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{2}{4} = \frac{-5}{-10} = \frac{b - 2a}{a - c}$

$\frac{b - 2a}{a - c} = \frac{1}{2} \Rightarrow a - c = 2b - 4a \Rightarrow 5a = 2b + c$

**Q.13** Let $A(1, 4)$ and $B(1, -5)$ be two points. Let P be a point on the circle $(x-1)^2+(y-1)^2=1$ such that $(PA)^2+(PB)^2$ have maximum value, then the points, P, A and B lie on :

**Options**
1. a parabola
2. a hyperbola
3. an ellipse
4. a straight line

**Ans:** 4

**Sol:** The general point on the circle is $P(1+\cos\theta, 1+\sin\theta)$

$PA^2 + PB^2 = (\cos\theta)^2 + (\sin\theta - 3)^2 + (\cos\theta)^2 + (\sin\theta + 6)^2$

(By distance formula)

$= 2\cos^2\theta + 2\sin^2\theta - 6\sin\theta + 12\sin\theta + 9 + 36$

$= 2(\cos^2\theta + \sin^2\theta) + 6\sin\theta + 45 = 47 + 6\sin\theta$ (To be maximum)

$\Rightarrow \theta = \dfrac{\pi}{2}$

$\Rightarrow P(1, 2), A(1, 4)$ and $B(1, -5)$

P, A and B lies on a line.

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**Q.14** Let $f(x) = \int_0^x e^t f(t)dt + e^x$ be a differentiable function for all $x \in \mathbb{R}$. Then $f(x)$ equals :

**Options**
1. $e^{(e^x-1)}$
2. $2e^{e^x}-1$
3. $2e^{(e^x-1)}-1$
4. $e^{e^x}-1$

**Ans:** 3

**Sol:** $f(x) = \int_0^x e^t f(t)\,dt + e^x \ldots\ldots(1)$

Differentiating both sides

$f'(x) = \frac{d}{dx}\left\{\int_0^x e^t. f(t)\,dt\right\} + e^x = e^x f(x) - 0 + e^x = e^x f(x) + e^x$

$\left[\text{Since } \dfrac{d}{dx}\left\{\int_{u(x)}^{v(x)} f(x)\,dx\right\} = f\{v(x)\}.v'(x) - f\{u(x)\}.u'(x)\right]$

Let $f(x) = y \Rightarrow f'(x) = \dfrac{dy}{dx}$

$\Rightarrow \dfrac{dy}{dx} = e^x y + e^x = e^x(y+1)$

$\Rightarrow \displaystyle\int \frac{dy}{y+1} = \int e^x dx$

$\log(y+1) = e^x + C$

$y + 1 = e^{e^x + C} = e^{e^x}.e^C = ke^{e^x}$

$f(x) = k.e^{e^x} - 1 \ldots\ldots(2)$

From (1), $f(0) = \displaystyle\int_0^0 e^t.f(t)\,dt + e^0 = 0 + 1 = 1$

(2) $\Rightarrow f(0) = k.e^{e^0} - 1 = ke - 1 \Rightarrow 1 = ke - 1$

$ke = 2 \Rightarrow k = \dfrac{2}{e}$

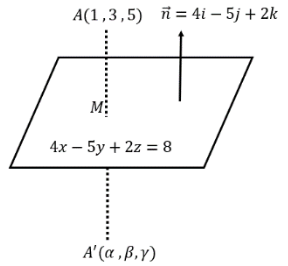
$f(x) = \dfrac{2}{e}.e^{e^x} - 1 = 2e^{e^x - 1} - 1$

**Q.15** If the mirror image of the point (1, 3, 5) with respect to the plane $4x-5y+2z=8$ is $(\alpha, \beta, \gamma)$, then $5(\alpha+\beta+\gamma)$ equals :

