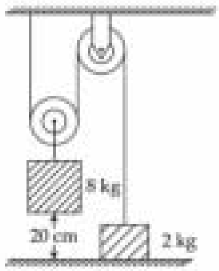
SOLUTIONS & ANSWERS FOR JEE MAINS-2021
27^{th} August Shift 2
[PHYSICS, CHEMISTRY & MATHEMATICS]

PART – A – PHYSICS

Section A

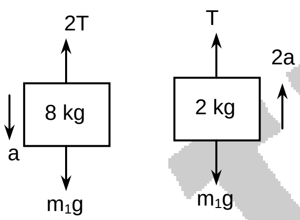
Q.1 The boxes of masses 2 kg and 8 kg are connected by a massless string passing over smooth pulleys. Calculate the time taken by box of mass 8 kg to strike the ground starting from rest. (use $g = 10\ m/s^2$) :



Options
1. 0.34 s
2. 0.25 s
3. 0.2 s
4. 0.4 s

Ans: 0.4 s

Sol:



$(m_1g - 2T) = m_1a$ – (1)

$T - m_2g = m_2(2a)$

$2T - 2m_2g = 4m_2 a$ – (2)

$m_1g - 2m_2g = (m_1 + 4m_2)\, a$

$$a = \frac{(8-4)g}{(8+8)} = \frac{4}{16}g = \frac{g}{4}$$

$$a = \frac{10}{4} m/s^2$$

$$S = \frac{1}{2}at^2$$

$$\frac{0.2 \times 2 \times 4}{10} = t^2$$

t = 0.4 sec

Q.2 Match **List - I** with **List - II**.

	List - I		List - II
(a)	R_H(Rydberg constant)	(i)	$kg\ m^{-1}s^{-1}$
(b)	h(Planck's constant)	(ii)	$kg\ m^2\ s^{-1}$
(c)	μ_B(Magnetic field energy density)	(iii)	m^{-1}
(d)	η(coefficient of viscocity)	(iv)	$kg\ m^{-1}\ s^{-2}$

Choose the **most appropriate** answer from the options given below :

Options
1. (a)-(iii), (b)-(ii), (c)-(i), (d)-(iv)
2. (a)-(iv), (b)-(ii), (c)-(i), (d)-(iii)
3. (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)
4. (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

Ans: (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)

Sol: SI unit of Rydberg constant = m^{-1}
SI unit of Plank's constant = $kg\ m^2\ s^{-1}$
SI unit of Magnetic field energy density = $kg\ m^{-1}s^{-2}$
SI unit of coefficient of viscosity = $kg\ m^{-1}\ s^{-1}$

Q.3 If the rms speed of oxygen molecules at 0°C is 160 m/s, find the rms speed of hydrogen molecules at 0°C.

Options
1. 40 m/s
2. 80 m/s
3. 332 m/s
4. 640 m/s

Ans: 640 m / s

Sol: $V_{rms} = \sqrt{\frac{3KT}{M}}$

$$\frac{(V_{rms})_{O_2}}{(V_{rms})_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{O_2}}} = \sqrt{\frac{2}{32}}$$

$$(V_{rms})_{H_2} = 4 \times (V_{rms})_{O_2} = 4 \times 160 = 640 \text{ m/s}$$

Q.4 A monochromatic neon lamp with wavelength of 670.5 nm illuminates a photo-sensitive material which has a stopping voltage of 0.48 V. What will be the stopping voltage if the source light is changed with another source of wavelength of 474.6 nm ?

Options
1. 0.96 V
2. 1.25 V
3. 0.24 V
4. 1.5 V

Ans: 1.25 V

Sol: $kE_{max} = \frac{hc}{\lambda_i} + \phi$

Or $eV_0 = \frac{hc}{\lambda_i} + \phi$

When λ_i = 670.5 nm; V_0 = 0.48
When λ_i = 474.6 nm; V_0 = ?

So, $e(0.48) = \frac{1240}{670.5} + \phi$ ----------(1)

$e(V_0) = \frac{1240}{474.6} + \phi$ -------- (2)

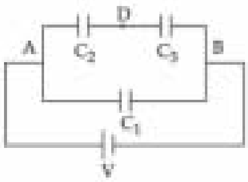
(2) – (1)

$e(V_0 - 0.48) = 1240\left(\frac{1}{474.6} - \frac{1}{670.5}\right)eV$

$V_0 = 0.48 + 1240\left(\frac{670.5 - 474.6}{474.6 \times 670.5}\right)$ Volts

V_0 = 0.48 + 0.76
V_0 = 1.24 V $\cong$ 1.25 V

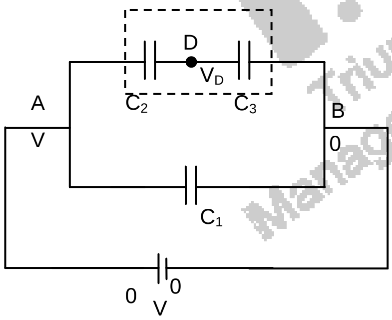
Q.5 Three capacitors $C_1 = 2\ \mu F$, $C_2 = 6\ \mu F$ and $C_3 = 12\ \mu F$ are connected as shown in figure. Find the ratio of the charges on capacitors C_1, C_2 and C_3 respectively :



Options
1. 3 : 4 : 4
2. 1 : 2 : 2
3. 2 : 1 : 1
4. 2 : 3 : 3

Ans: 1 : 2 : 2

Sol:



$(V_D - V)\ C_2 + (V_D - 0)\ C_3 = 0$
$(V_D - V)\ 6 + (V_D - 0)\ 12 = 0$
$V_D - V + 2V_D = 0$

$V_D = \frac{V}{3}$

$q_2 = (V - V_D)C_2 = \left(V - \frac{V}{3}\right)(6\mu F)$

$q_2 = (4V)\ \mu F$

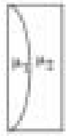
$q_3 = (V_D - 0)\ C_3 = \frac{V}{3} \times 12\ \mu F = 4V\ \mu F$

$q_1 = (V - 0)\, C_1 = V\,(2\mu F)$

$q_1 : q_2 : q_3 = 2 : 4 : 4$

$q_1 : q_2 : q_3 = 1 : 2 : 2$

Q.8 Curved surfaces of a plano-convex lens of refractive index μ_1 and a plano-concave lens of refractive index μ_2 have equal radius of curvature as shown in figure. Find the ratio of radius of curvature to the focal length of the combined lenses.

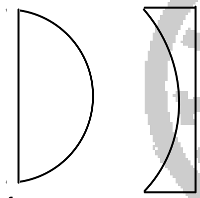


Options

1. $\mu_1 - \mu_2$
2. $\dfrac{1}{\mu_2 - \mu_1}$
3. $\dfrac{1}{\mu_1 - \mu_2}$
4. $\mu_2 - \mu_1$

Ans: $\mu_1 - \mu_2$

Sol:



$$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{R}\right)$$

$$\frac{1}{f_2} = (\mu_2 - 1)\left(-\frac{1}{R}\right)$$

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f_{eq}} = \frac{(\mu_1 - 1) - (\mu_2 - 1)}{R}$$

$$\frac{1}{f_{eq}} = \frac{(\mu_1 - \mu_2)}{R}$$

$$\frac{R}{f_{eq}} = (\mu_1 - \mu_2)$$

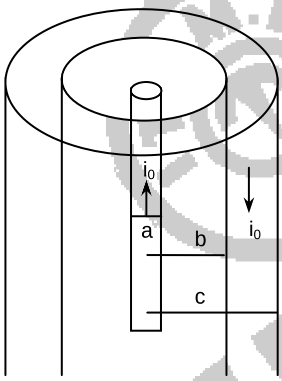
Q.7 A coaxial cable consists of an inner wire of radius 'a' surrounded by an outer shell of inner and outer radii 'b' and 'c' respectively. The inner wire carries an electric current i_0 which is distributed uniformly across cross-sectional area. The outer shell carries an equal current in opposite direction and distributed uniformly. What will be the ratio of the magnetic field at a distance x from the axis when (i) x < a and (ii) a < x < b ?

Options

1. $\dfrac{x^2}{b^2 - a^2}$
2. $\dfrac{b^2 - a^2}{x^2}$
3. $\dfrac{x^2}{a^2}$
4. $\dfrac{a^2}{x^2}$

Ans: $\dfrac{x^2}{a^2}$

Sol:



When x < a

$$B_1(2\pi x) = \mu_0\left(\frac{i_0}{\pi a^2}\right)\pi x^2$$

$$B(2\pi x) = \frac{\mu_0 i_0 x^2}{a^2}$$

$$B_1 = \frac{\mu_0 i_0 x}{2\pi a^2} \text{ ------------------(1)}$$

When a < x < b

$B_2(2\pi x) = \mu_0 i_0$

$$B_2 = \frac{\mu_0 i_0}{2\pi x} \text{ ----------------(2)}$$

$$\frac{B_1}{B_2} = \frac{\mu_0 i_0 \dfrac{x}{2\pi a^2}}{\dfrac{\mu_0 i_0}{2\pi x}} = \frac{x^2}{a^2}$$

Q.8 Two discs have moments of intertia I_1 and I_2 about their respective axes perpendicular to the plane and passing through the centre. They are rotating with angular speeds, ω_1 and ω_2 respectively and are brought into contact face to face with their axes of rotation coaxial. The loss in kinetic energy of the system in the process is given by :

Options

1. $\dfrac{I_1 I_2}{(I_1 + I_2)} (\omega_1 - \omega_2)^2$
2. $\dfrac{(I_1 - I_2)^2 \, \omega_1 \omega_2}{2(I_1 + I_2)}$
3. $\dfrac{I_1 I_2}{2(I_1 + I_2)} (\omega_1 - \omega_2)^2$
4. $\dfrac{(\omega_1 - \omega_2)^2}{2(I_1 + I_2)}$

Ans: $\dfrac{I_1 I_2}{2(I_1 + I_2)} (\omega_1 - \omega_2)^2$

Sol: From conservation of angular momentum we get

$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

$$k_i = \frac{1}{2} I_1\omega_1^2 + \frac{1}{2} I_2\omega_2^2$$

$$k_f = \frac{1}{2}(I_1 + I_2)\omega^2$$

$$k_i - k_f = \frac{1}{2}\left[I_1\omega_1^2 + I_2\omega_2^2 - \frac{(I_1\omega_1 + I_2\omega_2)^2}{I_1 + I_2}\right]$$

Solving above we get

$$k_i - k_f = \frac{1}{2}\left(\frac{I_1 I_2}{I_1 + I_2}\right)(\omega_1 - \omega_2)^2$$

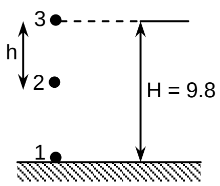
Q.9 Water drops are falling from a nozzle of a shower onto the floor, from a height of 9.8 m. The drops fall at a regular interval of time. When the first drop strikes the floor, at that instant, the third drop begins to fall. Locate the position of second drop from the floor when the first drop strikes the floor.

