

**SOLUTIONS & ANSWERS FOR JEE MAINS-2021**  
**26<sup>th</sup> February Shift 1**  
**[PHYSICS, CHEMISTRY & MATHEMATICS]**

**PART – A – PHYSICS**

**SECTION A**

**Q.1** Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

**Assertion A :** An electron microscope can achieve better resolving power than an optical microscope.

**Reason R :** The de Broglie's wavelength of the electrons emitted from an electron gun is much less than wavelength of visible light.

In the light of the above statements, choose the correct answer from the options given below :

**Options 1.**

Both A and R are true but R is NOT the correct explanation of A.

2. A is false but R is true.

3. A is true but R is false.

4.

Both A and R are true and R is the correct explanation of A.

**Ans:** 4

**Sol:** Option (4)

**Q.2** If two similar springs each of spring constant  $K_1$  are joined in series, the new spring constant and time period would be changed by a factor :

**Options**

1.  $\frac{1}{4}, 2\sqrt{2}$

2.  $\frac{1}{2}, 2\sqrt{2}$

3.  $\frac{1}{4}, \sqrt{2}$

4.  $\frac{1}{2}, \sqrt{2}$

**Ans:** 4

**Sol:**

$$\left[ \text{Spring } K \text{ --- } m \right] \Rightarrow T = 2\pi \sqrt{\frac{m}{K}}$$

$$\left[ \text{Spring } K \text{ --- Spring } K \text{ --- } m \right] \Rightarrow T' = 2\pi \sqrt{\frac{M}{K'}} = 2\pi \sqrt{\frac{M}{K/2}} = 2\pi \sqrt{\frac{2m}{K}}$$

$$K' = K/2$$

$$T' = \sqrt{2}T$$

**Q.3** Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

**Assertion A :** Body 'P' having mass M moving with speed 'u' has head-on collision elastically with another body 'Q' having mass 'm' initially at rest. If  $m \ll M$ , body 'Q' will have a maximum speed equal to '2u' after collision.

**Reason R :** During elastic collision, the momentum and kinetic energy are both conserved.

In the light of the above statements, choose the most appropriate answer from the options given below :

**Options** 1.

Both A and R are correct but R is NOT the correct explanation of A.

2. A is not correct but R is correct.

3.

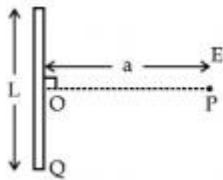
Both A and R are correct and R is the correct explanation of A.

4. A is correct but R is not correct.

**Ans: 3**

**Sol:** Option (3)

**Q.4** Find the electric field at point P (as shown in figure) on the perpendicular bisector of a uniformly charged thin wire of length L carrying a charge Q. The distance of the point P from the centre of the rod is  $a = \frac{\sqrt{3}}{2} L$ .



**Options**

1.  $\frac{Q}{2\sqrt{3}\pi\epsilon_0 L^2}$

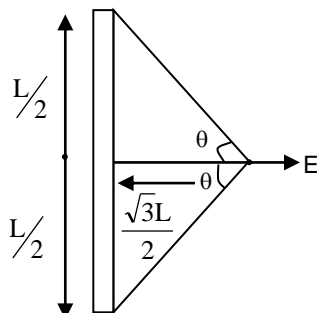
2.  $\frac{\sqrt{3}Q}{4\pi\epsilon_0 L^2}$

3.  $\frac{Q}{4\pi\epsilon_0 L^2}$

4.  $\frac{Q}{3\pi\epsilon_0 L^2}$

**Ans: 1**

**Sol:**



$$\tan \theta = \frac{L/2}{L \cdot \sqrt{3}/2} = \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$

$$E = \frac{KQ}{\left(\frac{\sqrt{3}L}{2}\right)L} (\sin 30 + \sin 30) = \frac{2KQ}{\sqrt{3}L^2}$$

$$E = \frac{Q}{2\sqrt{3}\pi\epsilon_0 L^2}$$

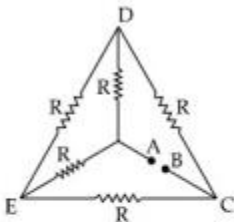
**Q.5** Consider the combination of 2 capacitors  $C_1$  and  $C_2$  with  $C_2 > C_1$ , when connected in parallel, the equivalent capacitance is  $\frac{15}{4}$  times the equivalent capacitance of the same connected in series. Calculate the ratio of capacitors,  $\frac{C_2}{C_1}$ .

- Options**
1.  $\frac{29}{15}$
  2.  $\frac{111}{80}$
  3.  $\frac{15}{11}$
  4.  $\frac{15}{4}$

**Ans:** NTA key is (2)

**Sol:** In sufficient data

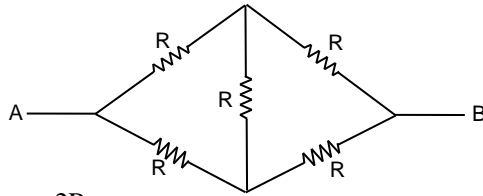
**Q.6** Five equal resistances are connected in a network as shown in figure. The net resistance between the points A and B is :



- Options**
1.  $\frac{R}{2}$
  2.  $2R$
  3.  $\frac{3R}{2}$
  4.  $R$

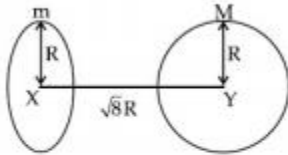
**Ans:** 4

**Sol:** It is a balanced wheatstone's bridge



$$R_{eq} = \frac{2R}{2} = R$$

**Q.7** Find the gravitational force of attraction between the ring and sphere as shown in the diagram, where the plane of the ring is perpendicular to the line joining the centres. If  $\sqrt{8}R$  is the distance between the centres of a ring (of mass 'm') and a sphere (mass 'M') where both have equal radius 'R'.

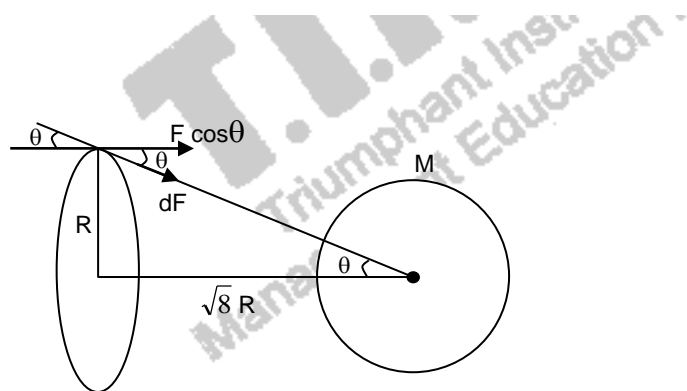


Options

1.  $\frac{1}{3\sqrt{8}} \cdot \frac{GMm}{R^2}$
2.  $\frac{\sqrt{8}}{9} \cdot \frac{GmM}{R}$
3.  $\frac{2\sqrt{2}}{3} \cdot \frac{GMm}{R^2}$
4.  $\frac{\sqrt{8}}{27} \cdot \frac{GmM}{R^2}$

**Ans:** 4

**Sol:**



$$\tan \theta = \frac{1}{\sqrt{8}}$$

$$F = \int dF \cos \theta = \cos \theta \int dF = \frac{\sqrt{8}}{3} \left[ \int (dm) \frac{G \cdot M}{(3R)^2} \right] = \frac{\sqrt{8}GM}{27R^2} \int dm = \frac{\sqrt{8}}{27} \cdot \frac{GMm}{R^2}$$

**OR**

$$F = ME = \frac{M \cdot GMx}{(R^2 + x^2)^{3/2}} = \frac{Mx Gm \sqrt{8}R}{(R^2 + 8R^2)^{3/2}} = \frac{\sqrt{8}GMm}{27R^2}$$

**Q.8** A particle is moving with uniform speed along the circumference of a circle of radius  $R$  under the action of a central fictitious force  $F$  which is inversely proportional to  $R^3$ . Its time period of revolution will be given by :

- Options**
1.  $T \propto R^{\frac{4}{3}}$
  2.  $T \propto R^2$
  3.  $T \propto R^{\frac{3}{2}}$
  4.  $T \propto R^{\frac{5}{2}}$

**Ans: 2**

**Sol:**  $F = \frac{mV^2}{r}$

$$\frac{K}{r^3} = \frac{mV^2}{r} \Rightarrow \frac{K}{r^2 m} = V^2$$

$$V = \frac{1}{r} \sqrt{\frac{K}{m}}$$

$$T = \frac{2\pi r}{V} = \frac{2\pi r}{\frac{1}{r} \sqrt{\frac{K}{m}}} \propto r^2$$

$$T \propto r^2$$

**Q.9** If  $\lambda_1$  and  $\lambda_2$  are the wavelengths of the third member of Lyman and first member of the Paschen series respectively, then the value of  $\lambda_1 : \lambda_2$  is :

- Options**
1. 7 : 108
  2. 1 : 9
  3. 7 : 135
  4. 1 : 3

**Ans: 3**

**Sol:**  $\frac{1}{\lambda_1} = RZ^2 \left[ 1 - \frac{1}{4^2} \right] = \frac{15}{16} RZ^2$

$$\frac{1}{\lambda_2} = RZ^2 \left[ \frac{1}{3^2} - \frac{1}{4^2} \right]$$

$$\frac{\lambda_1}{\lambda_2} = \frac{7}{135}$$

**Q.10** In a typical combustion engine the workdone by a gas molecule is given by

$W = \alpha^2 \beta e^{-\frac{\beta x^2}{kT}}$ , where  $x$  is the displacement,  $k$  is the Boltzmann constant and  $T$  is the temperature. If  $\alpha$  and  $\beta$  are constants, dimensions of  $\alpha$  will be :