Options
1. 43
2. 39
3. 47
4. 41

**Ans:** 3

**Sol:**




$\overrightarrow{AA'}$ is parallel to $\vec{n}$

$\overrightarrow{AA'} = (\alpha-1)i + (\beta-3)j + (\gamma-5)k$

$\vec{n} = 4i - 5j + 2k$

$$\frac{\alpha-1}{4} = \frac{\beta-3}{-5} = \frac{\gamma-5}{2} = \lambda$$

$\Rightarrow \alpha = 4\lambda+1, \beta = -5\lambda+3, \gamma = 2\lambda+5$

$M\left(\frac{\alpha+1}{2}, \frac{\beta+3}{2}, \frac{\gamma+5}{2}\right)$ lies on $4x-5y+2z=8$

$$\Rightarrow 4\left(\frac{\alpha+1}{2}\right) - 5\left(\frac{\beta+3}{2}\right) + 2\left(\frac{\gamma+5}{2}\right) = 8$$

$$2(4\lambda+2) - 5\left(\frac{-5\lambda+6}{2}\right) + (2\lambda+10) = 8$$

$\Rightarrow \lambda = 2/5$

$$\alpha = \frac{8}{5} + 1 = \frac{13}{5}$$

$$\beta = -\frac{10}{5} + 3 = \frac{5}{5}$$

$$\gamma = \frac{4}{5} + 5 = \frac{29}{5}$$

$5(\alpha+\beta+\gamma) = 47$

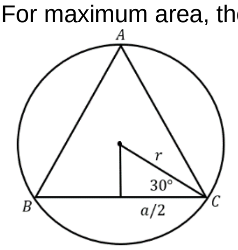


**Q.16** The triangle of maximum area that can be inscribed in a given circle of radius 'r' is :

Options
1. An equilateral triangle of height $\frac{2r}{3}$.
2. An isosceles triangle with base equal to 2r.
3. A right angle triangle having two of its sides of length 2r and r.
4. An equilateral triangle having each of its side of length $\sqrt{3}$ r.

**Ans:** 4

**Sol:** For maximum area, the triangle must be an equilateral.



$\cos 30 = \frac{a/2}{r} \Rightarrow \frac{\sqrt{3}}{2} = \frac{a}{2r} \Rightarrow a = \sqrt{3}r$

**Q.17** The sum of the series $\sum_{n=1}^{\infty} \frac{n^2 + 6n + 10}{(2n+1)!}$ is equal to :

**Options**

1. $\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$
2. $-\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$
3. $\frac{41}{8}e + \frac{19}{8}e^{-1} + 10$
4. $\frac{41}{8}e - \frac{19}{8}e^{-1} - 10$

**Ans:** 4

**Sol:** Let $n^2 + 6n + 10 = a(2n+1).2n + b(2n+1) + c$

$n = -\frac{1}{2} \Rightarrow \frac{1}{4} - 3 + 10 = c \Rightarrow c = \frac{29}{4}$

$n = 0 \Rightarrow 10 = b + \frac{29}{4} \Rightarrow b = 10 - \frac{29}{4} = \frac{11}{4}$

Coeff of $n^2$ : $4a = 1 \Rightarrow a = \frac{1}{4}$

$\frac{n^2 + 6n + 10}{(2n+1)!} = \frac{\frac{1}{4}(2n+1).2n + \frac{11}{4}(2n+1) + \frac{29}{4}}{(2n+1)!}$

$= \frac{1/4}{(2n-1)!} + \frac{11/4}{(2n)!} + \frac{29/4}{(2n+1)!}$

$\text{Sum} = \frac{1}{4}\sum_{n=1}^{\infty}\frac{1}{(2n-1)!} + \frac{11}{4}\sum_{n=1}^{\infty}\frac{1}{(2n)!} + \frac{29}{4}\sum_{n=1}^{\infty}\frac{1}{(2n+1)!}$

We know that $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ and $e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots$

$\Rightarrow \frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots$ and $\frac{e^x - e^{-x}}{2} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots$