Options

1. 4.18 m
2. 2.94 m
3. 2.45 m
4. 7.35 m

Ans: 7.35 m

Sol:



$$H = \frac{1}{2}gt^2$$

$$\frac{9.8 \times 2}{9.8} = t^2$$

$$t = \sqrt{2}\ \text{sec}$$

Δt: time interval between drops

$$h = \frac{1}{2}g\left(\sqrt{2} - \Delta t\right)^2$$

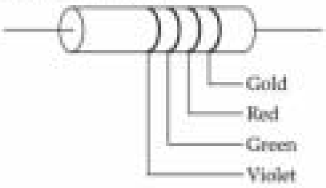
$$0 = \frac{1}{2}g\left(\sqrt{2} - 2\Delta t\right)^2$$

$$\Delta t = \frac{1}{\sqrt{2}}$$

$$h = \frac{1}{2}g\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} \times 9.8 \times \frac{1}{2} = \frac{9.8}{4} = 2.45\ \text{m}$$

H – h = 9.8 – 2.45 = 7.35 m

Q.10 The colour coding on a carbon resistor is shown in the given figure. The resistance value of the given resistor is :



Options
1. $(7500 \pm 375)\ \Omega$
2. $(5700 \pm 285)\ \Omega$
3. $(5700 \pm 375)\ \Omega$
4. $(7500 \pm 750)\ \Omega$

Ans: $(7500 \pm 375)\ \Omega$

Sol: $R = 75 \times 10^2 \pm 5\%$ of 7500

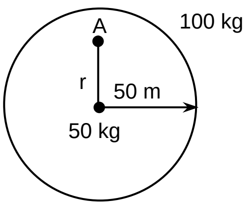
$R = (7500 \pm 375)\ \Omega$

Q.11 A mass of 50 kg is placed at the centre of a uniform spherical shell of mass 100 kg and radius 50 m. If the gravitational potential at a point, 25 m from the centre is V kg/m. The value of V is :

Options
1. $+2\ G$
2. $-60\ G$
3. $-20\ G$
4. $-4\ G$

Ans: –4 G

Sol:



$$V_A = \left[\frac{-GM_1}{r} - \frac{GM_2}{R}\right] = \left[-\frac{50}{25}G - \frac{100}{50}G\right] = -4\ G$$

Q.12 For a transistor α and β are given as $\alpha = \frac{I_C}{I_E}$ and $\beta = \frac{I_C}{I_B}$. Then the correct relation between α and β will be :

Options

1. $\alpha\beta = 1$
2. $\alpha = \frac{\beta}{1-\beta}$
3. $\alpha = \frac{1-\beta}{\beta}$
4. $\beta = \frac{\alpha}{1-\alpha}$

Ans: $\beta = \frac{\alpha}{1-\alpha}$

Sol: $\alpha = \frac{I_C}{I_E}, \beta = \frac{I_C}{I_E}; I_E = I_C + I_B$

$$\alpha = \frac{I_C}{I_C + I_E} = \frac{\frac{I_C}{I_B}}{\frac{I_C}{I_B}+1} = \frac{\beta}{\beta+1} + 1 + \frac{1}{\beta} = \frac{1}{\alpha}$$

$$\frac{1}{\beta} = \frac{1-\alpha}{\alpha}$$

$$\beta = \frac{\alpha}{1-\alpha}$$

Q.13 For full scale deflection of total 50 divisions, 50 mV voltage is required in galvanometer. The resistance of galvanometer if its current sensitivity is 2 div/mA will be :

Options

1. 1 Ω
2. 4 Ω
3. 2 Ω
4. 5 Ω

Ans: 2 Ω

Sol: $I_{max} = \frac{50}{2} = 25mA$

$$R = \frac{V}{I} = \frac{50mV}{25mA} = 2\Omega$$

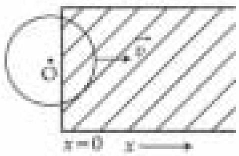
Q.14 The light waves from two coherent sources have same intensity $I_1 = I_2 = I_0$. In interference pattern the intensity of light at minima is zero. What will be the intensity of light at maxima ?

Options
1. I_0
2. $2\ I_0$
3. $4\ I_0$
4. $5\ I_0$

Ans: $4\ I_0$

Sol: $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = 4I_0$

Q.15 A constant magnetic field of 1 T is applied in the $x > 0$ region. A metallic circular ring of radius 1 m is moving with a constant velocity of 1 m/s along the x-axis. At $t = 0$ s, the centre O of the ring is at $x = -1$ m. What will be the value of the induced emf in the ring at $t = 1$ s ? (Assume the velocity of the ring does not change.)



Options
1. 1 V
2. 2π V
3. 2 V
4. 0 V

Ans: 2 V

Sol: emf = BLV = 1.(2R) . 1 = 2 V

Q.16 An antenna is mounted on a 400 m tall building. What will be the wavelength of signal that can be radiated effectively by the transmission tower upto a range of 44 km ?

Options
1. 302 m
2. 75.6 m
3. 605 m
4. 37.8 m

Ans: 605 m

Sol: h : height of antenna
λ : wavelength of signal
$h < \lambda$
$\lambda > h$
$\lambda > 400$ m

Q.17 A player kicks a football with an initial speed of 25 ms^{-1} at an angle of 45° from the ground. What are the maximum height and the time taken by the football to reach at the highest point during motion ? (Take g = 10 ms^{-2})

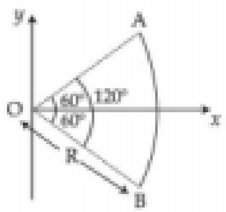
Options
1. $h_{max} = 3.54$ m $T = 0.125$ s
2. $h_{max} = 15.625$ m $T = 3.54$ s
3. $h_{max} = 10$ m $T = 2.5$ s
4. $h_{max} = 15.625$ m $T = 1.77$ s

Ans: $h_{max} = 15.625$ m, T = 1.77 s

Sol: $H = \frac{U^2 \sin^2\theta}{2g} = \frac{(25)^2 (\sin 45)^2}{2 \times 10} = 15.625$ m

$T = \frac{U \sin\theta}{g} = \frac{25 \times \sin 45°}{10} = 2.5 \times 0.7 = 1.77$ s

Q.18 Figure shows a rod AB, which is bent in a 120° circular arc of radius R. A charge (−Q) is uniformly distributed over rod AB. What is the electric field $\vec{E}$ at the centre of curvature O ?



Options
1. $\frac{3\sqrt{3}\,Q}{16\,\pi^2 \varepsilon_0 R^2} (\hat{i})$
2. $\frac{3\sqrt{3}\,Q}{8\,\pi^2 \varepsilon_0 R^2} (-\hat{i})$
3. $\frac{3\sqrt{3}\,Q}{8\,\pi \varepsilon_0 R^2} (\hat{i})$
4. $\frac{3\sqrt{3}\,Q}{8\,\pi^2 \varepsilon_0 R^2} (\hat{i})$

Ans: $\frac{3\sqrt{3}Q}{8\pi^2 \varepsilon_0 R^2} (\hat{i})$

Sol: $\varepsilon = \frac{2k\lambda}{R}\sin\left(\frac{\theta}{2}\right)\left(-\hat{i}\right)$

$$\lambda = \left(\frac{-Q}{R\theta}\right) = \left(\frac{-Q}{R\frac{2\pi}{3}}\right)$$

$$\lambda = \frac{-3Q}{2\pi R}$$

$$\varepsilon = \frac{2k}{R}\frac{-3Q}{2\pi R}\sin(60°)\left(-\hat{i}\right)$$

$$\varepsilon = \frac{3\sqrt{3}Q}{8\pi^2 \in_0 R^2}\left(+\hat{i}\right)$$

Q.19 The height of victoria falls is 63 m. What is the difference in temperature of water at the top and at the bottom of fall ?
[Given 1 cal = 4.2 J and specific heat of water = 1 cal g^{-1} °C^{-1}]

Options
1. 0.147°C
2. 1.476°C
3. 0.014°C
4. 14.76°C

Ans: 0.147°C

Sol: Change in P.E. = heat energy
mgh = mSΔT

$$\Delta T = \frac{gh}{S} = \frac{10\times 63}{4200J/kgC} = 0.147°C$$

Q.20 If force (F), length (L) and time (T) are taken as the fundamental quantities. Then what will be the dimension of density :

Options
1. $[FL^{-5}T^2]$
2. $[FL^{-3}T^3]$
3. $[FL^{-4}T^2]$
4. $[FL^{-3}T^2]$

Ans: $[F L^{-4} T^2]$

Sol: Density = $[F^aL^bT^c]$
$[ML^{-3}] = [M^aL^aT^{-2a}L^bT^c]$
$[M^1L^{-3}] = [M^aL^{a+b}T^{-2a+c}]$
a = 1 ; a + b = -3 ; -2a + c = 0
1 + b = -3   c = 2a
b = -4   c = 2
so, density = $[F^1 L^{-4}T^2]$

Section B

Q.1 X different wavelengths may be observed in the spectrum from a hydrogen sample if the atoms are exited to states with principal quantum number $n = 6$? The value of X is ________.

Given 5
Answer :

Ans: 15

Sol: No. of different wavelengths = $\frac{n(n-1)}{2} = \frac{6\times(6-1)}{2} = \frac{6\times 5}{2} = 15$

Q.2 Two simple harmonic motion, are represented by the equations

$$y_1 = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$$

$$y_2 = 5\left(\sin 3\pi t + \sqrt{3}\cos 3\pi t\right)$$

Ratio of amplitude of y_1 to $y_2 = x : 1$. The value of x is ________.

Given 2
Answer :

Ans: 1

Sol: $y_1 = 10\sin\left(3\pi t + \frac{\pi}{3}\right) \Rightarrow \text{Amplitude} = 10$

$$y_2 = 5\left(\sin 3\pi t + \sqrt{3}\cos 3\pi t\right)$$

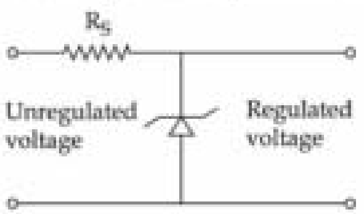
$$y_2 = 10\left(\frac{1}{2}\sin 3\pi t + \frac{\sqrt{3}}{2}\cos 3\pi t\right)$$

$$y_2 = 10\left(\cos\frac{\pi}{3}\sin 3\pi t + \sin\frac{\pi}{3}\cos 3\pi t\right)$$

$$y_2 = 10\sin\left(3\pi t + \frac{\pi}{3}\right) \Rightarrow \text{Amplitude} = 10$$

So ratio of amplitudes = $\frac{10}{10} = 1$

Q.3 A zener diode of power rating 2 W is to be used as a voltage regulator. If the zener diode has a breakdown of 10 V and it has to regulate voltage fluctuated between 6 V and 14 V, the value of R_S for safe operation should be ________ Ω.

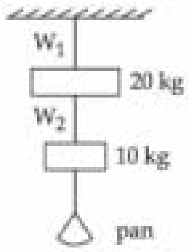


Given --
Answer :

Ans: 20

Sol: When unregulated voltage is 14 V voltage across zener diode must be 10V so potential difference across resistor $\Delta V_{Rs} = 4V$
And $P_{zener} = 2W$
$VI = 2$
$I = \frac{2}{10} = 0.2\,A$
$\Delta V_{Rs} = I\,R_s$
$4 \times 0.2\,R_s \Rightarrow R_s = \frac{40}{2} = 20\,\Omega$

Q.4 Wires W_1 and W_2 are made of same material having the breaking stress of $1.25 \times 10^9\ N/m^2$. W_1 and W_2 have cross-sectional area of $8 \times 10^{-7}\ m^2$ and $4 \times 10^{-7}\ m^2$, respectively. Masses of 20 kg and 10 kg hang from them as shown in the figure. The maximum mass that can be placed in the pan without breaking the wires is _________ kg. (Use $g = 10\ m/s^2$)



Given Answer : **50**

Ans: 50

Sol: $B.S_1 = \frac{T_{1max}}{8\times10^{-7}} \Rightarrow T_{1max} = 8\times1.25\times100 = 1000\ N$

$B.S_2 = \frac{T_{2max}}{4\times10^{-7}} \Rightarrow T_{2max} = 4\times1.25\times100 = 500\ N$

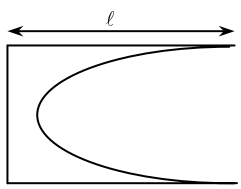
$m = \frac{1000-500}{10} = 50\ kg$

Q.5 A tuning fork is vibrating at 250 Hz. The length of the shortest closed organ pipe that will resonate with the tuning fork will be __________ cm.
(Take speed of sound in air as $340\ ms^{-1}$)

Given Answer : **34**

Ans: **34**

Sol:



$\frac{\lambda}{4} = \ell \Rightarrow \lambda = 4\ell$

$f = \frac{V}{\lambda} = \frac{V}{4\ell}$

$\Rightarrow 250 = \frac{340}{4\ell}$

$$\Rightarrow \ell = \frac{34}{4 \times 25} = 0.34\,\text{m}$$

ℓ = 34 cm

Q.6 A plane electromagnetic wave with frequency of 30 MHz travels in free space. At particular point in space and time, electric field is 6 V/m. The magnetic field at this point will be $x \times 10^{-8}$ T. The value of x is ________.