- Options**
1.  $[M L T^{-1}]$
  2.  $[M^2 L T^{-2}]$
  3.  $[M^0 L T^0]$
  4.  $[M L T^{-2}]$

**Ans: 3**

**Sol:**  $[kT] = [\beta] [x^2]$   
 $M L^2 T^{-2} = [\beta] [L^2]$   
 $[\beta] = M T^{-2}$   
 $\alpha^2 \beta = \text{energy} = M L^2 T^{-2}$   
 $\alpha^2 = L^2$   
 $[\alpha] = [L]$

**Q.11** Four identical solid spheres each of mass 'm' and radius 'a' are placed with their centres on the four corners of a square of side 'b'. The moment of inertia of the system about one side of square where the axis of rotation is parallel to the plane of the square is :

- Options**
1.  $\frac{4}{5} m a^2$
  2.  $\frac{4}{5} m a^2 + 2 m b^2$
  3.  $\frac{8}{5} m a^2 + m b^2$
  4.  $\frac{8}{5} m a^2 + 2 m b^2$

**Ans: 4**

**Sol:**  $I = I_1 + I_2 + I_3 + I_4$   $[I_1 = I_2, I_3 = I_4]$   
 $= 2I_1 + 2I_3$   
 $= (2 \times \frac{2}{5} m a^2) + 2(\frac{2}{5} m a^2 + m b^2)$   
 $= \frac{8}{5} m a^2 + 2 m b^2.$

**Q.12** A short straight object of height 100 cm lies before the central axis of a spherical mirror whose focal length has absolute value  $|f| = 40$  cm. The image of object produced by the mirror is of height 25 cm and has the same orientation of the object. One may conclude from the information :

- Options**
1. Image is real, same side of convex mirror.
  2. Image is virtual, opposite side of concave mirror.
  3. Image is real, same side of concave mirror.
  4. Image is virtual, opposite side of convex mirror.

**Ans: 4**

**Sol:** As image has same orientation as that of the object, image is virtual.

$$m = \frac{25}{100} = \frac{1}{4}$$

So, virtual and diminished image will form by only convex mirror on opposite side.

- Q.13** A planet revolving in elliptical orbit has :
- A. a constant velocity of revolution.
  - B. has the least velocity when it is nearest to the sun.
  - C. its areal velocity is directly proportional to its velocity.
  - D. areal velocity is inversely proportional to its velocity.
  - E. to follow a trajectory such that the areal velocity is constant.
- Choose the correct answer from the options given below :

- Options**
- 1. E only
  - 2. C only
  - 3. D only
  - 4. A only

**Ans:** 1

**Sol:** Areal velocity =  $\frac{L}{2m} = \text{constant}$

- Q.14** The normal density of a material is  $\rho$  and its bulk modulus of elasticity is  $K$ . The magnitude of increase in density of material, when a pressure  $P$  is applied uniformly on all sides, will be :

- Options**
- 1.  $\frac{K}{\rho P}$
  - 2.  $\frac{\rho P}{K}$
  - 3.  $\frac{\rho K}{P}$
  - 4.  $\frac{PK}{\rho}$

**Ans:** 2

**Sol:**  $K = -V \cdot \frac{\Delta p}{\Delta V}$   
 $\frac{\Delta V}{V} = \frac{-\Delta p}{K}$  as  $\rho V = \text{constant}$   
 $K = \frac{\rho \Delta P}{\Delta \rho} \Rightarrow \Delta \rho = \frac{\rho \Delta P}{K}$   
 $\Delta \rho = \frac{\rho P}{K}$

**Q.15** A large number of water drops, each of radius  $r$ , combine to have a drop of radius  $R$ . If the surface tension is  $T$  and mechanical equivalent of heat is  $J$ , the rise in heat energy per unit volume will be :

**Options**

1.  $\frac{2T}{rJ}$

2.  $\frac{2T}{J} \left( \frac{1}{r} - \frac{1}{R} \right)$

3.  $\frac{3T}{J} \left( \frac{1}{r} - \frac{1}{R} \right)$

4.  $\frac{3T}{rJ}$

**Ans: 3**

**Sol:**  $R^3 = nr^3$

$$n = \frac{R^3}{r^3}$$

$$\Delta A = n \times 4\pi r^2 - 4\pi R^2 \left( \frac{R}{r} - 1 \right)$$

$$JH = \Delta A \times T$$

$$\text{Heat per unit vol} = \frac{4\pi R^2 \left( \frac{R}{r} - 1 \right) T}{\frac{4}{3}\pi R^3 J} = \frac{3T}{J} \left( \frac{1}{r} - \frac{1}{R} \right)$$

**Q.16** An alternating current is given by the equation  $i = i_1 \sin \omega t + i_2 \cos \omega t$ . The rms current will be :

**Options**

1.  $\frac{1}{\sqrt{2}} (i_1 + i_2)^2$

2.  $\frac{1}{\sqrt{2}} (i_1^2 + i_2^2)^{\frac{1}{2}}$

3.  $\frac{1}{2} (i_1^2 + i_2^2)^{\frac{1}{2}}$

4.  $\frac{1}{\sqrt{2}} (i_1 + i_2)$

**Ans: 2**

**Sol:**  $I = \sqrt{I_1^2 + I_2^2} \left[ \frac{I_2}{\sqrt{I_1^2 + I_2^2}} \cos \theta + \frac{I_1}{\sqrt{I_1^2 + I_2^2}} \sin \theta \right] = \sqrt{I_1^2 + I_2^2} \sin(\theta + \alpha)$

$$\tan \alpha = \frac{I_2}{I_1}$$

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \sqrt{\frac{I_1^2 + I_2^2}{2}} = \frac{1}{\sqrt{2}} (I_1^2 + I_2^2)^{\frac{1}{2}}$$

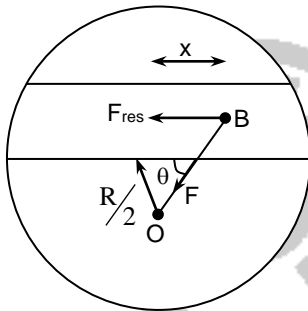
**Q.17** Assume that a tunnel is dug along a chord of the earth, at a perpendicular distance ( $R/2$ ) from the earth's centre, where ' $R$ ' is the radius of the Earth. The wall of the tunnel is frictionless. If a particle is released in this tunnel, it will execute a simple harmonic motion with a time period :

Options

1.  $\frac{g}{2\pi R}$
2.  $2\pi \sqrt{\frac{R}{g}}$
3.  $\frac{1}{2\pi} \sqrt{\frac{g}{R}}$
4.  $\frac{2\pi R}{g}$

Ans: 2

Sol:



$$F = mg'$$

$$mg' = \frac{GM}{R^3} (OB)m$$

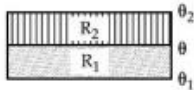
$$F_R = F \cos \theta = \frac{GM(OB)m}{R^3} \times \frac{x}{(OB)}$$

$$= \frac{GMmx}{R^3}$$

$$a_R = \frac{GM}{R^3} x$$

$$T = 2\pi \sqrt{\frac{R^3}{GM}} = 2\pi \sqrt{\frac{R}{g}}$$

**Q.18** The temperature  $\theta$  at the junction of two insulating sheets, having thermal resistances  $R_1$  and  $R_2$  as well as top and bottom temperatures  $\theta_1$  and  $\theta_2$  (as shown in figure) is given by :



Options

1.  $\frac{\theta_1 R_2 - \theta_2 R_1}{R_2 - R_1}$
2.  $\frac{\theta_1 R_1 + \theta_2 R_2}{R_1 + R_2}$
3.  $\frac{\theta_1 R_2 + \theta_2 R_1}{R_1 + R_2}$
4.  $\frac{\theta_2 R_2 - \theta_1 R_1}{R_2 - R_1}$

**Ans: 3**

$$\begin{aligned}\text{Sol: } H &= \frac{\theta_1 - \theta}{R_1} = \frac{\theta - \theta_2}{R_2} \\ \theta &= \frac{\frac{\theta_1}{R_1} + \frac{\theta_2}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_2\theta_1 + R_1\theta_2}{R_1 + R_2}\end{aligned}$$

**Q.19** LED is constructed from Ga-As-P semiconducting material. The energy gap of this LED is 1.9 eV. Calculate the wavelength of light emitted and its colour.  
[ $h = 6.63 \times 10^{-34}$  Js and  $c = 3 \times 10^8$  ms<sup>-1</sup>]

- Options**
1. 1046 nm and blue colour
  2. 654 nm and orange colour
  3. 654 nm and red colour
  4. 1046 nm and red colour

**Ans: 3**

$$\begin{aligned}\text{Sol: } E_g &= \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E_g} \\ &= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.9 \text{ eV}} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.9 \times 1.6 \times 10^{-19}} \\ &= 654 \text{ nm and red colour}\end{aligned}$$

**Q.20** In a Young's double slit experiment two slits are separated by 2 mm and the screen is placed one meter away. When a light of wavelength 500 nm is used, the fringe separation will be :

- Options**
1. 0.75 mm
  2. 0.50 mm
  3. 0.25 mm
  4. 1 mm

**Ans: 3**

$$\begin{aligned}\text{Sol: } \beta &= \frac{\lambda D}{d} = \frac{500 \times 10^{-9} \times 1}{2 \times 10^{-3}} \\ &= 250 \times 10^{-6} \text{ m} \\ &= 250 \mu\text{m} = 0.25 \text{ mm}\end{aligned}$$

## SECTION B

**Q.1** In a series LCR resonant circuit, the quality factor is measured as 100. If the inductance is increased by two fold and resistance is decreased by two fold, then the quality factor after this change will be \_\_\_\_\_.