$\therefore 1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \cdots = \frac{e + e^{-1}}{2}$ and $1 + \frac{1}{3!} + \frac{1}{5!} + \frac{1}{7!} + \cdots = \frac{e - e^{-1}}{2}$

$\text{Sum} = \frac{1}{4}\left(1 + \frac{1}{3!} + \frac{1}{5!} + \cdots\right) + \frac{11}{4}\left(\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \cdots\right) + \frac{29}{4}\left(\frac{1}{3!} + \frac{1}{5!} + \frac{1}{7!} + \cdots\right)$

$= \frac{1}{4}\left(\frac{e - e^{-1}}{2}\right) + \frac{11}{4}\left(\frac{e + e^{-1}}{2} - 1\right) + \frac{29}{4}\left(\frac{e - e^{-1}}{2} - 1\right) = \frac{41}{8}e - \frac{19}{8}e^{-1} - 10$

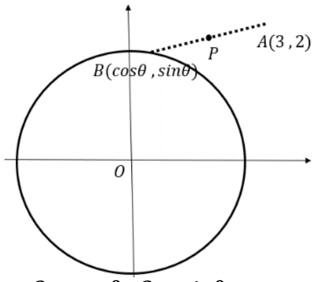
**Q.18** If the locus of the mid-point of the line segment from the point (3, 2) to a point on the circle, $x^2 + y^2 = 1$ is a circle of radius r, then r is equal to :

**Options**

1. 1
2. $\frac{1}{4}$
3. $\frac{1}{3}$
4. $\frac{1}{2}$

**Ans:** 4

**Sol:**




$P\left(\frac{3+\cos\theta}{2}, \frac{2+\sin\theta}{2}\right) = (x, y)$ (assume)

$\cos\theta = 2x - 3$ and $\sin\theta = 2y - 2$

Squaring and adding

$(2x-3)^2 + (2y-2)^2 = 1 \quad \Rightarrow 4x^2 + 4y^2 - 12x - 8y + 12 = 0$

$x^2 + y^2 - 3x - 2y + 3 = 0$

$g = -\frac{3}{2}, f = -1$ and $c = 3$

$r = \sqrt{g^2 + f^2 - c} = \sqrt{\frac{9}{4} + 1 - 3} = \frac{1}{2}$

**Q.19** Let $f(x)$ be a differentiable function at $x = a$ with $f'(a) = 2$ and $f(a) = 4$. Then $\lim_{x \to a} \frac{xf(a) - af(x)}{x - a}$ equals :

**Options**

1. $a + 4$
2. $2a + 4$
3. $2a - 4$
4. $4 - 2a$

**Ans:** 4

**Sol:** $\lim_{x \to a}\left(\frac{x.f(a) - a.f(x)}{x - a}\right) = \lim_{x \to a}\left(\frac{1.f(a) - a.f'(x)}{1}\right) = f(a) - a.f'(a) = 4 - 2a$



**Q.20** Let A = {1, 2, 3, ..., 10} and $f$ : A → A be defined as

$$f(k) = \begin{cases} k+1 & \text{if k is odd} \\ k & \text{if k is even} \end{cases}$$

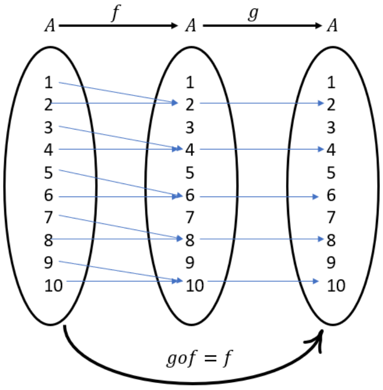
Then the number of possible functions $g$ : A → A such that $gof = f$ is :

**Options**

1. $10^5$
2. ${}^{10}C_5$
3. $5^5$
4. $5!$

**Ans:** 1

**Sol:**




The remaining elements in A ( second set ) = 5
Number of elements in A $(3^{rd}\text{set}) = 10$
We know that the number of functions from A to B is $\{n(B)\}^{n(A)}$
$\therefore$ Number of functions $= 10^5$

## SECTION B

**Q.1** Let L be a common tangent line to the curves $4x^2+9y^2=36$ and $(2x)^2+(2y)^2=31$. Then the square of the slope of the line L is ________.