Given --
Answer :

Ans: 2

Sol: $|B| = \frac{|E|}{C} = \frac{6}{3 \times 10^8} = 2 \times 10^{-8}$ T

$\therefore$ x = 2

Q.7 A heat engine operates between a cold reservoir at temperature $T_2 = 400$ K and a hot reservoir at temperature T_1. It takes 300 J of heat from the hot reservoir and delivers 240 J of heat to the cold reservoir in a cycle. The minimum temperature of the hot reservoir has to be ________ K.

Given --
Answer :

Ans: 500

Sol: Q_{in} = 300 J; Q_{out} = 240 J

Work done = $Q_{in} - Q_{out}$ = 300 – 240 = 60 J

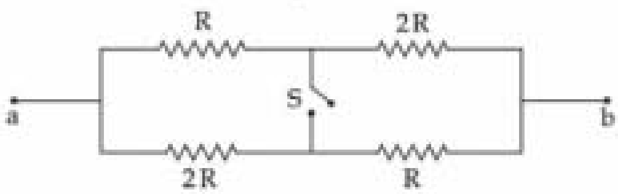
$$\text{Efficiency} = \frac{W}{Q_{in}} = \frac{60}{300} = \frac{1}{5}$$

$$\text{Efficiency} = 1 - \frac{T_2}{T_1}$$

$$\frac{1}{5} = 1 - \frac{400}{T_1} \Rightarrow \frac{400}{T_1} = \frac{4}{5}$$

T_1 = 500 K

Q.8 The ratio of the equivalent resistance of the network (shown in figure) between the points a and b when switch is open and switch is closed is x : 8. The value of x is ________.



Given 8
Answer :

Ans: **9**

Sol: $$R_{\text{eq open}} = \frac{3R}{2}$$

$$R_{\text{eq closed}} = 2 \times \frac{R \times 2R}{3R} = \frac{4R}{3}$$

$$\frac{R_{\text{eq open}}}{R_{\text{eq colsed}}} = \frac{3R}{2} \times \frac{3}{4R} = \frac{9}{8}$$

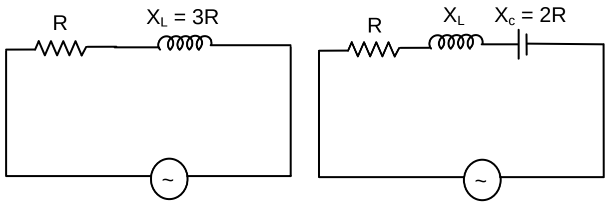
$\therefore$ x = 9

Q.9 An ac circuit has an inductor and a resistor of resistance R in series, such that $X_L = 3R$. Now, a capacitor is added in series such that $X_C = 2R$. The ratio of new power factor with the old power factor of the circuit is $\sqrt{5} : x$. The value of x is _________.

Given --
Answer :

Ans: 1

Sol:



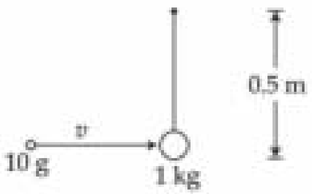
$$\cos\phi = \frac{R}{\sqrt{R^2 + 3R^2}} \qquad \cos\phi' = \frac{R}{\sqrt{R^2 + R^2}}$$

$$= \frac{1}{\sqrt{10}} \qquad = \frac{1}{\sqrt{2}}$$

$$\frac{\cos\phi}{\cos\phi'} = \frac{\sqrt{10}}{\sqrt{2}} = \frac{\sqrt{5}}{1}$$

$\therefore x = 1$

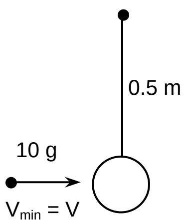
Q.10 A bullet of 10 g, moving with velocity v, collides head-on with the stationary bob of a pendulum and recoils with velocity 100 m/s. The length of the pendulum is 0.5 m and mass of the bob is 1 kg. The minimum value of $v =$ _________ m/s so that the pendulum describes a circle.
(Assume the string to be inextensible and $g = 10\ m/s^2$)



Given --
Answer :

Ans: 400

Sol:



$$V' = \sqrt{5gR} = \sqrt{5 \times 10 \times 0.5}$$

V' = 5 m/s

$m_1 V = m_2 \times 5 - m_1 \times 100$

$$\frac{10}{1000} \times V = 5 - \frac{10}{1000} \times 100$$

V = 400 m/s

PART – B – CHEMISTRY

Section A

Q.1 The correct order of ionic radii for the ions, P^{3-}, S^{2-}, Ca^{2+}, K^{+}, Cl^{-} is :

Options

1. $Cl^{-} > S^{2-} > P^{3-} > Ca^{2+} > K^{+}$
2. $K^{+} > Ca^{2+} > P^{3-} > S^{2-} > Cl^{-}$
3. $P^{3-} > S^{2-} > Cl^{-} > K^{+} > Ca^{2+}$
4. $P^{3-} > S^{2-} > Cl^{-} > Ca^{2+} > K^{+}$

Ans: $P^{3-} > S^{2-} > Cl^{-} > K^{+} > Ca^{2+}$

Sol: All the given species are isoelectronic. Among isoelectronic species, as atomic number increases, ionic radii decreases.

Q.2 Which one of the following reactions will **not** yield propionic acid ?

Options

1. $CH_3CH_2CH_2Br + Mg, CO_2$ dry ether$/H_3O^+$
2. $CH_3CH_2CCl_3 + OH^-/H_3O^+$
3. $CH_3CH_2COCH_3 + OI^-/H_3O^+$
4. $CH_3CH_2CH_3 + KMnO_4(\text{Heat}), OH^-/H_3O^+$

Ans: $CH_3CH_2CH_2Br$ + Mg, CO_2 dry ether / H_3O^+

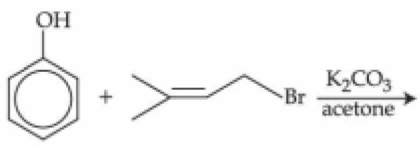
Sol:

$$CH_3-CH_2-CH_2-Br \xrightarrow[\text{Dry ether}]{Mg} CH_3-CH_2-CH_2MgBr \xrightarrow{CO_2} CH_3-CH_2-CH_2-\overset{O}{\overset{\|}{C}}-MgBr$$

$$\xrightarrow{H_3O^+} CH_3-CH_2-CH_2-\overset{O}{\overset{\|}{C}}-OH$$

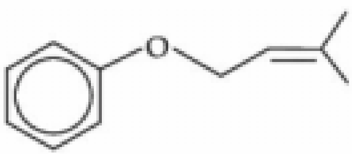
Butanoic acid

Q.3 The major product of the following reaction, if it occurs by S_N2 mechanism is :



Options

1.



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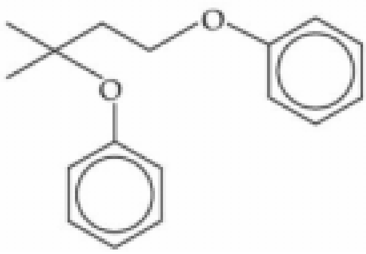
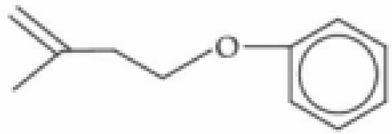
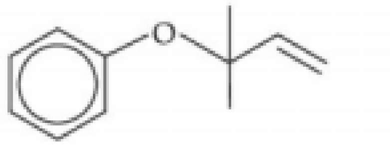
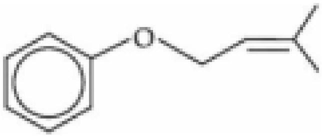
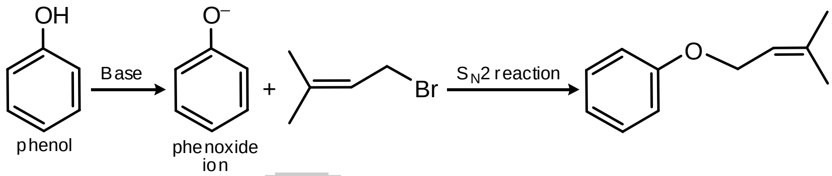


3. 

4. 

Ans: 

Sol: 

Q.4 Which one of the following tests used for the identification of functional groups in organic compounds does not use copper reagent ?

Options
1. Seliwanoff's test
2. Biuret test for peptide bond
3. Benedict's test
4. Barfoed's test

Ans: Seliwanoff's test

Sol: Biuret reagent → Sodium hydroxide + hydrated copper (II) sulphate
Benedict reagent → Sodium carbonate + sodium citrate + copper (II) sulphate pentahydrate
Barfoed's reagent → Copper (II) acetate in a dilute solution of acetic acid
Seliwanoff's reagent → Resorcinol + conc.HCl (No copper present)

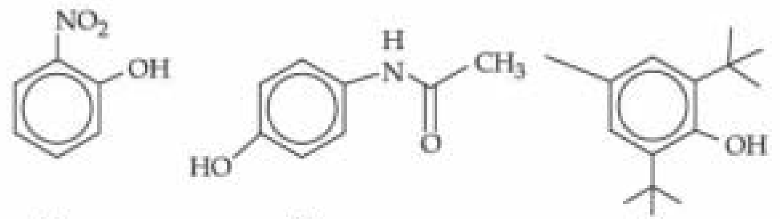
Q.5 Lyophilic sols are more stable than lyophobic sols because,

Options
1. the colloidal particles have no charge.
2. the colloidal particles have positive charge.
3. there is a strong electrostatic repulsion between the negatively charged colloidal particles.
4. the colloidal particles are solvated.

Ans: the colloidal particles are solvated

Sol: In the lyophilic colloids, the colloidal particles are extensively solvated.

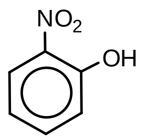
Q.6 The compound/s which will show significant intermolecular H-bonding is/are :



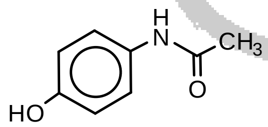
(a) (b) (c)

Options
1. (b) only
2. (c) only
3. (a), (b) and (c)
4. (a) and (b) only

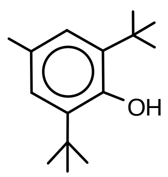
Ans: (b) only

Sol: 

shows intramolecular H-bonding



shows intermolecular H-bonding



It do not show intermolecular H-bonding due to steric hindrance

Q.7 Choose the **correct** statement from the following :

Options
1. Among the alkali metal halides, LiF is least soluble in water.
2. The low solubility of CsI in water is due to its high lattice enthalpy.
3. LiF has least negative standard enthalpy of formation among alkali metal fluorides.
4. The standard enthalpy of formation for alkali metal bromides becomes less negative on descending the group.