**Ans: 400.00**

$$\text{Sol: } Q = \frac{\omega L}{R}$$

$$\frac{Q'}{Q} = \frac{L'}{L} \times \frac{R}{R_1} = 2 \times 2 = 4$$

$$Q' = 4 Q = 4 \times 100 = 400$$

**Q.2**

A boy pushes a box of mass 2 kg with a force  $\vec{F} = (20\hat{i} + 10\hat{j})$  N on a frictionless surface.

If the box was initially at rest, then \_\_\_\_\_ m is displacement along the x-axis after 10 s.

**Ans: 500.00**

**Sol:**  $F = 20\hat{i} + 10\hat{j}$

$$a = \frac{F}{m} = \frac{20\hat{i} + 10\hat{j}}{2} = 10\hat{i} + 5\hat{j}$$

$$x = \frac{1}{2} a_x t^2 = \frac{1}{2} \times 10 \times 10^2 = 500$$

**Q.3**

A container is divided into two chambers by a partition. The volume of first chamber is 4.5 litre and second chamber is 5.5 litre. The first chamber contain 3.0 moles of gas at pressure 2.0 atm and second chamber contain 4.0 moles of gas at pressure 3.0 atm. After the partition is removed and the mixture attains equilibrium, then, the common equilibrium pressure existing in the mixture is  $x \times 10^{-1}$  atm. Value of x is \_\_\_\_\_.

**Ans: 25.00**

**Sol:** By energy conservation

$$\frac{f}{2} n_1 RT_1 + \frac{f}{2} n_2 RT_2 = \frac{f}{2} (n_1 + n_2) RT$$

$$\frac{f}{2} P_1 V_1 + \frac{f}{2} P_2 V_2 = \frac{f}{2} P (V_1 + V_2) \quad [PV = nRT]$$

$$P = \frac{P_1 V_1 + P_2 V_2}{V_1 + V_2} = \frac{2 \times 4.5 + 3 \times 5.5}{10} = 25.5 \times 10^{-1} \text{ atm}$$

**Q.4**

The mass per unit length of a uniform wire is 0.135 g/cm. A transverse wave of the form  $y = -0.21 \sin(x + 30t)$  is produced in it, where x is in meter and t is in second. Then, the expected value of tension in the wire is  $x \times 10^{-2}$  N. Value of x is \_\_\_\_\_. (Round-off to the nearest integer)

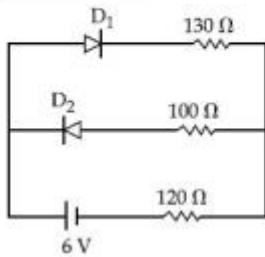
**Ans: 1215.00**

**Sol:**  $\mu = 0.135 \text{ g/cm} = \frac{0.135 \times 10^{-3}}{10^{-2}} \text{ kg/m} = 135 \times 10^{-4} \text{ kg/m}$

$$V = \frac{\omega}{K} = \frac{30}{1} = 30$$

$$V = \sqrt{\frac{T}{\mu}} \Rightarrow T = \mu v^2 = 135 \times 10^{-4} \times 900 = 1215 \times 10^{-2} \text{ N}$$

- Q.5** The circuit contains two diodes each with a forward resistance of  $50\ \Omega$  and with infinite reverse resistance. If the battery voltage is  $6\text{ V}$ , the current through the  $120\ \Omega$  resistance is \_\_\_\_\_ mA.



**Ans: 20.00**

**Sol:**  $R = 130 + 50 + 120 = 300\ \Omega$   
 $i = \frac{V}{R} = \frac{6}{300} = 20 \times 10^{-3}\text{ A} = 20\text{ mA}$

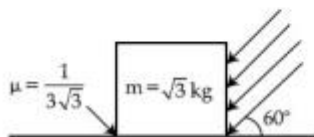
- Q.6** In an electrical circuit, a battery is connected to pass  $20\text{ C}$  of charge through it in a certain given time. The potential difference between two plates of the battery is maintained at  $15\text{ V}$ . The workdone by the battery is \_\_\_\_\_ J.

**Ans: 300.00**

**Sol:**  $w = QV = 15 \times 20 = 300\text{ J}$

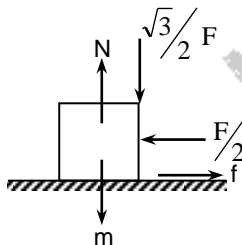
- Q.7** As shown in the figure, a block of mass  $\sqrt{3}\text{ kg}$  is kept on a horizontal rough surface of coefficient of friction  $\frac{1}{3\sqrt{3}}$ . The critical force to be applied on the vertical surface as shown at an angle  $60^\circ$  with horizontal such that it does not move, will be  $3x$ . The value of  $x$  will be \_\_\_\_\_.

$[g = 10\text{ m/s}^2; \sin 60^\circ = \frac{\sqrt{3}}{2}; \cos 60^\circ = \frac{1}{2}]$



**Ans: 10.00**

**Sol:**



$$N = \frac{\sqrt{3}F}{2} + mg$$

For no slipping,  $\frac{F}{2} \leq f_{\max}$

$$\frac{F}{2} \leq \mu \left( \frac{\sqrt{3}}{2} F + mg \right) \leq \frac{1}{3\sqrt{3}} \left( \frac{\sqrt{3}}{2} F + \sqrt{3}g \right)$$

$$\frac{F}{2} \leq \frac{F}{6} + \frac{g}{3}$$

$$\frac{F}{2} - \frac{F}{6} \leq \frac{g}{3}$$

$$\frac{F}{3} \leq \frac{g}{3}$$

$$F \leq 10 \Rightarrow x = \frac{10}{3}$$

**Q.8** The maximum and minimum amplitude of an amplitude modulated wave is 16 V and 8 V respectively. The modulation index for this amplitude modulated wave is  $x \times 10^{-2}$ . The value of  $x$  is \_\_\_\_\_.

**Ans: 33.00**

**Sol:**  $A_m = \frac{A_{\max} - A_{\min}}{2}$

$$A_c = \frac{A_{\max} + A_{\min}}{2}$$

$$m_i = \frac{A_m}{A_c} = \frac{16 - 8}{16 + 8} = \frac{8}{24} = \frac{1}{3}$$

$$= 33 \times 10^{-2}$$

**Q.9** A person standing on a spring balance inside a stationary lift measures 60 kg. The weight of that person if the lift descends with uniform downward acceleration of  $1.8 \text{ m/s}^2$  will be \_\_\_\_\_ N. [ $g = 10 \text{ m/s}^2$ ]

**Ans: 492.00**

**Sol:**  $mg - N = 1.8 \times m$   
 $N = m(10 - 1.80) = 60 \times 8.2 = 492 \text{ N}$

**Q.10** A radiation is emitted by 1000 W bulb and it generates an electric field and magnetic field at P, placed at a distance of 2 m. The efficiency of the bulb is 1.25%. The value of peak electric field at P is  $x \times 10^{-1} \text{ V/m}$ . Value of  $x$  is \_\_\_\_\_. (Rounded-off to the nearest integer)  
 [Take  $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ ,  $c = 3 \times 10^8 \text{ ms}^{-1}$ ]

**Ans: 137.00**

**Sol:**  $I = \frac{1}{2}(\epsilon_0 c)E_0^2 = \frac{\text{Power}}{4\pi r^2} \times \text{Efficiency}$

$$\Rightarrow \frac{1}{2} \times (4\pi\epsilon_0)(c)E_0^2 = \frac{1000 \times 1.25}{4} \times \frac{1}{100}$$

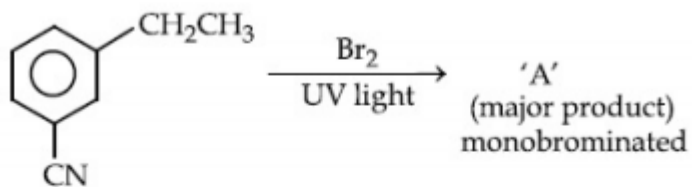
$$\therefore \frac{1}{2} \times \frac{3 \times 10^8}{9 \times 10^9} \times E_0^2 = \frac{125}{40}$$

$$\therefore E_0 = 136.9 \times 10^{-1} = 137 \times 10^{-1} \text{ v/m}$$

PART – B – CHEMISTRY

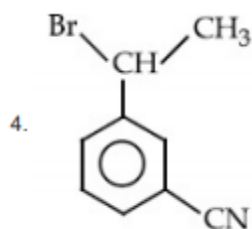
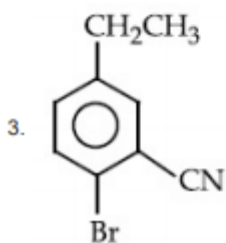
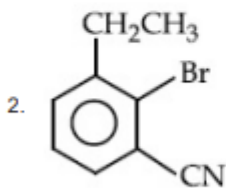
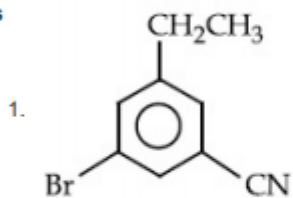
SECTION A

Q.1 For the given reaction :

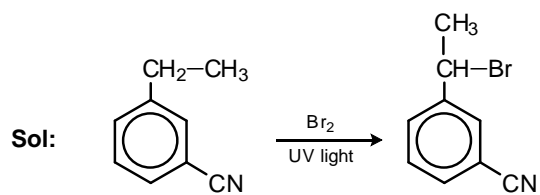


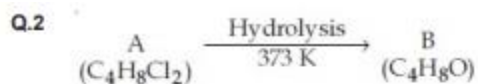
What is 'A'?