**Ans:** 3

**Sol:** Ellipse : $\frac{x^2}{9}+\frac{y^2}{4}=1$

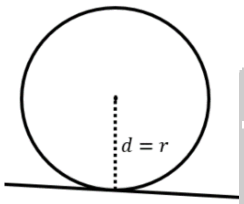
$a^2=9, b^2=4$

If $y = mx + c$ is a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then $c = \pm\sqrt{a^2m^2+b^2}$

$c = \pm\sqrt{9m^2+4}$

$\Rightarrow$ Tangent : $y = mx \pm \sqrt{9m^2+4}$

This is also tangent to the circle $x^2+y^2 = 31/4$




$$d = r \Rightarrow \left|\frac{\sqrt{9m^2+4}}{\sqrt{m^2+1}}\right| = \frac{\sqrt{31}}{2}$$

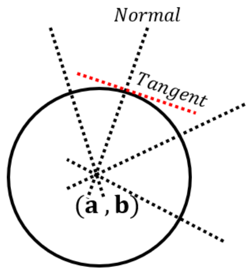
$$\frac{9m^2+4}{m^2+1} = \frac{31}{4} \Rightarrow 36m^2+16 = 31m^2+31$$

$$5m^2 = 15 \Rightarrow m^2 = 3$$

**Q.2** Let the normals at all the points on a given curve pass through a fixed point (a, b). If the curve passes through (3, −3) and $(4, -2\sqrt{2})$, and given that $a-2\sqrt{2}b = 3$, then $(a^2+b^2+ab)$ is equal to ________.

**Ans:** 9

**Sol:**



The curve must be a circle with centre at (a , b)

$\therefore$ Equation of the circle is $x^2 + y^2 - 2ax - 2by + c = 0$

The circle passes through $(3 , -3)$ and $(4 , -2\sqrt{2})$

$\therefore 9 + 9 - 6a + 6b + c = 0 \Rightarrow 6a - 6b - c = 18 \ldots . (1)$

$16 + 8 - 8a + 4\sqrt{2}b + c = 0 \Rightarrow 8a - 4\sqrt{2}b - c = 24 \ldots \ldots (2)$

$(2) - (1) \Rightarrow 2a + (6 - 4\sqrt{2}b) = 6$

$\Rightarrow a + (3 - 2\sqrt{2}b) = 3 \ \ldots \ (A)$ and $a - 2\sqrt{2}b = 3 \ldots . (B)$ [ Given]

Solving (A) and (B) , $a = 3$ and $b = 0$

$\therefore a^2 + b^2 + ab = 9$

---

**Q.3** Let a be an integer such that all the real roots of the polynomial $2x^5 + 5x^4 + 10x^3 + 10x^2 + 10x + 10$ lie in the interval $(a, a+1)$. Then, $|a|$ is equal to ________.

**Ans:** 2

**Sol:** If $x > 0$ , then $f(x) > 0$

Hence no positive roots

By trial , $f(-2) < 0$ and $f(-1) > 0$

$\therefore$ There is a root between $-2$ and $-1$

$\Rightarrow a = -2 \quad \Rightarrow |a| = 2$

**Q.4** The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is ________.

**Ans:** 1000

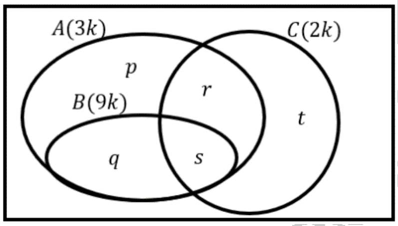
**Sol:** $18 = 2 \times 3 \times 3$

The number must be a multiple of 3 but not multiple of 9 and not a multiple of 2.