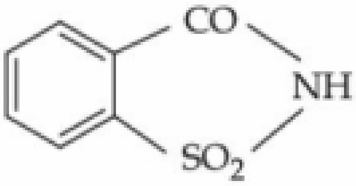
Ans: Among the alkali metal halides, LiF is least soluble in water

Sol:
* For alkali metal fluoride the solubility in water increases from Lithium to Caesium. LiF is least soluble in water.
* In CsI, lattice energy is less, but Cs^+ is having less hydration enthalpy due to which it is less soluble in water.
* Standard enthalpy of formation for LiF is most negative among alkali metal fluorides.

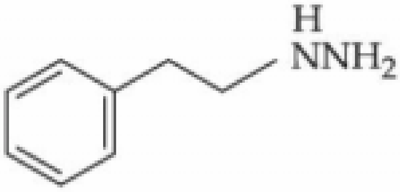
* Standard enthalpy formation for alkali metal bromide becomes more negative on descending the group

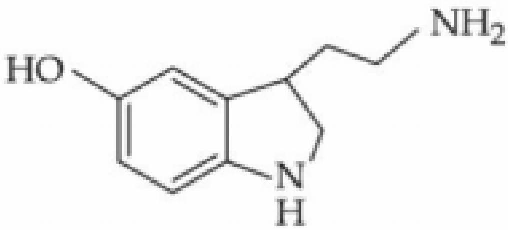
Q.8 Which one of the following chemicals is responsible for the production of HCl in the stomach leading to irritation and pain ?

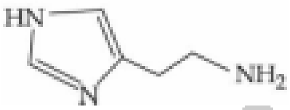
Options

1. 

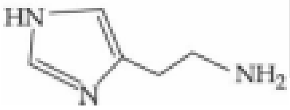
2. 

3. 

4. 

Ans: 

Sol: Histamine stimulates secretion of HCl in stomach leading to irritation and pain. Structure of Histamine is



Q.9 The addition of dilute NaOH to Cr^{3+} salt solution will give :

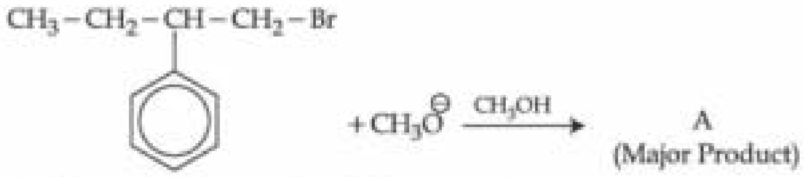
Options

1. precipitate of $Cr_2O_3(H_2O)_n$
2. precipitate of $[Cr(OH)_6]^{3-}$
3. precipitate of $Cr(OH)_3$
4. a solution of $[Cr(OH)_4]^-$

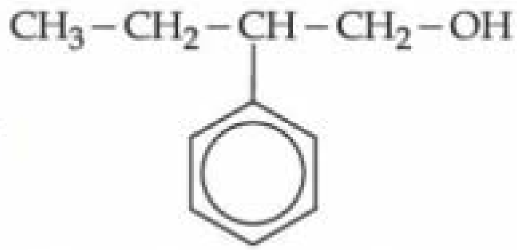
Ans: precipitate of $Cr_2O_3(H_2O)_n$

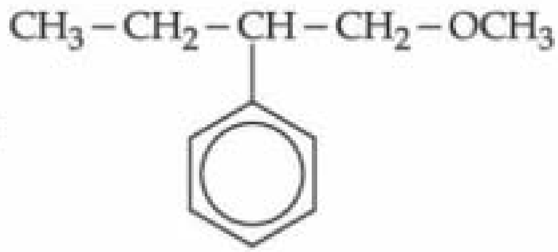
Sol: $Cr^{3+} + \text{dil.NaOH} \longrightarrow \underset{\text{precipitate}}{Cr_2O_3.(H_2O)_n}$

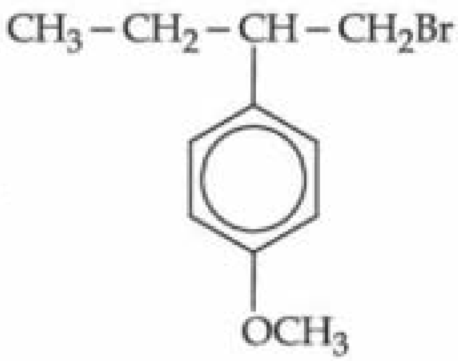
Q.10 The major product (A) formed in the reaction given below is :

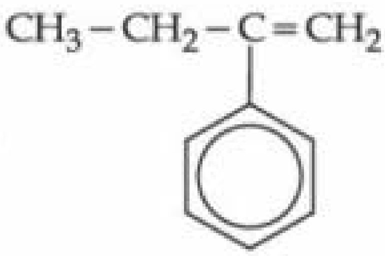


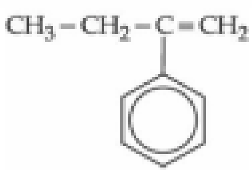
Options

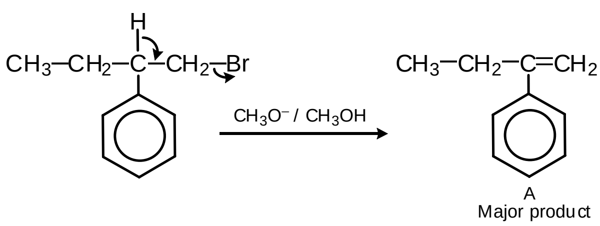
1. 

2. 

3. 

4. 

Ans: 

Sol: 

Q.11 Which one of the following is formed (mainly) when red phosphorus is heated in a sealed tube at 803 K ?

Options
1. Yellow phosphorus
2. α-Black phosphorus
3. White phosphorus
4. β-Black phosphorus

Ans: α-Black phosphorus

Sol: When red phosphorus is heated in a sealed tube at 803 K, α-black phosphorus is formed

Q.12 Potassium permanganate on heating at 513 K gives a product which is :

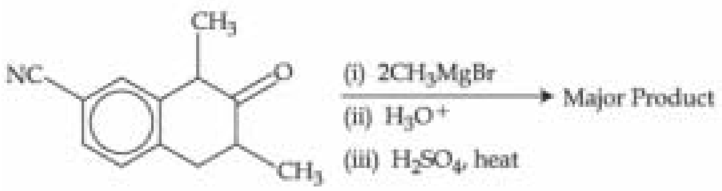
Options
1. diamagnetic and colourless
2. diamagnetic and green
3. paramagnetic and green
4. paramagnetic and colourless

Ans: paramagnetic and green

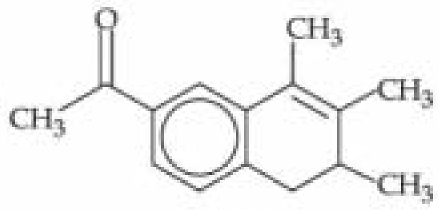
Sol: $2KMnO_4 \xrightarrow[513K]{\Delta} \underset{Green}{K_2MnO_4} + \underset{Black}{MnO_2} + O_2$

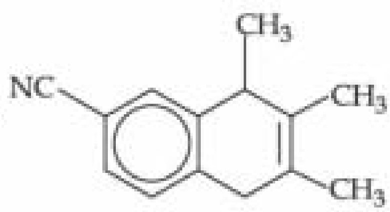
K_2MnO_4 has one unpaired electron hence it is paramagnetic

Q.13 Which one of the following is the major product of the given reaction ?



Options

1. 

2. 

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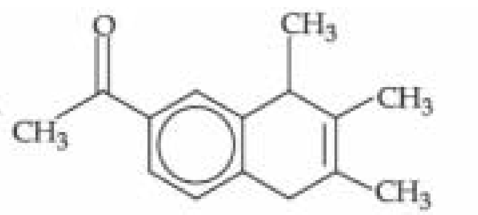

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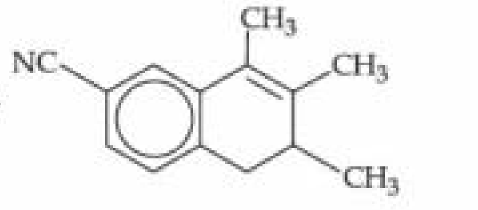
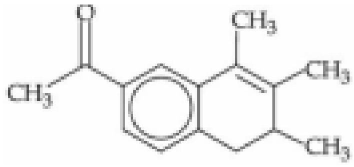
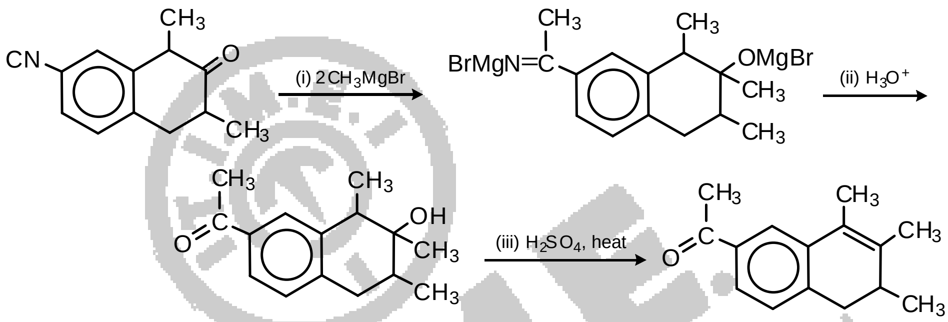


Ans: 

Sol:



Q.14 Match **List - I** with **List - II** :

	List - I (Name of ore/mineral)		List - II (Chemical formula)
(a)	Calamine	(i)	ZnS
(b)	Malachite	(ii)	$FeCO_3$
(c)	Siderite	(iii)	$ZnCO_3$
(d)	Sphalerite	(iv)	$CuCO_3 \cdot Cu(OH)_2$

Choose the **most appropriate** answer from the options given below :

Options
1. (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)
2. (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)
3. (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)
4. (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)

Ans: (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

Sol:
(a) Calamine − $ZnCO_3$
(b) Malachite − $CuCO_3.Cu(OH)_2$
(c) Siderite − $FeCO_3$
(d) Sphalerite − ZnS

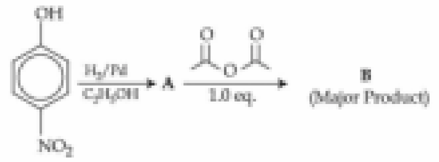
Q.15 The oxide that gives H_2O_2 most readily on treatment with H_2O is :

Options
1. SnO_2
2. Na_2O_2
3. PbO_2
4. $BaO_2 \cdot 8H_2O$

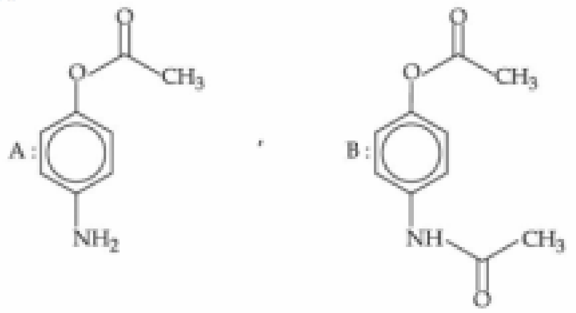
Ans: Na_2O_2

Sol: 1. $SnO_2 + 2H_2O \rightarrow Sn(OH)_4$
2. $Na_2O_2 + 2H_2O \rightarrow 2NaOH + H_2O_2$
3. $PbO_2 + 2H_2O \rightarrow Pb(OH)_4$
4. Acidified $BaO_2.8H_2O$ gives H_2O_2 only after evaporation

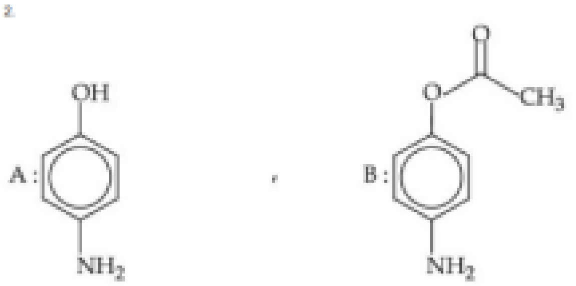
Q.16 The correct structures of **A** and **B** formed in the following reactions are :



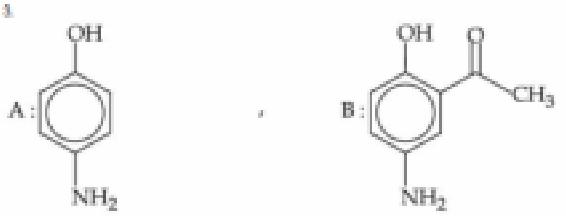
Options 1.