Options



Ans: 4

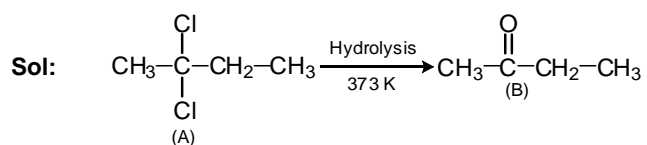




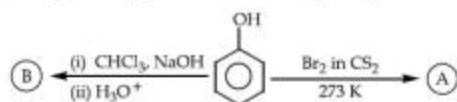
B reacts with Hydroxyl amine but does not give Tollen's test. Identify A and B.

- Options**
- 1, 2,2-Dichlorobutane and Butanal
  - 2, 1,1-Dichlorobutane and Butanal
  - 3, 1,1-Dichlorobutane and 2-Butanone
  - 4, 2,2-Dichlorobutane and Butan-2-one

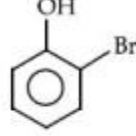

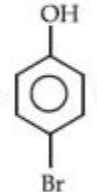
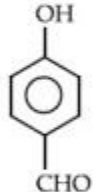
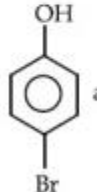
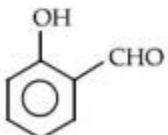
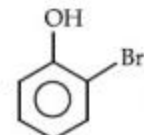
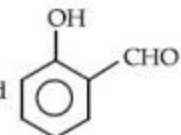
**Ans:** 4



**Q.3** Identify the major products A and B respectively in the following reactions of phenol :

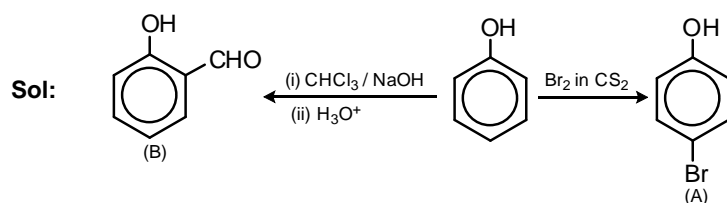


**Options**

1.  and 
2.  and 
3.  and 
4.  and 

**Ans:** 3

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**Q.4** Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

**Assertion A :** Dipole-dipole interactions are the only non-covalent interactions, resulting in hydrogen bond formation.

**Reason R :** Fluorine is the most electronegative element and hydrogen bonds in HF are symmetrical.

In the light of the above statements, choose the most appropriate answer from the options given below :

- Options**
1. A is false but R is true
  2. Both A and R are true and R is the correct explanation of A
  3. A is true but R is false
  4. Both A and R are true but R is NOT the correct explanation of A

**Ans:** 1

**Sol:** A is false but R is true  
Dipole-Dipole interactions are non-covalent in nature but ion-dipole interactions can also result in Hydrogen bond formation.

**Q.5** Which one of the following lanthanoids does not form  $\text{MO}_2$  ?  
[M is lanthanoid metal]

- Options**
1. Yb
  2. Pr
  3. Dy
  4. Nd

**Ans:** 1

**Sol:** Yb does not form  $\text{MO}_2$  type oxide

**Q.6** Given below are two statements :

**Statement I :** *o*-Nitrophenol is steam volatile due to intramolecular hydrogen bonding.

**Statement II :** *o*-Nitrophenol has high melting due to hydrogen bonding.

In the light of the above statements, choose the most appropriate answer from the options given below :

- Options**
1. Statement I is true but Statement II is false
  2. Both Statement I and Statement II are false
  3. Statement I is false but Statement II is true
  4. Both Statement I and Statement II are true

**Ans:** 1

**Sol:** o-Nitrophenol has low melting point (44.8°C). The melting point depend on packing efficiency and not hydrogen bonding.

**Q.7** The orbital having two radial as well as two angular nodes is :

**Options** 1. 4f

2. 3p

3. 4d

4. 5d

**Ans:** 4

**Sol:** No. of radial nodes =  $n - \ell - 1$   
No. of angular nodes =  $\ell$   
For 5d orbital, radial nodes =  $5 - 2 - 1 = 2$   
angular nodes = 2

**Q.8** Match List-I with List-II.

List-I (Ore)	List-II (Element Present)
(a) Kernite	(i) Tin
(b) Cassiterite	(ii) Boron
(c) Calamine	(iii) Fluorine
(d) Cryolite	(iv) Zinc

Choose the most appropriate answer from the options given below :

**Options** 1. (a) → (ii), (b) → (iv), (c) → (i), (d) → (iii)

2. (a) → (i), (b) → (iii), (c) → (iv), (d) → (ii)

3. (a) → (iii), (b) → (i), (c) → (ii), (d) → (iv)

4. (a) → (ii), (b) → (i), (c) → (iv), (d) → (iii)

**Ans:** 4

**Sol:** Kernite –  $\text{Na}_2\text{B}_4\text{O}_7 \cdot 4\text{H}_2\text{O}$   
Cassiterite –  $\text{SnO}_2$   
Calamine –  $\text{ZnCO}_3$   
Cryolite –  $\text{Na}_3\text{AlF}_6$

**Q.9** The presence of ozone in troposphere :

**Options** 1. protects us from the X-ray radiation

2. protects us from greenhouse effect

3. protects us from the UV radiation

4. generates photochemical smog

**Ans:** 4

**Sol:**  $\text{O}_3$  generate photochemical smog in troposphere

**Q.10** Given below are two statements :

**Statement I :** A mixture of chloroform and aniline can be separated by simple distillation.

**Statement II :** When separating aniline from a mixture of aniline and water by steam distillation aniline boils below its boiling point.

In the light of the above statements, choose the most appropriate answer from the options given below :

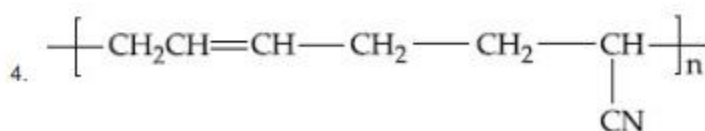
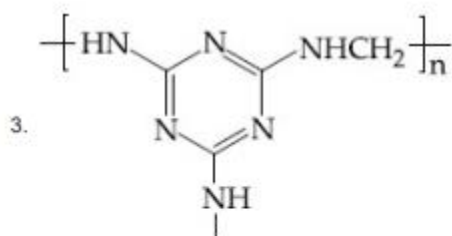
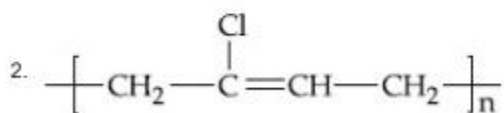
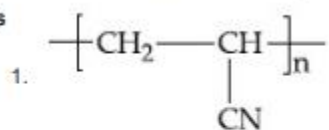
- Options**
1. Both Statement I and Statement II are true
  2. Statement I is false but Statement II is true
  3. Both Statement I and Statement II are false
  4. Statement I is true but Statement II is false

**Ans:** 1

**Sol:** Both statements are true

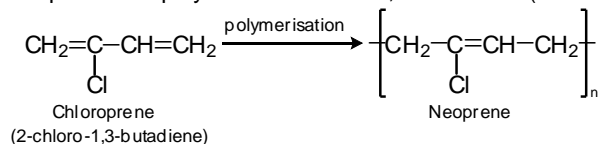
**Q.11** The structure of Neoprene is :

**Options**

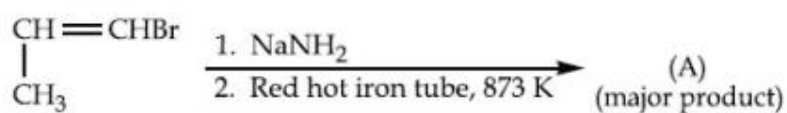


**Ans:** 2

**Sol:** Neoprene is a polymer of 2-chloro-1,3-butadiene (chloroprene)

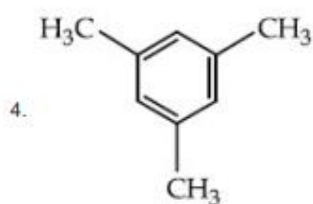
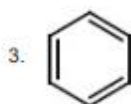
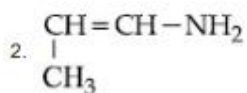


Q.12 For the given reaction :

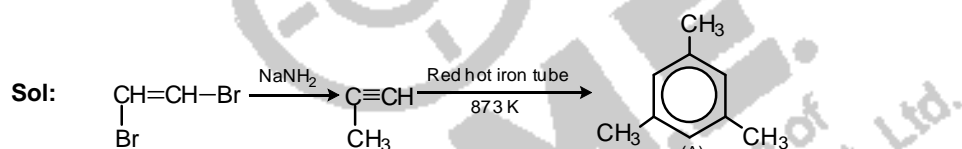


What is 'A' ?