A = set of multiples of 3

B = Set of multiples of 9

C = Set of multiples of 2




$A = \{1002 , 1005 , 1008 , \ldots \ldots 9999\}$

$$n(A) = \frac{9999 - 1002}{3} + 1 = 3000$$

$\Rightarrow p + q + r + s = 3000 \ldots \ldots (1)$

$B = \{1008 , 1017 , \ldots \ldots , 9999\}$

$$n(B) = \frac{9999 - 1008}{9} + 1 = 1000$$

$\Rightarrow q + s = 1000 \ldots . (2)$

$C = \{1000 , 1002 , 1004 , \ldots \ldots 9998\}$

$$n(C) = \frac{9998 - 1000}{2} + 1 = 4500$$

$\Rightarrow r + s + t = 4500 \ldots \ldots (3)$

$r + s =$ Number of multiples of 2 and 3 (i.e , multiples of 6)

$\Rightarrow \{1002 , 1008 , \ldots \ldots 9996\}$

$$r + s = \frac{9996 - 1002}{6} + 1 = 1500 \ldots \ldots (4)$$

$s =$ Number of multiples of 9 and 2 (ie , Multiples 18)

$\Rightarrow \{1008 , 1026 , \ldots \ldots 9990\}$

$$s = \frac{9990 - 1008}{18} + 1 = 500 \ldots \ldots (5)$$

$(4) - (5) \Rightarrow r = 1000$

$(2) - (5) \Rightarrow q = 500$

$(1) \Rightarrow p + 500 + 1000 + 500 = 3000$
$\therefore p = 1000$

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**Q.5** Let $X_1, X_2, \ldots, X_{18}$ be eighteen observations such that $\sum_{i=1}^{18}(X_i - \alpha) = 36$ and $\sum_{i=1}^{18}(X_i - \beta)^2 = 90$, where $\alpha$ and $\beta$ are distinct real numbers. If the standard deviation of these observations is 1, then the value of $|\alpha - \beta|$ is ________.

**Ans:** **0**

**Sol:**

$$\sum_{i-1}^{18}(x_i - \alpha) = 36 \Rightarrow \sum x_i - 18\alpha = 36$$

$$\sum_{i-1}^{18}(x_i - \beta)^2 = 90 \Rightarrow \sum x_i^2 - 2\beta \sum x_i + 18\beta^2 = 90$$

$$\Rightarrow \sum x_i^2 = 90 + 72\beta + 36\alpha\beta - 18\beta^2$$

$$\sigma = \sqrt{\frac{\sum x_i^2}{18} - \left(\frac{\sum x_i}{18}\right)^2}$$

$\Rightarrow (5 + 4\beta + 2\alpha\beta - \beta^2) - (2 + \alpha)^2 = 1$
$\Rightarrow 4(\beta - \alpha) = \alpha^2 - 2\alpha\beta + \beta^2$
$\Rightarrow -4(\alpha - \beta) = (\alpha - \beta)^2$
$\Rightarrow \alpha - \beta = -4$
$\Rightarrow |\alpha - \beta| = 4$

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**Q.6** If the arithmetic mean and geometric mean of the $p^{th}$ and $q^{th}$ terms of the sequence $-16, 8, -4, 2, \ldots$ satisfy the equation $4x^2 - 9x + 5 = 0$, then $p + q$ is equal to ________.

**Ans:** **10**

**Sol:** GP : $-16, 8, -4, 2, -1, \frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, \ldots\ldots$

Consider 2 and $\frac{1}{2}$

$AM = \frac{2 + 1/2}{2} = \frac{5}{4}$ and $GM = \sqrt{2 \times 1/2} = 1$

Both are the roots of the equation $4x^2 - 9x + 5 = 0$

$\Rightarrow$ $p = 4$ and $q = 6$ $\Rightarrow$ $p + q = 10$

**Q.7** Let $z$ be those complex numbers which satisfy

$|z + 5| \le 4$ and $z(1 + i) + \bar{z}(1 - i) \ge -10$, $i = \sqrt{-1}$.