2.



3.



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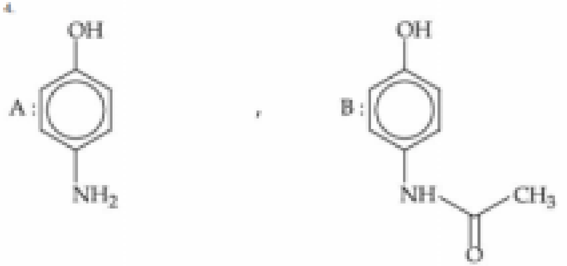
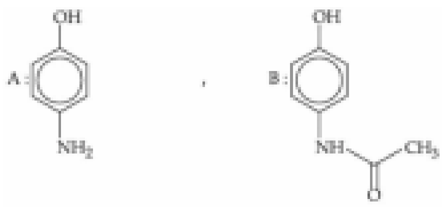
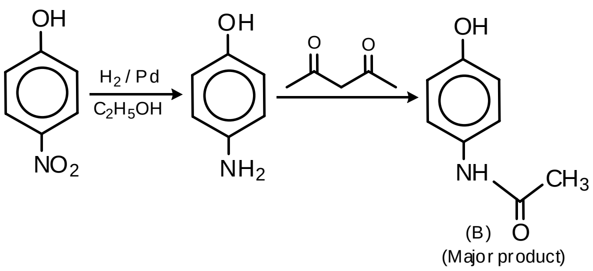


Ans: 

Sol: 

Q.17 In stratosphere most of the ozone formation is assisted by :

Options
1. ultraviolet radiation.
2. visible radiations.
3. γ-rays.
4. cosmic rays.

Ans: ultraviolet radiation

Sol: $O_{2(g)} \xrightarrow{uv} O_{(g)} + O_{(g)}$

$O_{(g)} + O_{2(g)} \overset{uv}{\rightleftharpoons} O_{3(g)}$

Q.18 Hydrolysis of sucrose gives :

Options
1. α-D-(+)-Glucose and α-D-(−)-Fructose
2. α-D-(−)-Glucose and α-D-(+)-Fructose
3. α-D-(−)-Glucose and β-D-(−)-Fructose
4. α-D-(+)-Glucose and β-D-(−)-Fructose

Ans: α-D-(+)-Glucose and β-D-(−)-Fructose

Sol: Hydrolysis of sucrose gives α-D-(+)-Glucose and β-D-(−)-Fructose

Q.19 Given below are two statements :

Statement I : Ethyl pent-4-yn-oate on reaction with CH_3MgBr gives a 3°-alcohol.

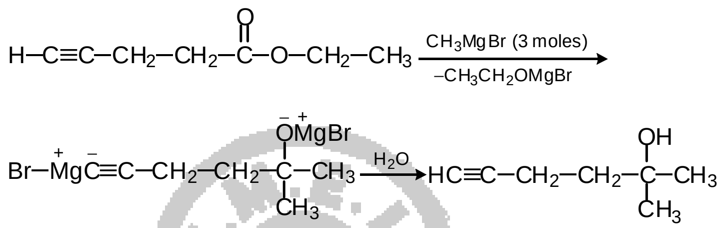
Statement II : In this reaction one mole of ethyl pent-4-yn-oate utilizes two moles of CH_3MgBr.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

Options
1. Both **Statement I** and **Statement II** are true
2. **Statement I** is true but **Statement II** is false
3. **Statement I** is false but **Statement II** is true
4. Both **Statement I** and **Statement II** are false

Ans: Statement I is true but statement II is false

Sol:



It consumes 3 moles of Grignard reagent so statement 2 is false

Q.20 Which one of the following is used to remove most of plutonium from spent nuclear fuel ?

Options
1. I_2O_5
2. O_2F_2
3. ClF_3
4. BrO_3

Ans: O_2F_2

Sol: O_2F_2 oxidises plutonium to PuF_6 and the reaction is used in removing plutonium as PuF_6 from spent nuclear fuel.

Section B

Q.1 The first order rate constant for the decomposition of $CaCO_3$ at 700 K is $6.36 \times 10^{-3}\ s^{-1}$ and activation energy is 209 kJ mol^{-1}. Its rate constant (in s^{-1}) at 600 K is $x \times 10^{-6}$. The value of x is ________. (Nearest integer)

[Given $R = 8.31\ J\ K^{-1}\ mol^{-1}$; $\log 6.36 \times 10^{-3} = -2.19$, $10^{-4.79} = 1.62 \times 10^{-5}$]

Given --
Answer :

Ans: 16

Sol: $k_2 = 6.36 \times 10^{-3}\ s^{-1}$

$E_a = 209\ kJ\ mol^{-1}$

$$\log \frac{k_2}{k_1} = \frac{E_a}{2.303R}\left[\frac{1}{T_2} - \frac{1}{T_1}\right]$$

$$\log\frac{k_2}{k_1}=\frac{E_a}{2.303R}\left[\frac{1}{700}-\frac{1}{600}\right]$$

$$\log\frac{6.36\times10^{-3}}{k_1}=\frac{209\times10^3}{2.303\times8.31}\times\frac{700-600}{700\times600}=\frac{209\times10^3}{2.303\times8.31}\times\frac{100}{700\times600}$$

$\log(6.36\times10^{-3})-\log k_1=2.6$

$\log k_2=\log(6.36\times10^{-3})-2.6=-2.19-2.6=-4.79$

$k_2=\text{Antilog}(-4.79)=1.62\times10^{-5}=16.2\times10^{-6}$

$\therefore x=16$

Q.2 The number of species having non-pyramidal shape among the following is ________.

(A) SO_3

(B) NO_3^-

(C) PCl_3

(D) CO_3^{2-}

Given Answer : **1**

Ans: 3

Sol: SO_3 – Trigonal planar

NO_3^- – Trigonal planar

PCl_3 – Pyramidal

CO_3^{2-} – Trigonal planar

Q.3 Data given for the following reaction is as follows :

$$FeO_{(s)}+C_{(graphite)}\longrightarrow Fe_{(s)}+CO_{(g)}$$

Substance	Δ_fH° (kJ mol^{-1})	ΔS° (J mol^{-1} K^{-1})
$FeO_{(s)}$	–266.3	57.49
$C_{(graphite)}$	0	5.74
$Fe_{(s)}$	0	27.28
$CO_{(g)}$	–110.5	197.6

The minimum temperature in K at which the reaction becomes spontaneous is ________. (Integer answer)

Given Answer : **1**

Ans: 964

Sol: $$T_{eqbm}=\frac{\Delta_rH^\circ}{\Delta_rS^\circ}$$

$\Delta_rH^\circ=\left[\Delta_fH^\circ_{(Fe)}+\Delta_fH^\circ_{(CO)}\right]-\left[\Delta_fH^\circ_{(FeO)}+\Delta_fH^\circ_{(graphite)}\right]$

$=[0-110.5]-[-266.3+0]$

$=155.8\text{ kJ mol}^{-1}$

$\Delta_rS^\circ=\left[\Delta S^\circ_{(Fe)}+\Delta S^\circ_{(CO)}\right]-\left[\Delta S^\circ_{(FeO)}+\Delta S^\circ_{(graphite)}\right]$

$=[27.28+197.6]-[57.49+5.74]=161.65\text{ J mol K}^{-1}$

$$T_{eqbm}=\frac{155.8\times10^3}{161.65}=963.8\text{K}\approx964$$

Since Δ_rH° and Δ_rS° are positive, the reaction is spontaneous if T > 963.8 K

Therefore the minimum temp at which the reaction becomes spontaneous is 964 K

Q.4 The number of photons emitted by a monochromatic (single frequency) infrared range finder of power 1 mW and wavelength of 1000 nm, in 0.1 second is $x \times 10^{13}$. The value of x is ________. (Nearest integer)
$(h = 6.63 \times 10^{-34}$ Js, $c = 3.00 \times 10^{8}$ ms$^{-1})$

Given --
Answer :

Ans: 50

Sol: E = nhν

$E = \frac{nhc}{\lambda}$ $\qquad P = 1mW = 1 \times 10^{-3}$ J/s

For 1s → E = 10^{-3} J

∴0.1 s → E = 10^{-4} J

$$n = \frac{E\lambda}{hc} = \frac{1 \times 10^{-4} \times 1000 \times 10^{-9}}{6.63 \times 10^{-34} \times 3 \times 10^{8}} = \frac{10^{-10}}{19.89 \times 10^{-26}}$$

$= 0.050 \times 10^{16}$

$= 50 \times 10^{13}$

∴ x = 50

Q.5 When 5.1 g of solid NH_4HS is introduced into a two litre evacuated flask at 27°C, 20% of the solid decomposes into gaseous ammonia and hydrogen sulphide. The K_p for the reaction at 27°C is $x \times 10^{-2}$. The value of x is ________. (Integer answer)
[Given R = 0.082 L atm K^{-1} mol^{-1}]

Given --
Answer :

Ans: 6

Sol: Molecular mass of NH_4HS = 14 + 5 + 32 = 51

Moles of NH_4HS initially taken $= \frac{5.1}{51} = 0.1$mol

Volume of vessel = 2L

	NH_4HS	⇌	NH_3	+	H_2S	
At t = 0	0.1 mol					
At t = ∞			0.1×0.2		0.1×0.2	[Since % decomposition = 20%]

Partial pressure of component, $p = \frac{nRT}{V}$

$$= \frac{0.1 \times 0.2 \times 0.082 \times 300}{2} = 0.246 \text{ atm}$$

$$K_p = p_{NH_3} . p_{H_2S} = (0.246) \times (0.246) = (0.246)^2 = 0.0605$$

$= 6.05 \times 10^{-2}$

x = 6

Q.6 100 g of propane is completely reacted with 1000 g of oxygen. The mole fraction of carbon dioxide in the resulting mixture is $x \times 10^{-2}$. The value of x is ________.
(Nearest integer)
[Atomic weight : H = 1.008; C = 12.00; O = 16.00]

Given 2
Answer :

Ans: 19

Sol: $C_3H_{8(g)} + 5O_{2(g)} \longrightarrow 3CO_{2(g)} + 4H_2O_{(\ell)}$

$n_{C_3H_8} = \frac{100}{44} = 2.25$

$$n_{O_2} = \frac{1000}{32} = 31.25$$

	C_3H_8	+	$5O_2$	$\longrightarrow$	$3CO_2$	$+ 4H_2O$
No. of moles initially	2.27 mol		31.25 mol		–	–
No. of moles left after completion	0		(31.27-5×2.37) =19.9		(3×2.37) = 6.81	(4×2.37) = 9.08

$$\chi_{CO_2} = \frac{6.81}{19.9 + 6.81 + 9.08}$$
$$= 0.1902 = 19.02 \times 10^{-2}$$

Q.7 40 g of glucose (Molar mass = 180) is mixed with 200 mL of water. The freezing point of solution is __________ K. (Nearest integer)

[Given : $K_f = 1.86$ K kg mol^{-1}; Density of water = 1.00 g cm^{-3} ; Freezing point of water = 273.15 K]

Given --
Answer :

Ans: 271

Sol: $\Delta T_f = K_f . m$

$$= K_f . \frac{1000\, W_B}{M_B . W_A}$$

$$W_A = V \times d = 200 \times 1 = 200\,g$$

$$\Delta T_f = \frac{1.86 \times 1000 \times 40}{180 \times 200} = 2.066\,K$$

$$\Delta T_f = T_f^{\circ} - T_f$$

$$T_f = T_f^{\circ} - \Delta T_f = 273.15 - 2.066 = 271.084 \approx 271K$$

Q.8 The resistance of a conductivity cell with cell constant 1.14 cm^{-1}, containing 0.001 M KCl at 298 K is 1500 Ω . The molar conductivity of 0.001 M KCl solution at 298 K in S cm^2 mol^{-1} is ___________. (Integer answer)

Given --
Answer :

Ans: 760

Sol: $\Lambda_m = \frac{1000\,\kappa}{M}$ $\quad \kappa = \frac{1}{R} \times \frac{\ell}{A} = \frac{1}{1500} \times 1.14 = 0.00076$

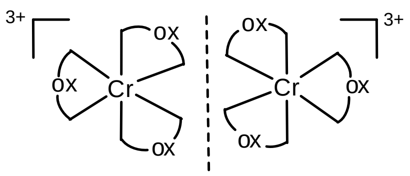
$$\Lambda_m = \frac{1000 \times 0.00076}{0.001} = \frac{0.76}{0.001} = \frac{0.76}{10^{-3}} = 0.76 \times 10^3 = 760\ \text{S cm}^2\ \text{mol}^{-1}$$

Q.9 The number of optical isomers possible for $[Cr(C_2O_4)_3]^{3-}$ is __________.