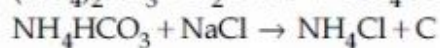
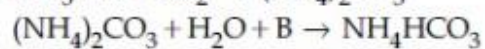
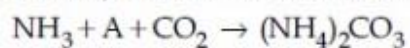
Options



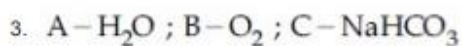
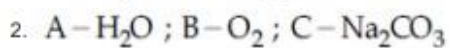
Ans : 4



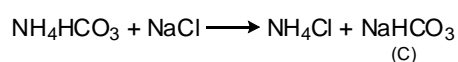
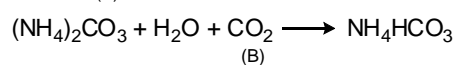
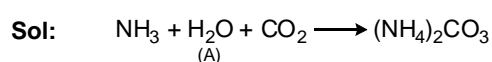
Q.13 Find A, B and C in the following reactions :



Options



Ans: 1



**Q.14** Statements about heavy water are given below.

- A. Heavy water is used in exchange reactions for the study of reaction mechanisms.
- B. Heavy water is prepared by exhaustive electrolysis of water.
- C. Heavy water has higher boiling point than ordinary water.
- D. Viscosity of  $\text{H}_2\text{O}$  is greater than  $\text{D}_2\text{O}$ .

Choose the most appropriate answer from the options given below :

- Options**
- 1. A and D only
  - 2. A and C only
  - 3. A, B and C only
  - 4. A and B only

**Ans:** 3

**Sol:** Viscosity of heavy water,  $\text{D}_2\text{O}$  (1.07 centipoise) is greater than ordinary water,  $\text{H}_2\text{O}$  (0.89 centipoise)

**Q.15** On treating a compound with warm dil.  $\text{H}_2\text{SO}_4$ , gas X is evolved which turns  $\text{K}_2\text{Cr}_2\text{O}_7$  paper acidified with dil.  $\text{H}_2\text{SO}_4$  to a green compound Y. X and Y respectively are :

- Options**
- 1.  $\text{X} = \text{SO}_3$ ,  $\text{Y} = \text{Cr}_2(\text{SO}_4)_3$
  - 2.  $\text{X} = \text{SO}_3$ ,  $\text{Y} = \text{Cr}_2\text{O}_3$
  - 3.  $\text{X} = \text{SO}_2$ ,  $\text{Y} = \text{Cr}_2\text{O}_3$
  - 4.  $\text{X} = \text{SO}_2$ ,  $\text{Y} = \text{Cr}_2(\text{SO}_4)_3$

**Ans:** 4

**Sol:** Compound + dil.  $\text{H}_2\text{SO}_4 \rightarrow \text{SO}_2$  (X)



**Q.16** Match List-I with List-II.

List-I		List-II	
Electronic configuration of elements		$\Delta_f H$ in $\text{kJ mol}^{-1}$	
(a)	$1s^2 2s^2$	(i)	801
(b)	$1s^2 2s^2 2p^4$	(ii)	899
(c)	$1s^2 2s^2 2p^3$	(iii)	1314
(d)	$1s^2 2s^2 2p^1$	(iv)	1402

Choose the most appropriate answer from the options given below :

- Options**
- 1. (a)  $\rightarrow$  (iv), (b)  $\rightarrow$  (i), (c)  $\rightarrow$  (ii), (d)  $\rightarrow$  (iii)
  - 2. (a)  $\rightarrow$  (i), (b)  $\rightarrow$  (iv), (c)  $\rightarrow$  (iii), (d)  $\rightarrow$  (ii)
  - 3. (a)  $\rightarrow$  (i), (b)  $\rightarrow$  (iii), (c)  $\rightarrow$  (iv), (d)  $\rightarrow$  (ii)
  - 4. (a)  $\rightarrow$  (ii), (b)  $\rightarrow$  (iii), (c)  $\rightarrow$  (iv), (d)  $\rightarrow$  (i)

**Ans:** 4

**Sol:**  $1s^2 2s^2$  – Be  
 $1s^2 2s^2 2p^1$ – B  
 $1s^2 2s^2 2p^3$ – N  
 $1s^2 2s^2 2p^4$ – O  
The correct increasing order of the first ionization enthalpy among these elements is of the order  
 $B < Be < O < N$

**Q.17** Which of the following is 'a' FALSE statement ?

**Options**

1. Kjeldahl's method is used for the estimation of nitrogen in an organic compound.

2.

Carius method is used for the estimation of nitrogen in an organic compound.

3.

Phosphoric acid produced on oxidation of phosphorus present in an organic compound is precipitated as  $Mg_2P_2O_7$  by adding magnesia mixture.

4.

Carius tube is used in the estimation of sulphur in an organic compound.

**Ans:** 2

**Sol:** Carius method is used for the estimation of halogens and sulphur in the organic compounds.

**Q.18** Which of the following vitamin is helpful in delaying the blood clotting ?

**Options**

1. Vitamin E

2. Vitamin C

3. Vitamin K

4. Vitamin B

**Ans:** 3

**Sol:** Vitamin K is helpful in delaying the clotting of blood.

**Q.19** Compound A used as a strong oxidizing agent is amphoteric in nature. It is the part of lead storage batteries. Compound A is :

**Options**

1.  $PbSO_4$

2.  $Pb_3O_4$

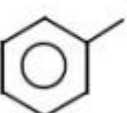
3.  $PbO$

4.  $PbO_2$

**Ans:** 4

**Sol:**  $PbO_2$  is an amphoteric oxide

**Q.20** An amine on reaction with benzenesulphonyl chloride produces a compound insoluble in alkaline solution. This amine can be prepared by ammonolysis of ethyl chloride. The correct structure of amine is :

- Options**
1.  $\text{CH}_3\text{CH}_2\text{CH}_2\text{NHCH}_3$
  2.  $\text{CH}_3\text{CH}_2\text{CH}_2\overset{\text{H}}{\text{N}}-\text{CH}_2\text{CH}_3$
  3.   $\text{NH}-\text{CH}_2\text{CH}_2\text{CH}_3$
  4.  $\text{CH}_3\text{CH}_2\text{NH}_2$

**Ans:** 2

**Sol:** 2° amine react with benzene sulphonyl chloride giving a product which is insoluble in alkali.  $\text{R}-\text{NH}-\text{CH}_2-\text{CH}_3$  can be prepared by ammonolysis of ethyl halide.

### SECTION B

**Q.1** 3.12 g of oxygen is adsorbed on 1.2 g of platinum metal. The volume of oxygen adsorbed per gram of the adsorbent at 1 atm and 300 K in L is \_\_\_\_\_.  
[ $R=0.0821 \text{ L atm K}^{-1} \text{ mol}^{-1}$ ]

**Ans:** 2.00

**Sol:**  $PV = nRT$

$$1 \times V = \frac{3.12}{32} \times 0.0821 \times 300 = 2.4 \text{ L}$$

1.2 g Pt metal absorb 2.4 L oxygen  
Volume adsorbed per gram = 2L

**Q.2** For a chemical reaction  $\text{A} + \text{B} = \text{C} + \text{D}$   
( $\Delta_r H^\ominus = 80 \text{ kJ mol}^{-1}$ ) the entropy change  $\Delta_r S^\ominus$  depends on the temperature T(in K) as  $\Delta_r S^\ominus = 2T \text{ (J K}^{-1} \text{ mol}^{-1})$ .  
Minimum temperature at which it will become spontaneous is \_\_\_\_\_ K. (Integer)

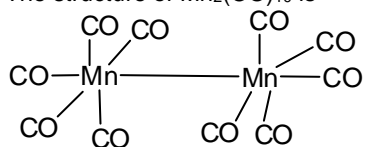
**Ans:** 200

**Sol:** At eqbm,  $\Delta G = 0 \quad \therefore \Delta H = T\Delta S$   
 $80 \times 10^3 \text{ J mol}^{-1} = T \times 2T \text{ K}^{-1} \text{ mol}^{-1}$   
 $2T^2 = 80 \times 10^3$   
 $T = 200 \text{ K}$

**Q.3** Number of bridging CO ligands in  $[\text{Mn}_2(\text{CO})_{10}]$  is \_\_\_\_\_ .

**Ans:** 0

**Sol:** The structure of  $\text{Mn}_2(\text{CO})_{10}$  is



**Q.4** Dichromate ion is treated with base, the oxidation number of Cr in the product formed is \_\_\_\_\_.

**Ans:** 6

**Sol:**  $\text{Cr}_2\text{O}_7^{2-} + 2\text{OH}^- \rightarrow 2\text{CrO}_4^{2-} + \text{H}_2\text{O}$   
Oxidation number of Cr in  $\text{CrO}_4^{2-}$  is +6

**Q.5** A homogeneous ideal gaseous reaction  $\text{AB}_{2(g)} = \text{A}_{(g)} + 2\text{B}_{(g)}$  is carried out in a 25 litre flask at 27°C. The initial amount of  $\text{AB}_2$  was 1 mole and the equilibrium pressure was 1.9 atm. The value of  $K_p$  is  $x \times 10^{-2}$ . The value of  $x$  is \_\_\_\_\_. (Integer answer)  
[R = 0.08206 dm<sup>3</sup>atm K<sup>-1</sup> mol<sup>-1</sup>]

**Ans:** 73

**Sol:**

	$\text{AB}_2$	$\rightleftharpoons$	$\text{A}$	$+$	$2\text{B}$
Initial no. of moles	1		0		0
At. eqbm	$1-\alpha$		$\alpha$		$2\alpha$

$PV = nRT$   
 $1.9 \times 25 = n \times 0.0821 \times 300$   
 $n = 1.93$   
 $1 + 2\alpha = 1.93$   
 $2\alpha = 0.93$   
 $\alpha = 0.465$   
 At eqbm,  $n_{\text{AB}_2} = 0.535$ ,  $n_{\text{A}} = 0.465$ ,  $n_{\text{B}} = 0.93$   
 $K_p = \frac{\left(\frac{0.465}{1.93} \times 1.9\right) \left(\frac{0.93}{1.93} \times 1.9\right)^2}{\frac{0.535}{1.93} \times 1.9} \cong 73 \times 10^{-2} \text{ atm}^2$