If the maximum value of $|z + 1|^2$ is $\alpha + \beta\sqrt{2}$, then the value of $(\alpha + \beta)$ is ________.

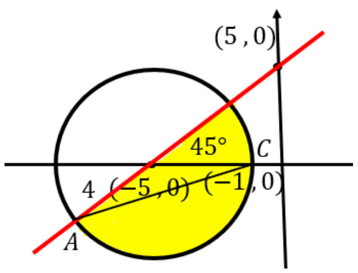
**Ans:** **48**

**Sol:** Let $z = x + iy$

$z(1 + i) + \bar{z}(1 - i) = (x + iy)(1 + i) + (x - iy)(1 - i) = (x - y) + (x + y)i + (x - y) + (-x - y)i$
$= 2(x - y) \ge -10 \quad \Rightarrow \quad x - y \ge -5$

$|z + 5| \le 4 \ \Rightarrow |z - (-5 + 0i)| \le 4$

which represents interior of the circle with centre at $(-5, 0)$ and radius $= 4$




$|z + 1| = |z - (-1)|$ is maximum

⇒ The position of z must be at A

Using cosines rule ,

$$AC = \sqrt{4^2 + 4^2 - 2(4)(4)\cos 135} = \sqrt{32 - 32\left(-\frac{1}{\sqrt{2}}\right)} = \sqrt{32 + 16\sqrt{2}}$$

$AC^2 = 32 + 16\sqrt{2}$

$\alpha + \beta = 32 + 16 = 48$

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**Q.8**

If the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$ satisfies the equation $A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ for some real numbers $\alpha$ and $\beta$, then $\beta - \alpha$ is equal to ________.

**Ans:** **4**

**Sol:**

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$A^2 = A.A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{20} = (A^2)^{10} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{10} = \begin{bmatrix} 1^{10} & 0 & 0 \\ 0 & 4^{10} & 0 \\ 0 & 0 & 1^{10} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4^{10} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{19} = (A^2)^9 . A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}^9 . \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4^9 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2(4^9) & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4^{10} & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} \alpha & 0 & 0 \\ 0 & 2\alpha 4^9 & 0 \\ 3\alpha & 0 & -\alpha \end{bmatrix} + \begin{bmatrix} \beta & 0 & 0 \\ 0 & 2\beta & 0 \\ 3\beta & 0 & -\beta \end{bmatrix}$$

$$= \begin{bmatrix} 1+\alpha+\beta & 0 & 0 \\ 0 & 4^{10} + 2\alpha 4^9 + 2\beta & 0 \\ 3(\alpha+\beta) & 0 & 1-(\alpha+\beta) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow \alpha + \beta = 0$ and $4^{10} + 2\alpha 4^9 + 2\beta = 4$

$2^{20} + \alpha . 2^{19} - 2\alpha = 4 \quad \Rightarrow \quad \alpha = -2$ (By back substitution)

$\beta = 2 \quad \Rightarrow \quad \beta - \alpha = 4$

---

**Q.9**

If $I_{m,n} = \int_0^1 x^{m-1}(1-x)^{n-1}\, dx$, for $m, n \geq 1$, and $\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}}\, dx = \alpha\, I_{m,n}$, $\alpha \in \mathbb{R}$, then $\alpha$ equals ________.