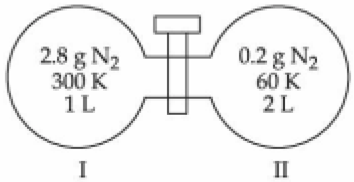
Given 0
Answer :

Ans: 2

Sol:



Q.10 Two flasks I and II shown below are connected by a valve of negligible volume.



When the valve is opened, the final pressure of the system in bar is $x \times 10^{-2}$. The value of x is __________. (Integer answer)
[Assume - Ideal gas; 1 bar $= 10^5$ Pa; Molar mass of $N_2 = 28.0$ g mol^{-1}; $R = 8.31$ J mol^{-1} K^{-1}]

Given **8**
Answer :

Ans: 84

Sol: Let the final temperature of the system be 'T'
Applying thermal equilibrium
$n_1 C\Delta T_1 = n_2 C\Delta T_2$
$\frac{2.8}{28} \times (300 - T) = \frac{0.2}{28} \times (T - 60)$
Solving, T = 284 K
Final pressure $P = \frac{nRT}{V} = \frac{3.0}{28} \times \frac{8.31 \times 284}{3 \times 10^{-3}}$
$= 84287$ Pa
$= 84.28 \times 10^{-2}$ bar
$\therefore$ x = 84

PART – C – MATHEMATICS

Section A

Q.1 The angle between the straight lines, whose direction cosines are given by the equations $2l + 2m - n = 0$ and $mn + nl + lm = 0$, is :

Options

1. $\frac{\pi}{3}$
2. $\cos^{-1}\left(\frac{8}{9}\right)$
3. $\frac{\pi}{2}$
4. $\pi - \cos^{-1}\left(\frac{4}{9}\right)$

Ans: $\frac{\pi}{2}$

Sol: $n = 2(l + m)$ ---- (1)
$lm + n(l + m) = 0$ ---- (2)

sub (1) in (2)

$lm + 2(l+m)^2 = 0$

$2l^2 + 2m^2 + 5ml = 0$

$\div$ by m^2

$2\left(\frac{l}{m}\right)^2 + 2 + 5\left(\frac{l}{m}\right) = 0$

Put $t = \frac{l}{m}$

$2t^2 + 5t + 2 = 0$

Solving

$\Rightarrow t = -2; -\frac{1}{2}$

When $\frac{l}{m} = -2$

$\frac{n}{m} = -2$

Dr's = $(-2m, m, -2m)$

$(-2, 1, -2)$

when $\frac{l}{m} = -\frac{1}{2}$

$n = -2l$

$(l, -2l, -2l)$

$(1, -2, -2)$

$\therefore$ required angle, $\cos\theta = \frac{-2-2+4}{\sqrt{9}} = 0 \Rightarrow \frac{\pi}{2}$

Q.2 Let M and m respectively be the maximum and minimum values of the function $f(x) = \tan^{-1}(\sin x + \cos x)$ in $\left[0, \frac{\pi}{2}\right]$. Then the value of $\tan(M - m)$ is equal to :

Options
1. $2 + \sqrt{3}$
2. $2 - \sqrt{3}$
3. $3 - 2\sqrt{2}$
4. $3 + 2\sqrt{2}$

Ans: $3 - 2\sqrt{2}$

Sol: sinx + cosx = $\sqrt{2}\sin x\left(x + \frac{\pi}{4}\right)$

$\Rightarrow \sin x + \cos x \in \left[1, \sqrt{2}\right]$ for $x \in \left[0, \frac{\pi}{2}\right]$

$f(x) = \tan^{-1}(\sin x + \cos x) \in \left[\frac{\pi}{4}, \tan^{-1}\sqrt{2}\right]$

$\tan\left(\tan^{-1}\sqrt{2} - \frac{\pi}{4}\right) = \frac{\sqrt{2}-1}{1+\sqrt{2}} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = 3 - 2\sqrt{2}$

Q.3 Let $A = \begin{pmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{pmatrix}$, where [t] denotes the greatest integer less than or equal to t. If $\det(A) = 192$, then the set of values of x is the interval :

Options
1. $[68, 69)$
2. $[65, 66)$
3. $[62, 63)$
4. $[60, 61)$

Ans: $[62, 63)$

Sol: $\begin{vmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{vmatrix} = 192$

$R_1 \to R_1 - R_2 \,\&\, R_2 \to R_2 - R_3$

$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ [x] & [x]+2 & [x]+4 \end{bmatrix} = 192$

$2[x] + 6 + [x] = 192 \Rightarrow [x] = 62$

Q.4 Let [λ] be the greatest integer less than or equal to λ. The set of all values of λ for which the system of linear equations $x + y + z = 4$, $3x + 2y + 5z = 3$, $9x + 4y + (28 + [\lambda])z = [\lambda]$ has a solution is :

Options
1. $\mathbf{R}$
2. $(-\infty, -9) \cup [-8, \infty)$
3. $(-\infty, -9) \cup (-9, \infty)$
4. $[-9, -8)$

Ans: R

Sol: $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & 5 \\ 9 & 4 & 28+[\lambda] \end{vmatrix} = -24 - [\lambda] + 15 = -[\lambda] - 9 \neq 0$

If $[\lambda] + 9 \neq 0$ then unique solution

If $[\lambda] + 9 = 0$ then $\Delta_1 = \Delta_2 = \Delta_3 = 0$

$\therefore \lambda$ can be any real number

Q.5 A differential equation representing the family of parabolas with axis parallel to y-axis and whose length of latus rectum is the distance of the point $(2, -3)$ form the line $3x+4y=5$, is given by :

Options

1. $10\dfrac{d^2y}{dx^2}=11$

2. $11\dfrac{d^2y}{dx^2}=10$

3. $10\dfrac{d^2x}{dy^2}=11$

4. $11\dfrac{d^2x}{dy^2}=10$

Ans: $11\dfrac{d^2y}{dx^2}=10$

Sol: Latus rectum $=\dfrac{|3(2)+4(-3)-5|}{5}=\dfrac{11}{5}$

$(x-h)^2=\dfrac{11}{5}(y-k)$

Differentiate with respect to x

$2(x-h)=\dfrac{11}{5}\dfrac{dy}{dx}$

Differentiating again,

$2-\dfrac{11}{5}\dfrac{d^2y}{dx^2}$

$11\dfrac{d^2y}{dx^2}=10$

Q.6 Each of the persons A and B independently tosses three fair coins. The probability that both of them get the same number of heads is :

Options

1. 1

2. $\dfrac{1}{8}$

3. $\dfrac{5}{16}$

4. $\dfrac{5}{8}$

Ans: $\frac{5}{16}$

Sol:

No. of heads	0	1	2	3
P (same)	$\frac{1}{8}\times\frac{1}{8}=\frac{1}{64}$	$\frac{3}{8}\times\frac{3}{8}=\frac{9}{64}$	$\frac{3}{8}\times\frac{3}{8}=\frac{9}{64}$	$\frac{1}{8}\times\frac{1}{8}=\frac{1}{64}$

Total probability $=\frac{1}{64}+\frac{9}{64}+\frac{9}{64}+\frac{1}{64}=\frac{5}{16}$

Q.7 If the solution curve of the differential equation $(2x-10y^3)dy+ydx=0$, passes through the points $(0, 1)$ and $(2, \beta)$, then β is a root of the equation :

Options

1. $2y^5-y^2-2=0$
2. $y^5-2y-2=0$
3. $y^5-y^2-1=0$
4. $2y^5-2y-1=0$

Ans: $y^5-y^2-1=0$

Sol: $(2x-10y^3)dy+ydx=0$

Rearranging, we get

$\Rightarrow \frac{dx}{dy}+\left(\frac{2}{y}\right)x=10y^2$

$I.F=e^{\int\frac{2}{y}dy=e^{2\ln(y)}=y^2}$

Solution of D.E is

$\therefore x.y=\int(10y^2)y^2.dy$

$xy^2=\frac{10y^5}{5}+C\Rightarrow xy^2=2y^5+C$

Since it passes through $(0,1)$; $0=2+C\Rightarrow C=-2$

$\therefore$ Curve is $xy^2=2y^5-2$

Given it passes through $(2,\beta)$

$2\beta^2=2\beta^5-2\Rightarrow\beta^5-\beta^2-1=0$

$\therefore\beta$ is root of an equation $y^5-y^2-1=0$

Q.8 If $\lim_{x\to\infty}\left(\sqrt{x^2-x+1}-ax\right)=b$, then the ordered pair (a, b) is :

Options

1. $\left(-1,\ \frac{1}{2}\right)$
2. $\left(1,\ \frac{1}{2}\right)$
3. $\left(-1,\ -\frac{1}{2}\right)$
4. $\left(1,\ -\frac{1}{2}\right)$

Ans: $\left(1,-\frac{1}{2}\right)$

Sol: $\lim_{x\to\infty}\left(\sqrt{x^2-x+1}\right)-ax=b \qquad (\infty,-\infty)$

$\Rightarrow a>0$

$\Rightarrow \lim_{x\to\infty}\frac{\left(x^2-x+1-a^2x^2\right)}{\sqrt{x^2-x+1}+ax}=b$

$\lim_{x\to\infty}\frac{\left(1-a^2\right)x^2-x+1}{\sqrt{x^2-x+1}+ax}=b$

$\lim_{x\to\infty}\frac{\left(1-a^2\right)x^2-x+1}{x\left(\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a\right)}=b$

$\Rightarrow 1-a^2=0\Rightarrow a=1$

Also, $\lim_{x\to\infty}\frac{-x+1}{x\left(\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a\right)}=b$

$\Rightarrow \frac{-1}{1+a}=b\Rightarrow b=-\frac{1}{2}$

$(a,b)=\left(1,-\frac{1}{2}\right)$

Q.9 The equation of the plane passing through the line of intersection of the planes $\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=1$ and $\vec{r}\cdot\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0$ and parallel to the x-axis is :