**Q.6** 224 mL of  $\text{SO}_2$  at 298 K and 1 atm is passed through 100 mL of 0.1 M NaOH solution. The non-volatile solute produced is dissolved in 36 g of water. The lowering of vapour pressure of solution (assuming the solution is dilute) ( $P_{(\text{H}_2\text{O})}^* = 24 \text{ mm of Hg}$ ) is  $x \times 10^{-2} \text{ mm of Hg}$ , the value of  $x$  is \_\_\_\_\_. (Integer answer)

**Ans:** 24

**Sol:**

$$\underset{0.01}{\text{SO}_2} + \underset{0.01}{\text{NaOH}} \rightarrow \underset{0.01 \text{ mol}}{\text{NaHSO}_3}$$

$$\frac{\Delta P}{P_A^*} = i \times \frac{n_B}{n_A}$$

$$\frac{\Delta P}{P_A^*} = 2 \times \frac{0.01}{2}$$

$$\frac{24 - x}{24} = 2 \times \frac{0.01}{2}$$

$$24 - x = 0.24$$

$$x = 23.76$$

$\therefore$  lowering of vapour pressure,  $\Delta P = 0.24 = 24 \times 10^{-2} \text{ mm of Hg}$

**Q.7** Consider the following reaction



The quantity of electricity required in Faraday to reduce five moles of  $\text{MnO}_4^-$  is \_\_\_\_\_. (Integer answer)

Ans: 25

Sol: 1 mole  $\text{MnO}_4^-$  required 5F  
 $\therefore$  5 mole  $\text{MnO}_4^-$  required 25 F

Q.8 A certain gas obeys  $P(V_m - b) = RT$ . The value of  $\left(\frac{\partial Z}{\partial P}\right)_T$  is  $\frac{xb}{RT}$ . The value of x is \_\_\_\_\_.  
(Integer answer) (Z : compressibility factor)

Ans: 1

Sol:  $z = 1 + \frac{Pb}{RT}$   
 $\left(\frac{\partial z}{\partial P}\right)_T = 0 + \frac{b}{RT} \times 1$   
 $\left(\frac{\partial z}{\partial P}\right)_T = \frac{1b}{RT}$   
X = 1

Q.9 The number of significant figures in  $50000.020 \times 10^{-3}$  is \_\_\_\_\_.

Ans: 8

Sol:  $50000.020 \times 10^{-3}$  Number of significant figures = 8

Q.10 An exothermic reaction  $X \rightarrow Y$  has an activation energy  $30 \text{ kJ mol}^{-1}$ . If energy change  $\Delta E$  during the reaction is  $-20 \text{ kJ}$ , then the activation energy for the reverse reaction in kJ is \_\_\_\_\_. (Integer answer)

Ans: 50

Sol:  $\Delta H = E_{a(f)} - E_{a(b)}$   
 $-20 = 30 - E_{a(b)}$   
 $\therefore E_{a(b)} = 30 + 20 = 50 \text{ kJ}$

## PART – C – MATHEMATICS

### SECTION A

Q.1 If  $(1, 5, 35)$ ,  $(7, 5, 5)$ ,  $(1, \lambda, 7)$  and  $(2\lambda, 1, 2)$  are coplanar, then the sum of all possible values of  $\lambda$  is:

- Options
1.  $-\frac{44}{5}$
  2.  $-\frac{39}{5}$
  3.  $\frac{39}{5}$
  4.  $\frac{44}{5}$

Ans: 4

**Sol:** A(1, 5, 35), B(7, 5, 5), C(1, λ, 7) and D(2λ, 1, 2)

$$[AB \ AC \ AD] = 0$$

$$\begin{vmatrix} 6 & 0 & -30 \\ 0 & \lambda - 5 & -28 \\ 2\lambda - 1 & -4 & -33 \end{vmatrix} = 0$$

$$16\lambda^2 - 88\lambda + 78 = 0$$

Sum of possible values =  $\frac{44}{5}$

**Q.2** The maximum slope of the curve  $y = \frac{1}{2}x^4 - 5x^3 + 18x^2 - 19x$  occurs at the point :

**Options**

1.  $\left(3, \frac{21}{2}\right)$
2. (0, 0)
3. (2, 2)
4. (2, 9)

**Ans:** 3

**Sol:**  $\frac{dy}{dx} = 2x^3 - 15x^2 + 36x - 19$   
 Let  $f(x) = 2x^3 - 15x^2 + 36x - 19$   
 $f'(x) = 6x^2 - 30x + 36 = 0$   
 $x^2 - 5x + 6 = 0$   
 $x = 2, 3$   
 $f'(x) = 12x - 30$   
 $f''(x) < 0$  for  $x = 2$   
 At  $x = 2$   
 $y = 8 - 40 + 72 - 19 = 21$   
 $y = 72 - 70 = 2$   
 $\Rightarrow (2, 2)$

**Q.3** Let  $f$  be any function defined on  $\mathbb{R}$  and let it satisfy the condition :

$$|f(x) - f(y)| \leq |x - y|^2, \forall (x, y) \in \mathbb{R}$$

If  $f(0) = 1$ , then :

**Options**

1.  $f(x) < 0, \forall x \in \mathbb{R}$
2.  $f(x) = 0, \forall x \in \mathbb{R}$
3.  $f(x) > 0, \forall x \in \mathbb{R}$
4.  $f(x)$  can take any value in  $\mathbb{R}$

**Ans:** 3

**Sol:**  $\left| \frac{f(x) - f(y)}{x - y} \right| \leq |x - y|$   
 $\lim_{x \rightarrow y} \left| \frac{f(x) - f(y)}{x - y} \right| \leq 0$   
 $|f'(y)| \leq 0 \Rightarrow f'(y) = 0$   
 $f(y) = c$   
 $\Rightarrow c = 1 \quad \forall f(0) = 1$   
 $\Rightarrow f(x) = 1$

Q.4

If  $\vec{a}$  and  $\vec{b}$  are perpendicular, then  $\vec{a} \times (\vec{a} \times (\vec{a} \times (\vec{a} \times \vec{b})))$  is equal to :

Options

1.  $\vec{0}$
2.  $\frac{1}{2} |\vec{a}|^4 \vec{b}$
3.  $\vec{a} \times \vec{b}$
4.  $|\vec{a}|^4 \vec{b}$

Ans: 4

Sol:  $\vec{a} \times (\vec{a} \times ((\vec{a} \cdot \vec{b})\vec{a} - |\vec{a}|^2\vec{b}))$

$$\vec{a} \times (-|\vec{a}|^2(\vec{a} \times \vec{b})) = -|\vec{a}|^2((\vec{a} \cdot \vec{b})\vec{a} - |\vec{a}|^2\vec{b}) = |\vec{a}|^4\vec{b} \quad (\because \vec{a} \cdot \vec{b} = 0)$$

Q.5

In an increasing geometric series, the sum of the second and the sixth term is  $\frac{25}{2}$  and the product of the third and fifth term is 25. Then, the sum of 4<sup>th</sup>, 6<sup>th</sup> and 8<sup>th</sup> terms is equal to :

Options

1. 32
2. 26
3. 30
4. 35

Ans: 4

Sol:

$$ar + ar^5 = \frac{25}{2}$$

$$ar^2 \times ar^4 = 25$$

$$ar^3 = 5$$

$$a = \frac{5}{r^3} \quad \text{--- (1)}$$

$$\frac{5r}{r^3} + \frac{5r^5}{r^3} = \frac{25}{2}$$

$$\frac{1}{r^2} + r^2 = \frac{5}{2}$$

Put  $r^2 = u$

$$\frac{u^2+1}{u} = \frac{5}{2}$$

$$u = \frac{1}{2}, 2 \Rightarrow r^2 = \frac{1}{2}, 2$$

$$r = \sqrt{2}$$

Sum of 4<sup>th</sup>, 6<sup>th</sup> and 8<sup>th</sup> terms is

$$= 5[1 + 2 + 4] = 35$$

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**Q.6** The sum of the infinite series  $1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \dots$  is equal to :

- Options**
1.  $\frac{9}{4}$
  2.  $\frac{15}{4}$
  3.  $\frac{13}{4}$
  4.  $\frac{11}{4}$

**Ans: 3**

**Sol:**

$$S = 1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \dots$$

$$\frac{S}{3} = \frac{1}{3} + \frac{2}{3^2} + \frac{7}{3^3} + \dots \dots \dots \infty$$

$$\frac{2S}{3} = 1 + \frac{1}{3} + \frac{5}{3^2} + \frac{5}{3^3} + \dots \dots \dots \infty$$

$$\frac{2S}{3} = \frac{4}{3} + \frac{5}{3} \left( \frac{1/3}{1-1/3} \right) = \frac{5}{6} + \frac{4}{3} = \frac{13}{6}$$

$$S = \frac{13}{4}$$

**Q.7** The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only is :

- Options**
1. 35
  2. 82
  3. 77
  4. 42

**Ans: 3**

**Sol:** One of possibility is : 1, 1, 1, 1, 1, 2, 3  
 Number of ways of arranging =  $\frac{7!}{5!} = 42$   
 Second possibility is : 1, 1, 1, 1, 2, 2, 2  
 Number of ways of arranging =  $\frac{7!}{4! \cdot 3!} = 42 = 35$   
 Total number of ways =  $42 + 35 = 77$

**Q.8** The value of  $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1 + 3^x} dx$  is :

- Options**
1.  $\frac{\pi}{4}$
  2.  $\frac{\pi}{2}$
  3.  $2\pi$
  4.  $4\pi$

**Ans: 1**

**Sol:** Let  $I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx$   
 $I = \int_{-\pi/2}^{\pi/2} \frac{3^x \cos^2 x}{1+3^x} dx$   
 $2I = \int_{-\pi/2}^{\pi/2} \cos^2 x dx$   
 $I = \int_0^{\pi/2} \cos^2 x dx = \frac{\pi}{4}$

**Q.9** The intersection of three lines  $x - y = 0$ ,  $x + 2y = 3$  and  $2x + y = 6$  is a :

- Options**
1. Isosceles triangle
  2. Right angled triangle
  3. Equilateral triangle
  4. None of the above

**Ans:** 1

**Sol:** The vertices of the triangle formed by the lines is (3,0), (1,1) and (2,2). The lengths of the sides are  $\sqrt{5}$ ,  $\sqrt{5}$  and  $\sqrt{2}$ . Therefore, the triangle is an isosceles triangle.