**Ans:** **1**

**Sol:**

$I_{m,n} = \int_0^1 x^{m-1} . (1-x)^{n-1}\, dx$

Let $x = \sin^2\theta \quad \Rightarrow \quad dx = 2\sin\theta\cos\theta$

$$I_{m,n} = \int_0^{\pi/2} \sin^{2m-2}\theta \cos^{2n-2}\theta \,.\, 2\sin\theta\cos\theta\, d\theta$$

$$= 2\int_0^{\pi/2} \sin^{2m-1}\theta . \cos^{2n-1}\theta \,.\, d\theta \ldots\ldots\ldots\ldots.(1)$$

$$2^{nd}\text{integral} = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}}\, dx$$

Let $x = \tan^2\theta \quad \Rightarrow \quad dx = 2\tan\theta\sec^2\theta \,.\, d\theta$

$$\Rightarrow \int_0^{\frac{\pi}{4}} \frac{(\tan^2\theta)^{m-1} + (\tan^2\theta)^{n-1}}{(1+\tan\theta)^{m+n}} . 2\tan\theta . \sec^2\theta . d\theta$$

Substituting $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sec\theta = \frac{1}{\cos\theta}$

$$\Rightarrow \quad 2\int_0^{\pi/4} \sin^{2m-1}\theta . \cos^{2n-1}\theta + \sin^{2n-1}\theta . \cos^{2m-1}\theta$$

$$= 2\int_0^{\pi/2} \sin^{2m-1}\theta . \cos^{2n-1}\theta . d\theta$$

$$\left[\text{Since } \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx = \int_0^{2a} f(x)\,dx\right]$$

$\Rightarrow\ I_{m,n} = 2^{nd}$ integral $\quad \Rightarrow \quad \alpha = 1$

**Aliter**

Put m =1 and n =1

$$I_{m,n} = \int_0^1 x^0 \,.\, (1-x)^0 dx = \int_0^1 1\,dx = 1$$

$$2^{nd}\text{integral} = \int_0^1 \frac{x^0 + x^0}{(1+x)^2}\,dx = \int_0^1 \frac{2}{(1+x)^2}\,dx = -2\left[\frac{1}{1+x}\right]_0^1 = 1$$

$\Rightarrow \quad \alpha = 1$

---

**Q.10** Let $\alpha$ and $\beta$ be two real numbers such that $\alpha+\beta=1$ and $\alpha\beta=-1$. Let $p_n=(\alpha)^n+(\beta)^n$, $p_{n-1}=11$ and $p_{n+1}=29$ for some integer $n \geq 1$. Then, the value of $p_n^2$ is __________.

**Ans:** **324**

**Sol:**

$\alpha + \beta = 1\,,\ \alpha\beta = -1$

$P_1 = \alpha + \beta = 1$

$\alpha$ and $\beta$ are the roots of the equation $x^2 - x - 1 = 0 \ldots (A)$

$\alpha^2 - \alpha - 1 = 0 \ldots.(1) \quad$ and $\beta^2 - \beta - 1 = 0 \ldots\ldots(2)$

$(1) + (2) \Rightarrow\ P_2 - 1 - 2 = 0 \quad \Rightarrow\ P_2 = 3$

$(A) \times x \Rightarrow\ x^3 - x^2 - x = 0$

$\Rightarrow\ \alpha^3 - \alpha^2 - \alpha = 0$ and $\beta^3 - \beta^2 - \beta = 0$

adding , $P_3 - P_2 - P_1 = 0 \quad \Rightarrow\ P_3 = 4$

$(A) \times x^2 \Rightarrow x^4 - x^3 - x^2 = 0$

Substituting $\alpha$ and $\beta$ and adding, $\quad$ P

$(A) \times x^{n-1} \Rightarrow\ x^{n+1} - x^n - x^{n-1} = 0$

$\alpha^{n+1} - \alpha^n - \alpha^{n-1} = 0$ and $\beta^{n+1} - \beta^n - \beta^{n-1} = 0$

Adding , $P_{n+1} - P_n - P_{n-1} = 0$

$\Rightarrow\ 29 - P_n - 11 = 0 \Rightarrow\ P_n = 18 \quad \Rightarrow\ (P_n)^2 = 324$