Options

1. $\vec{\mathbf{r}}\cdot\left(\hat{j}-3\hat{k}\right)-6=0$
2. $\vec{\mathbf{r}}\cdot\left(\hat{i}+3\hat{k}\right)+6=0$
3. $\vec{\mathbf{r}}\cdot\left(\hat{i}-3\hat{k}\right)+6=0$
4. $\vec{\mathbf{r}}\cdot\left(\hat{j}-3\hat{k}\right)+6=0$

Ans: $\vec{r}.(\hat{j} - 3\hat{k}) + 6 = 0$

Sol: Equation of planes through line of intersection of these planes is:-
(x + y + z – 1) + λ (2x + 3y – z + 4) = 0
$\Rightarrow (1 + 2\lambda)\,x + (1 + 3\lambda)y + (1 - \lambda)z - 1 + 4\lambda = 0$
Since parallel to x–axis whose direction ratios are (1, 0, 0)
$\therefore (1 + 2\lambda)1 + (1 + 3\lambda)0 + (1 - \lambda)0 = 0$

$\lambda = -\frac{1}{2}$

$\therefore$ Required plane is $0x + \left(1 - \frac{3}{2}\right)y + \left(1 + \frac{1}{2}\right)z - 1 + 4\left(-\frac{1}{2}\right) = 0$

$\Rightarrow -\frac{y}{2} + \frac{3}{2}z - 3 = 0$

$\Rightarrow \vec{r}.(\hat{j} - 3\hat{k}) + 6 = 0$

Q.10 The area of the region bounded by the parabola $(y-2)^2 = (x-1)$, the tangent to it at the point whose ordinate is 3 and the x-axis is :

Options
1. 10
2. 4
3. 6
4. 9

Ans: 9

Sol: When $y = 3 \Rightarrow x = 2$
$\therefore$ Point is (2, 3)
Differentiating $(y - 2)^2 = x - 1$ with respect to x
$2(y - 2)y' = 1$

$\Rightarrow y' = \frac{1}{2(y-2)}$

At (2, 3), $y' = \frac{1}{2}$

$\Rightarrow \frac{y-3}{x-2} = \frac{1}{2} \Rightarrow x - 2y + 4 = 0$

Area = $\int_0^3 \left((y-2)^2 + 1 - (2y - 4)\right)dy$ = 9 sq. units

Q.11 If $y(x) = \cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right), x \in \left(\frac{\pi}{2}, \pi\right)$, then $\frac{dy}{dx}$ at $x = \frac{5\pi}{6}$ is :

Options
1. $-\frac{1}{2}$
2. $\frac{1}{2}$
3. -1
4. 0

Ans: $-\frac{1}{2}$

Sol: $y(x)=\cot^{-1}\left[\frac{\cos\frac{x}{2}+\sin\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}-\sin\frac{x}{2}+\cos\frac{x}{2}}\right]$

$y(x)=\cot^{-1}\left(\tan\frac{x}{2}\right)=\frac{\pi}{2}-\frac{x}{2}$

$y'(x)=-\frac{1}{2}$

Q.12 If $0 < x < 1$ and $y = \frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \ldots$, then the value of e^{1+y} at $x = \frac{1}{2}$ is :

Options

1. $\frac{1}{2}e^2$
2. $\frac{1}{2}\sqrt{e}$
3. $2e$
4. $2e^2$

Ans: $\frac{1}{2}e^2$

Sol: $y=\left(1-\frac{1}{2}\right)x^2+\left(1-\frac{1}{3}\right)x^3+\ldots\ldots$

$=\left(x^2+x^3+x^4+\ldots\right)-\left(\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\ldots\right)=\frac{x^2}{1-x}+x-\left(x+\frac{x^2}{2}+\frac{x^2}{3}+\ldots\right)$

$x=\frac{1}{2}\Rightarrow y=1-\ln 2$

$e^{1+y}=e^{1+1-\ln 2}=e^{2-\ln 2}=\frac{e^2}{2}$

Q.13 Let $\mathbb{Z}$ be the set of all integers,

$A=\{(x,y)\in\mathbb{Z}\times\mathbb{Z}:(x-2)^2+y^2\le 4\}$,

$B=\{(x,y)\in\mathbb{Z}\times\mathbb{Z}:x^2+y^2\le 4\}$ and

$C=\{(x,y)\in\mathbb{Z}\times\mathbb{Z}:(x-2)^2+(y-2)^2\le 4\}$

If the total number of relations from $A\cap B$ to $A\cap C$ is 2^p, then the value of p is :

Options

1. 25
2. 9
3. 16
4. 49

Ans: 25

Sol: $(x-2)^2 + y^2 \le 4$

$x^2 + y^2 \le 4$

No. of points common in A & B is 5.

(0, 0), (1, 0), (2, 0), (1, 1), (1, –1)

Similarly in B & C is 5

No. of relations = $2^{5\times5} = 2^{25}$

Q.14 The set of all values of $k > -1$, for which the equation $(3x^2+4x+3)^2-(k+1)(3x^2+4x+3)(3x^2+4x+2)+k(3x^2+4x+2)^2=0$ has real roots, is :

Options

1. $\left(\frac{1}{2}, \frac{3}{2}\right] - \{1\}$
2. $[2, 3)$
3. $\left[-\frac{1}{2}, 1\right)$
4. $\left(1, \frac{5}{2}\right]$

Ans: $\left(1, \frac{5}{2}\right]$

Sol: $(3x^2+4x+3)^2-(k+1)(3x^2+4x+3)(3x^2+4x+2)+k(3x^2+4x+2)^2=0$

Let $3x^2+4x+3 = a$ and $b = 3x^2+4x+2 = b \Rightarrow b = a - 1$

Given equation becomes

$\Rightarrow a^2-(k+1)ab+kb^2=0$

$\Rightarrow a = kb$ or $a = b$

$\therefore a = kb$ (a = b is impossible)

$\Rightarrow 3x^2+4x+3 = k(3x^2+4x+2)$

$\Rightarrow 3(k-1)x^2+4(k-1)x+(2k-3)=0$

For real roots $D \ge 0$

$\Rightarrow 16(k-1)^2-4(3(k-1))(2k-3)\ge 0$

$\Rightarrow -4(k-1)\{2k-5\} \ge 0$

$\Rightarrow (k-1)(2k-5) \le 0$

$\Rightarrow k \in \left[1, \frac{5}{2}\right]$

But $k \ne 1$

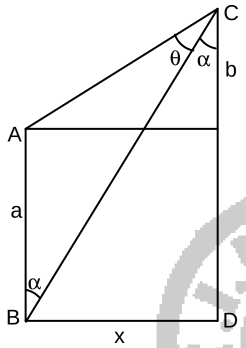
$\Rightarrow k \in \left(1, \frac{5}{2}\right]$

Q.15 Two poles, AB of length a metres and CD of length a + b (b ≠ a) metres are erected at the same horizontal level with bases at B and D. If BD = x and $\tan\angle ACB = \frac{1}{2}$, then :

Options
1. $x^2 + 2(a+2b)x - b(a+b) = 0$
2. $x^2 - 2ax + b(a+b) = 0$
3. $x^2 + 2(a+2b)x + a(a+b) = 0$
4. $x^2 - 2ax + a(a+b) = 0$

Ans: $x^2 - 2ax + b(a+b) = 0$

Sol:



$\tan\theta = \frac{1}{2}$

$\tan(\theta+\alpha) = \frac{x}{b}, \tan\alpha = \frac{x}{a+b}$

$\Rightarrow \dfrac{\frac{1}{2} + \frac{x}{a+b}}{1 - \frac{1}{2} \times \frac{x}{a+b}} = \frac{x}{b}$

Rearranging

$\Rightarrow x^2 - 2ax + ab + b^2 = 0$

Q.16 A box open from top is made from a rectangular sheet of dimension a × b by cutting squares each of side x from each of the four corners and folding up the flaps. If the volume of the box is maximum, then x is equal to :

Options
1. $\dfrac{a + b - \sqrt{a^2 + b^2 + ab}}{6}$
2. $\dfrac{a + b - \sqrt{a^2 + b^2 - ab}}{12}$
3. $\dfrac{a + b - \sqrt{a^2 + b^2 - ab}}{6}$
4. $\dfrac{a + b + \sqrt{a^2 + b^2 - ab}}{6}$

Ans: $\dfrac{a+b-\sqrt{a^2+b^2-ab}}{6}$

Sol: $V=(a-2x)(b-2x)x$

$\Rightarrow \dfrac{dV(x)}{dx}=12x^2-4(a+b)x+ab$

$\dfrac{dV(x)}{dx}=0 \Rightarrow 12x^2-4(a+b)x+ab=0$

Solving, x = $\dfrac{(a+b)\pm\sqrt{a^2+b^2-ab}}{6}$

Let $x=\dfrac{(a+b)+\sqrt{a^2+b^2-ab}}{6}$

or $\dfrac{(a+b)-\sqrt{a^2+b^2-ab}}{6}$

We can see that the maximum point is

$\therefore x=\dfrac{a+b-\sqrt{a^2+b^2-ab}}{b}$

Q.17 If two tangents drawn from a point P to the parabola $y^2=16(x-3)$ are at right angles, then the locus of point P is :

Options
1. $x+3=0$
2. $x+4=0$
3. $x+1=0$
4. $x+2=0$

Ans: $x+1=0$

Sol: Locus is directrix of parabola

$\Rightarrow x+1=0$

Q.18 The Boolean expression $(p\wedge q)\Rightarrow((r\wedge q)\wedge p)$ is equivalent to :

Options
1. $(p\wedge q)\Rightarrow(r\wedge q)$
2. $(p\wedge q)\Rightarrow(r\vee q)$
3. $(p\wedge r)\Rightarrow(p\wedge q)$
4. $(q\wedge r)\Rightarrow(p\wedge q)$

Ans: $(p\wedge q)\Rightarrow(r\wedge q)$

Sol: $(p\wedge q)\Rightarrow((r\wedge q)\wedge p)$

$\sim(p\wedge q)\vee((r\wedge q)\wedge p)$

$\sim(p\wedge q)\vee((r\wedge q)\wedge(p\wedge q))$

$\Rightarrow[\sim(p\wedge q)\vee(p\wedge q)]\wedge(\sim(p\wedge q)\vee(r\wedge p))$

$\Rightarrow T\wedge[\sim(p\wedge q)\vee(r\wedge p)]$

$\Rightarrow\sim(p\wedge q)\vee(r\wedge p)$

$\Rightarrow(p\wedge q)\Rightarrow(r\wedge q)$

Q.19

The value of the integral $\int_0^1 \frac{\sqrt{x}\,dx}{(1+x)(1+3x)(3+x)}$ is :

Options

1. $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{2}\right)$

2. $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{2}\right)$

3. $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{6}\right)$

4. $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{6}\right)$

Ans: $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{2}\right)$

Sol: $I=\int_0^1 \frac{\sqrt{x}}{(1+x)(1+3x)(3+x)}dx$

$I=\int_0^1 \frac{t(2t)}{(t^2+1)(1+3t^2)(3+t^2)}dt$

Simplifying, we get

$$=\frac{1}{2}\int_0^1 \frac{dt}{(1+t^2)}-\frac{3}{8}\int_0^1 \frac{dt}{(t^2+3)}-\frac{3}{8}\int_0^1 \frac{dt}{(1+3t^2)}=\frac{1}{2}\left(\tan^{-1}(t)\right)_0^1-\frac{3}{8\sqrt{3}}\left(\tan^{-1}\left(\frac{t}{\sqrt{3}}\right)\right)_0^1-\frac{3}{8\sqrt{3}}\left(\tan^{-1}\left(\sqrt{3}t\right)\right)_0^1$$

$$=\frac{1}{2}\left(\frac{\pi}{4}\right)-\frac{\sqrt{3}}{8}\left(\frac{\pi}{6}\right)-\frac{\sqrt{3}}{8}\left(\frac{\pi}{3}\right)=\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{2}\right)$$

Q.20 Let A(a, 0), B(b, 2b + 1) and C(0, b), $b \neq 0$, $|b| \neq 1$, be points such that the area of triangle ABC is 1 sq. unit, then the sum of all possible values of a is :

Options

1. $\frac{2b}{b+1}$

2. $\frac{-2b}{b+1}$

3. $\frac{2b^2}{b+1}$

4. $\frac{-2b^2}{b+1}$

Ans: $\frac{-2b^2}{b+1}$

Sol: $\left|\frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ b & 2b+1 & 1 \\ 0 & b & 1 \end{vmatrix}\right| = 1$

$\Rightarrow \begin{vmatrix} a & 0 & 1 \\ b & 2b+1 & 1 \\ 0 & b & 1 \end{vmatrix} = \pm 2$

$\Rightarrow a(2b+1-b) - 0 + 1(b^2 - 0) = \pm 2$

$\Rightarrow a = \frac{\pm 2 - b^2}{b+1}$

$\therefore a = \frac{2-b^2}{b+1}$ and $a = -\frac{2-b^2}{b+1}$

Sum of possible values of 'a' is = $\frac{-2b^2}{b+1}$

Q.1 If $\int \frac{2e^x + 3e^{-x}}{4e^x + 7e^{-x}} dx = \frac{1}{14}(ux + v \log_e(4e^x + 7e^{-x})) + C$, where C is a constant of integration, then u + v is equal to __________.