**Q.10**

The value of  $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$  is :

- Options**
1. -2
  2. 0
  3.  $(a+1)(a+2)(a+3)$
  4.  $(a+2)(a+3)(a+4)$

**Ans:** 1

**Sol:**  $C_1 \rightarrow C_1 - C_2, C_2 \rightarrow C_2 - C_3$   
 $= \begin{vmatrix} (a+2)a & a+1 & 1 \\ (a+3)(a+1) & a+2 & 1 \\ (a+4)(a+2) & a+3 & 1 \end{vmatrix}$   
 $R_2 \rightarrow R_2 - R_1$  and  $R_3 \rightarrow R_3 - R_1$   
 $= \begin{vmatrix} a^2 + 2a & a+1 & 1 \\ 2a+3 & 1 & 0 \\ 4a+8 & 2 & 0 \end{vmatrix}$   
 $= 6 - 8 = -2$

**Q.11** Consider the three planes

$$P_1 : 3x + 15y + 21z = 9,$$

$$P_2 : x - 3y - z = 5, \text{ and}$$

$$P_3 : 2x + 10y + 14z = 5$$

Then, which one of the following is true ?

- Options**
1.  $P_2$  and  $P_3$  are parallel.
  2.  $P_1$  and  $P_3$  are parallel.
  3.  $P_1, P_2$  and  $P_3$  all are parallel.
  4.  $P_1$  and  $P_2$  are parallel.

**Ans: 2**

**Sol:** The direction ratios of  $P_1$  and  $P_3$  are proportional. Therefore  $P_1$  and  $P_3$  are parallel.

**Q.12**

The maximum value of the term independent of 't' in the expansion of  $\left( tx^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t} \right)^{10}$

where  $x \in (0, 1)$  is :

- Options**
1.  $\frac{10!}{3(5!)^2}$
  2.  $\frac{10!}{\sqrt{3}(5!)^2}$
  3.  $\frac{2 \cdot 10!}{3(5!)^2}$
  4.  $\frac{2 \cdot 10!}{3\sqrt{3}(5!)^2}$

**Ans: 4**

**Sol:**  $T_{r+1} = {}^{10}C_r \left( tx^{\frac{1}{5}} \right)^{10-r} \left[ \frac{(1-x)^{\frac{1}{10}}}{t} \right]^r = {}^{10}C_r t^{(10-2r)} \times x^{\frac{10-r}{5}} \times (1-x)^{\frac{r}{10}}$   
 $\Rightarrow 10 - 2r = 0 \Rightarrow r = 5$

$$T_6 = {}^{10}C_5 x^{\sqrt{1-x}}$$
$$\frac{dT_6}{dx} = {}^{10}C_5 \left[ \sqrt{1-x} - \frac{x}{2\sqrt{1-x}} \right] = 0$$

$$= 1x = \frac{x}{2} \Rightarrow 3x = 2$$

$$\Rightarrow x = \frac{2}{3}$$

$$\text{Max } T_6 = \frac{10!}{5! 5! 3\sqrt{3}}$$

**Q.13** Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of  $A^2$  is 1, then the possible number of such matrices is :

- Options**
1. 4
  2. 6
  3. 12
  4. 1

Ans: 1

Sol: Let  $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$   
 $a^2 = \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} a^2 + b^2 & ab + bc \\ ab + bc & c^2 + b^2 \end{bmatrix}$   
 $= a^2 + 2b^2 + c^2 = 1$   
 $a = 1, b = 0, c = 0$   
 $a = 0, b = 0, c = 1$   
 $a = -1, b = 0, c = 0$   
 $c = -1, b = 0, a = 0$

Q.14 The value of  $\sum_{n=1}^{100} \int_{n-1}^n e^{x-[x]} dx$ , where  $[x]$  is the greatest integer  $\leq x$ , is :

- Options
1.  $100(e-1)$
  2.  $100(1-e)$
  3.  $100e$
  4.  $100(1+e)$

Ans: 1

Sol:  $\sum_{n=1}^{100} \int_{n-1}^n e^{x-[x]} dx$   
 $= \int_0^1 e^{(x)} dx + \int_1^2 e^{(x)} dx + \dots + \int_{99}^{100} e^{(x)} dx$  ( $\because [x] = x - [x]$ )  
 $= (e-1) + (e-1) + (e-1) + \dots + (e-1)$   
 $= 100(e-1)$

Q.15 A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is :

- Options
1.  $\frac{15}{2^{14}}$
  2.  $\frac{15}{2^{12}}$
  3.  $\frac{15}{2^{13}}$
  4.  $\frac{15}{2^8}$

Ans: 3

Sol:  $P(X=9) = P(X=7)$   
 $nC_9 \left(\frac{1}{2}\right)^{n-9} \left(\frac{1}{2}\right)^9 = nC_7 \left(\frac{1}{2}\right)^{n-7} \left(\frac{1}{2}\right)^7$   
 $nC_9 = nC_7$   
 $n = 9 + 7 = 16$   
 $P(X=2) = {}^{16}C_2 \left(\frac{1}{2}\right)^{16-2} \left(\frac{1}{2}\right)^2 = {}^{16}C_2 \left(\frac{1}{2}\right)^{16} = \frac{15}{2^{11}}$

Q.16

The value of  $\lim_{h \rightarrow 0} 2 \left\{ \frac{\sqrt{3} \sin\left(\frac{\pi}{6} + h\right) - \cos\left(\frac{\pi}{6} + h\right)}{\sqrt{3}h(\sqrt{3}\cos h - \sin h)} \right\}$  is :

Options

1.  $\frac{2}{\sqrt{3}}$
2.  $\frac{3}{4}$
3.  $\frac{4}{3}$
4.  $\frac{2}{3}$

Ans: 3

Sol:  $\lim_{h \rightarrow 0} 2 \times 2 \left\{ \frac{\sin\left(\frac{\pi}{6} + h - \frac{\pi}{6}\right)}{2\sqrt{3}h\cos\left(h + \frac{\pi}{6}\right)} \right\} = \lim_{h \rightarrow 0} 4 \frac{\sinh}{2\sqrt{3}h} \times \frac{1}{\cos\left(h + \frac{\pi}{6}\right)} = \frac{2}{\sqrt{3}} \times 1 \times \frac{2}{\sqrt{3}} = \frac{4}{3}$

Q.17 Let  $R = \{(P, Q) \mid P \text{ and } Q \text{ are at the same distance from the origin}\}$  be a relation, then the equivalence class of  $(1, -1)$  is the set :

Options

1.  $S = \{(x, y) \mid x^2 + y^2 = \sqrt{2}\}$
2.  $S = \{(x, y) \mid x^2 + y^2 = 1\}$
3.  $S = \{(x, y) \mid x^2 + y^2 = 4\}$
4.  $S = \{(x, y) \mid x^2 + y^2 = 2\}$

Ans: 4

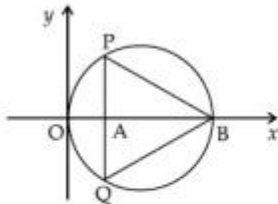
Sol: Let  $P(a, b), Q(c, d), PO = QO$

$$\Rightarrow a^2 + b^2 = c^2 + d^2$$

Let  $R(x, y), S(1, -1), RO = SO$

$$x^2 + y^2 = 2$$

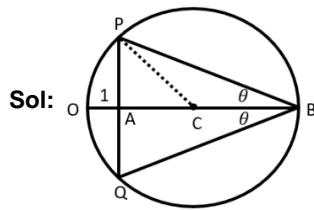
Q.18 In the circle given below, let  $OA = 1$  unit,  $OB = 13$  unit and  $PQ \perp OB$ . Then, the area of the triangle  $PQB$  (in square units) is :



Options

1.  $24\sqrt{2}$
2.  $26\sqrt{3}$
3.  $26\sqrt{2}$
4.  $24\sqrt{3}$

Ans: 4



$$OC = \frac{13}{2} = 6.5$$

$$AC = 6.5 - 1 = 5.5$$

$$PA = \sqrt{(6.5)^2 - (5.5)^2} = \sqrt{12}$$

$$PQ = 2 \cdot PA = 2\sqrt{12}$$

$$\text{Area of } \Delta PQB = \frac{1}{2} \times 2\sqrt{12} \times 12 = 12\sqrt{12} = 24\sqrt{3}$$

Sol:

**Q.19** If  $\frac{\sin^{-1} x}{a} = \frac{\cos^{-1} x}{b} = \frac{\tan^{-1} y}{c}$ ;  $0 < x < 1$ , then the value of  $\cos\left(\frac{\pi c}{a+b}\right)$  is :

Options

1.  $\frac{1-y^2}{2y}$

2.  $1-y^2$

3.  $\frac{1-y^2}{1+y^2}$

4.  $\frac{1-y^2}{y\sqrt{y}}$

Ans: 3

Sol: Applying componendo dividend,

$$\frac{\sin^{-1} x}{a} = \frac{\cos^{-1} x}{b} = \frac{\sin^{-1} x + \cos^{-1} x}{a+b} = \frac{\pi}{2(a+b)}$$

$$\text{Now, } \frac{\tan^{-1} y}{c} = \frac{\pi}{2(a+b)}$$

$$2 \tan^{-1} y = \frac{\pi c}{a+b} \Rightarrow \cos\left(\frac{\pi c}{a+b}\right) = \cos(2 \tan^{-1} y) = \frac{1-y^2}{1+y^2}$$

**Q.20** The rate of growth of bacteria in a culture is proportional to the number of bacteria present and the bacteria count is 1000 at initial time  $t=0$ . The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after  $\frac{k}{\log_e\left(\frac{6}{5}\right)}$  hours, then  $\left(\frac{k}{\log_e 2}\right)^2$  is equal to :

Options 1. 16

2. 8

3. 4

4. 2

Ans: 3

**Sol:**  $\int_{1000}^x \frac{dx}{dt} = \int_0^t \lambda dt$   
 $\ln x - \ln 1000 = \lambda t$   
 $\ln\left(\frac{x}{1000}\right) = \lambda t$   
 Put  $t = 2$ , and  $x = 1200$   
 $\ln\left(\frac{12}{10}\right) = 2\lambda \Rightarrow \lambda = \frac{1}{2} \ln \frac{6}{5}$   
 Now  $\ln\left(\frac{x}{1000}\right) = \frac{t}{2} \ln\left(\frac{6}{5}\right)$   
 $x = 1000 e^{\frac{t}{2} \ln\left(\frac{6}{5}\right)}$   
 $x = 2000$  at  $t = \frac{k}{\ln\left(\frac{6}{5}\right)}$   
 $\Rightarrow 2000 = 1000 e^{\frac{k}{2 \ln\left(\frac{6}{5}\right)} \times \ln\left(\frac{6}{5}\right)}$   
 $\Rightarrow 2 = e^{k/2} \Rightarrow \ln 2 = \frac{k}{2} \Rightarrow \frac{k}{\ln 2} = 2 \Rightarrow \left(\frac{k}{\ln 2}\right)^2 = 4$

**SECTION B**

**Q.1** If  $y=y(x)$  is the solution of the equation  $e^{\sin y} \cos y \frac{dy}{dx} + e^{\sin y} \cos x = \cos x$ ,  $y(0)=0$ ; then

$1 + y\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right) + \frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)$  is equal to \_\_\_\_\_.

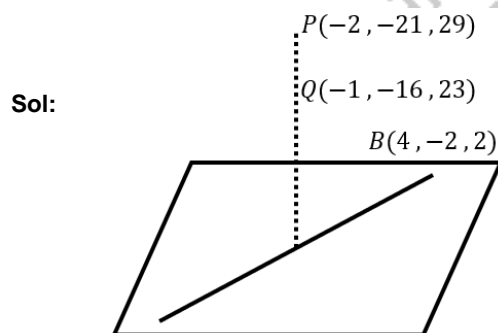
**Ans: 1**

**Sol:**  $e^{\sin y} \cos y \frac{dy}{dx} + e^{\sin y} \cos x = \cos x$   
 Solution is given by,  
 $e^{\sin y} \cdot e^{\sin x} = e^{\sin x} + C$   
 Put  $x = 0$  and  $y = 0$   
 $1 = 1 + C \Rightarrow C = 0$   
 $e^{\sin x + \sin y} = e^{\sin x}$   
 $\sin x + \sin y = \sin x \Rightarrow y = 0$   
 $\Rightarrow y\left(\frac{\pi}{6}\right) = y\left(\frac{\pi}{3}\right) = y\left(\frac{\pi}{4}\right) = 0$   
 $\Rightarrow 1 + 0 + 0 + 0 = 1$

**Q.2** Let  $(\lambda, 2, 1)$  be a point on the plane which passes through the point  $(4, -2, 2)$ . If the plane is perpendicular to the line joining the points  $(-2, -21, 29)$  and  $(-1, -16, 23)$ , then

$\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$  is equal to \_\_\_\_\_.

**Ans: 8**



$A(\lambda, 2, 1)$   
 $B(4, -2, 2)$   
 $AB$  is perpendicular to  $PQ$   
 $\{(4 - \lambda)i - 4j + k\} \cdot \{-i + 5j - 6k\} = 0$   
 $\Rightarrow 4 - \lambda - 20 - 6 = 0 \Rightarrow \lambda = -22$   
 $\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4 = 8$

**Q.3** The difference between degree and order of a differential equation that represents the family of curves given by  $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right)$ ,  $a > 0$  is \_\_\_\_\_.

**Ans: 2**

**Sol:**  $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right) \Rightarrow 2yy' = a$   
 $y^2 = 2yy'\left(x + \frac{\sqrt{2yy'}}{2}\right) \Rightarrow y = 2y'\left(x + \frac{\sqrt{yy'}}{\sqrt{2}}\right)$   
 $y - 2xy' = \sqrt{2}y' \cdot \sqrt{yy'} \Rightarrow \left(y - 2x \cdot \frac{dy}{dx}\right)^2 = 2y\left(\frac{dy}{dx}\right)^3$   
 Here, order is 1 and degree 3. Therefore, difference is 2.

**Q.4** The number of solutions of the equation  $\log_4(x-1) = \log_2(x-3)$  is \_\_\_\_\_.

**Ans: 1**

**Sol:**  $\frac{1}{2} \log_2(x-1) = \log_2(x-3)$   
 $\log_2(x-1) = 2 \cdot \log_2(x-3)$   
 $x-1 = (x-3)^2 \Rightarrow x^2 - 7x + 10 = 0$   
 $x = 5$  (if  $x = 2$ ,  $\log(x-1)$  is not defined)  
 Therefore only 1 solution

**Q.5** The value of the integral  $\int_0^{\pi} |\sin 2x| dx$  is \_\_\_\_\_.

**Ans: 2**

**Sol:**  $\int_0^{\pi} |\sin 2x| dx = 2 \int_0^{\pi/2} |\sin 2x| dx = 2 \int_0^{\pi/2} \sin 2x dx = 2 \left[-\frac{\cos 2x}{2}\right]_0^{\pi/2} = 2$   
 $\therefore n = 1.00$

**Q.6** Let  $m, n \in \mathbb{N}$  and  $\gcd(2, n) = 1$ . If  $30 \binom{30}{0} + 29 \binom{30}{1} + \dots + 2 \binom{30}{28} + 1 \binom{30}{29} = n \cdot 2^m$ , then  $n+m$  is equal to \_\_\_\_\_.

(Here  $\binom{n}{k} = {}^n C_k$ )

**Ans: 45**

**Sol:**  $S = \sum_{r=0}^{30} (30-r) \cdot 30 C_r = 30 \sum_{r=0}^{30} 30 C_r - \sum_{r=0}^{30} r 30 C_r$   
 $= 20 \times 2^{30} - \sum_{r=1}^{30} r \cdot \frac{30}{4} \cdot 29 C_{r-1} = 30 \times 2^{30} - 30 \times 2^{29}$   
 $= (60 - 30) \cdot 2^{29} = 15 \cdot 2^{30}$   
 $n = 15, m = 30 \Rightarrow n + m = 45$

**Q.7** If  $\sqrt{3}(\cos^2 x) = (\sqrt{3} - 1)\cos x + 1$ , the number of solutions of the given equation when  $x \in \left[0, \frac{\pi}{2}\right]$  is \_\_\_\_\_.

**Ans: 1**

**Sol:** Solving we get  $\cos x = 1$  and  $\cos x = \frac{1}{\sqrt{3}}$   
 But  $-\frac{1}{\sqrt{3}}$  is not possible as  $x \in \left[0, \frac{\pi}{2}\right]$   
 Therefore, number of solutions is 1.

**Q.8** The number of integral values of 'k' for which the equation  $3\sin x + 4\cos x = k + 1$  has a solution,  $k \in \mathbb{R}$  is \_\_\_\_\_.

**Ans: 11**

**Sol:**  $3\sin x + 4\cos x = k + 1 - \sqrt{3^2 + 4^2} \leq k + 1 \leq \sqrt{3^2 + 4^2} - 5 \leq k + 1 \leq 5 - 6 \leq k \leq 4$   
 $k = -6, -5, -4, -3, -2, 1, 0, 1, 2, 3, 4 \Rightarrow 11$  solutions

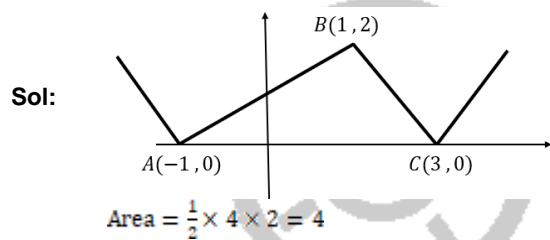
**Q.9** The sum of 162<sup>th</sup> power of the roots of the equation  $x^3 - 2x^2 + 2x - 1 = 0$  is \_\_\_\_\_.

**Ans: 3**

**Sol:**  $(x - 1)(x^2 - x + 1) = 0$   
 $x = 1, -\omega, -\omega^2$   
**Ans**  $= 1 + \omega^{162} + (\omega^2)^{162} = 1 + (\omega^3)^{54} + (\omega^3)^{108} = 1 + 1 + 1 = 3$

**Q.10** The area bounded by the lines  $y = ||x - 1| - 2|$  is \_\_\_\_\_.

**Ans: 4**



**TIME.**  
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