Given --
Answer :

Ans: 7

Sol: $2e^x + 3e^{-x} = A(4e^x + 7e^{-x}) + B(4e^x - 7e^{-x})$

Equating coefficient of e^x and e^{-x} on both sides

$2 = 4A + 4B; 3 = 7A - 7B$

Solving, we get $A = \frac{13}{28}, B = \frac{1}{28}$ \

i.e., Required integral is

$\int \frac{13}{28} dx + \frac{1}{28} \int \frac{4e^x - 7e^{-x}}{4e^x + 7e^{-x}} dx$

Comparing, u = $\frac{13}{2}$, v = $\frac{1}{2}$

$\Rightarrow$ u + v= 7

Q.2 Let S be the mirror image of the point Q(1, 3, 4) with respect to the plane $2x - y + z + 3 = 0$ and let R (3, 5, γ) be a point of this plane. Then the square of the length of the line segment SR is __________.

Given --
Answer :

Ans: 72

Sol: Since R (3,5, λ) lies on the plane 2x – y + z + 3 = 0.

Therefore, 6 – 5 + λ+ 3 = 0

$\Rightarrow \lambda = -4$

dr's of line QS are 2, –1,1

equation of line QS is

$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = \lambda$

Let P be the foot of the ⊥

$\Rightarrow P(2\lambda + 1, -\lambda + 3, \lambda + 4)$

P lies in the plane

$\Rightarrow 2(2\lambda + 1) - (-\lambda + 3) + (\lambda + 4) + 3 = 0$

$\Rightarrow 4\lambda + 2 + 3 + \lambda + 7 = 0$

$\Rightarrow 6\lambda + 6 = 0 \Rightarrow \lambda = -1.$

$\Rightarrow P(-1, 4, 3)$

Now, P is mid-point of QS.

Therefore, co-ordinated of S are (–3, 5, 2).

$\therefore SR = \sqrt{36+0+36} = \sqrt{72}$

$(SR)^2 = 72$

Q.3 Let z_1 and z_2 be two complex numbers such that $\arg(z_1 - z_2) = \frac{\pi}{4}$ and z_1, z_2 satisfy the equation $|z-3| = \text{Re}(z)$. Then the imaginary part of $z_1 + z_2$ is equal to ________.

Given 6
Answer :

Ans: 6

Sol: $|z - 3| = \text{Re}(z)$

Put $z = x + iy$

Simplifying we get $y^2 = 6x - 9$

$\Rightarrow y^2 = 6\left(x - \frac{3}{2}\right)$

Given z_1 and z_2 lie on $|z - 3| = \text{Re}z$

and $\text{Arg}(z_1 - z_2) = \frac{\pi}{4}$

$\Rightarrow$ Slope of line joining z_1 and z_2 is 1

Let $z_1\left(\frac{3}{2} + \frac{3}{2}t_1^2, 3t_1\right)$ and $z_2\left(\frac{3}{2} + \frac{3}{2}t_2^2, 3t_2\right)$

$\Rightarrow \frac{2}{t_2 + t_2} = 1$

$\Rightarrow t_2 + t_1 = 2$

Imaginary$(z_1 - z_2) = 3t_1 + 3t_2 = 3(t_1 + t_2) = 3(2) = 6$

Q.4 The probability distribution of random variable X is given by :

X	1	2	3	4	5
P(X)	K	2K	2K	3K	K

Let $p = P(1 < X < 4 \mid X < 3)$. If $5p = \lambda K$, then λ is equal to ________.

Given 2
Answer :

Ans: 30

Sol: $\sum P(X) = 1 \Rightarrow k + 2k + 2k + 3k + k = 1$

$\Rightarrow k = \frac{1}{9}$

Now, $p = P\left(\frac{1 < X < 4}{X < 3}\right) = \frac{P(X=2)}{P(X<3)} = \frac{\frac{2}{9}}{\frac{1}{9} + \frac{2}{9}} = \frac{2}{3}$

$\Rightarrow p = \frac{2}{3}$

Given $5p = \lambda k$

$$\frac{10}{3} = \frac{\lambda}{9}$$
$\Rightarrow \lambda = 30$

Q.5 $3 \times 7^{22} + 2 \times 10^{22} - 44$ when divided by 18 leaves the remainder _________.

Given --
Answer :

Ans: 15

Sol: $3(1 + 6)^{22} + 2 \times (1 + 9)^{22} - 44 = (3 + 2 - 44) = 18\,k$
$= -39 + 18.k = -3 - 36 + 18\,k = 18\,k + 15$
$\Rightarrow$Remainder = 15.

Q.6 Let $S = \{1, 2, 3, 4, 5, 6, 9\}$. Then the number of elements in the set $T = \{A \subseteq S : A \neq \phi$ and the sum of all the elements of A is not a multiple of 3} is __________.

Given **47**
Answer :

Ans: 80

Sol: The number will be of the form
[3x, 3x – 1, 3x – 2]
Number of subset of S containing one element which are not divisible by 3
$= {}^2C_1 \times {}^2C_1 = 4$
Number of subset of S containing two elements whose some is not divisible by 3
$= {}^3C_1 \times {}^2C_1 + {}^3C_1 \times {}^2C_1 + {}^2C_2 + {}^2C_2 = 14$
Number of subsets containing 3 elements whose sum is not divisible by 3
$= {}^3C_2 \times {}^4C_1 + \left({}^2C_2 \times {}^2C_1\right)2 + {}^3C_1\left({}^2C_2 + {}^2C_2\right) = 22$
Number of subsets containing 4 elements whose sum is not divisible by 3
$= {}^3C_1[({}^2C_1\,{}^2C_1) + ({}^2C_1\;{}^2C_2)] + {}^3C_2[{}^2C_2 + {}^2C_2] + {}^3C_3[{}^2C_1 + {}^2C_1] = 22$
Number of subsets of S containing 5 elements whose sum is not divisible by 3.
$= {}^3C_3\left({}^2C_2 \times {}^2C_2\right) + \left({}^3C_2\,{}^2C_1 + {}^2C_2\right) \times 2 \times {}^3C_2 = 2 + 12 = 14$
Number of subsets of S containing 6 elements whose sum is not divisible by 3 = 4
$\Rightarrow$Total subsets of Set A whose sum of digits is not divisible by 3
= 4 + 14 + 22 + 22 + 14 + 4 = 80.

Q.7 An online exam is attempted by 50 candidates out of which 20 are boys. The average marks obtained by boys is 12 with a variance 2. The variance of marks obtained by 30 girls is also 2. The average marks of all 50 candidates is 15. If μ is the average marks of girls and σ^2 is the variance of marks of 50 candidates, then $\mu + \sigma^2$ is equal to __________.

Given **19**
Answer :

Ans: 25

Sol: $\sigma^2_{boys} = 2$ $\quad$ n_1 = no. of boys
$\bar{x}_{boys} = 12$
$\sigma^2_{girls} = 2$
$$\bar{x}_{girls} = \frac{50 \times 15 - 12 \times \sigma_{boys}}{30} = \frac{750 - 12 \times 20}{30} = 17 = \mu$$
Variance of combined series

$$\sigma^2 = \frac{n_1\sigma^2_{boys} + n_2\sigma^2_{girls}}{n_1+n_2} + \frac{n_1.n_2}{(n_1+n_2)^2}(\bar{x}_{boys} - \bar{x}_{girls})^2$$

$$\sigma^2 = \frac{20\times 2 + 30\times 2}{20+30} + \frac{20\times 30}{(20+30)^2}(12-17)^2$$

$$\sigma^2 = 8$$

$$\Rightarrow \mu + \sigma^2 = 17 + 8 = 25$$

Q.8 Two circles each of radius 5 units touch each other at the point (1, 2). If the equation of their common tangent is $4x + 3y = 10$, and $C_1(\alpha, \beta)$ and $C_2(\gamma, \delta)$, $C_1 \neq C_2$ are their centres, then $|(\alpha+\beta)(\gamma+\delta)|$ is equal to _________.

Given --
Answer :

Ans: 40

Sol: Slope of line joining centres of circles = $\frac{4}{3} = \tan\theta$

Using this

$\Rightarrow \cos\theta = \frac{3}{5}, \sin\theta = \frac{4}{5}$

parametric form is given by

$\frac{x-1}{\cos\theta} = \frac{y-2}{\sin\theta} = \pm 5$

Comparing, $(x, y) = (1 + 5\cos\theta, 2 + 5\sin\theta)$

$\Rightarrow (\alpha, \beta) = (4, 6)$

$(x, y) = (\gamma, \delta) = (1 - 5\cos\theta, 2 - 5\sin\theta)$

$\Rightarrow (\gamma, s) = (-2, 2)$

$\therefore |(\alpha+\beta)(\gamma+\delta)| = |10x - 4| = 40$

Q.9 Let S be the sum of all solutions (in radians) of the equation $\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$ in $[0, 4\pi]$. Then $\frac{8S}{\pi}$ is equal to _________.

Given **2**
Answer :

Ans: 56

Sol: $\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$

$\Rightarrow 1 - \sin^2\theta\cos^2\theta - \sin\theta\cos\theta = 0$

$\Rightarrow (\sin 2\theta + 2)(\sin 2\theta - 1) = 0$

$\Rightarrow \sin 2\theta = 1$ or $\sin 2\theta = -2$ (not possible)

$\Rightarrow \theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$

$\Rightarrow S = \frac{\pi}{4} + \frac{5\pi}{4} + \frac{9\pi}{4} + \frac{13\pi}{4} = 7\pi$

$\Rightarrow \frac{8S}{\pi} = \frac{8\times 7\pi}{\pi} = 56.00$

Q.10 Let A ($\sec\theta$, $2\tan\theta$) and B ($\sec\phi$, $2\tan\phi$), where $\theta+\phi=\pi/2$, be two points on the hyperbola $2x^2-y^2=2$. If (α, β) is the point of the intersection of the normals to the hyperbola at A and B, then $(2\beta)^2$ is equal to __________.

Given --
Answer :

Ans: -----

Sol: Given A ($\sec\theta$, $2\tan2\theta$)
lie on the hyperbola
$2x^2-y^2=2$
$\Rightarrow 2\sec^2\theta - 4\tan^2\theta = 2$
$\Rightarrow \theta = 0$
Similarly, since B lie on the hyperbola, we will get $\phi = 0$.
But $\theta+\phi=\frac{\pi}{2}$
Impossible so no answer