SOLUTIONS & ANSWERS FOR JEE MAINS-2021
31st August Shift 2
[PHYSICS, CHEMISTRY & MATHEMATICS]

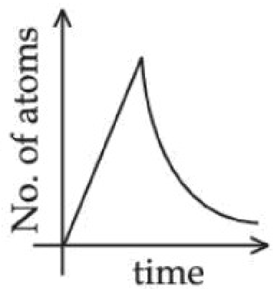
PART – A – PHYSICS

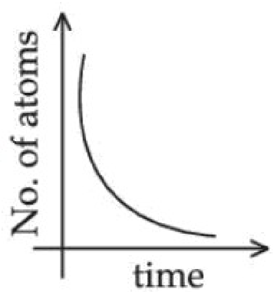
Section A

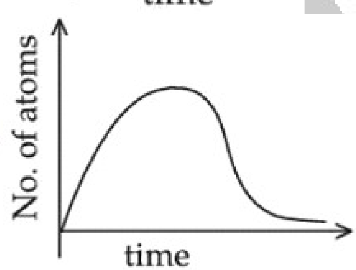
Q.1 A sample of a radioactive nucleus A disintegrates to another radioactive nucleus B, which in turn disintegrates to some other stable nucleus C. Plot of a graph showing the variation of number of atoms of nucleus B vesus time is :

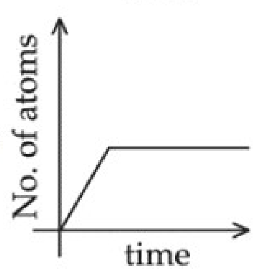
(Assume that at $t = 0$, there are no B atoms in the sample)

Options

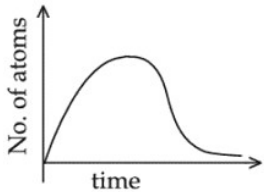
1. 

2. 

3. 

4. 

Ans:



Sol: A → B → C (stable)

Initially no. of atoms of B = 0 after t = 0, no. of atoms of B will starts increasing and reaches maximum value when rate of decay of B = rate of formation of B.

After that maximum value, no. of atoms will starts decreasing as growth and decay both are exponential functions, so best possible graph is (3).

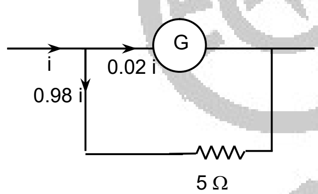
Q.2 Consider a galvanometer shunted with 5 Ω resistance and 2% of current passes through it. What is the resistance of the given galvanometer ?

Options

1. 300 Ω
2. 226 Ω
3. 344 Ω
4. 245 Ω

Ans: 245 Ω

Sol:



$0.02i\, R_g = 0.98\, i \times 5$ R_g : Resistance of the Galvanometer

$R_g = 245\ \Omega$

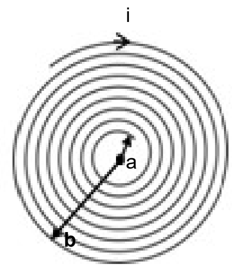
Q.3 A coil having N turns is wound tightly in the form of a spiral with inner and outer radii 'a' and 'b' respectively. Find the magnetic field at centre, when a current I passes through coil :

Options

1. $\dfrac{\mu_0 I}{4(a-b)}\left[\dfrac{1}{a}-\dfrac{1}{b}\right]$
2. $\dfrac{\mu_0 I}{8}\left[\dfrac{a+b}{a-b}\right]$
3. $\dfrac{\mu_0 I}{8}\left(\dfrac{a-b}{a+b}\right)$
4. $\dfrac{\mu_0 IN}{2(b-a)}\log_e\left(\dfrac{b}{a}\right)$

Ans: $\frac{\mu_0 IN}{2(b-a)}\log_e\left(\frac{b}{a}\right)$

Sol:



No. of turns in dx width = $\frac{N}{b-a}dx$

$$\int dB = \int_a^b \left(\frac{N}{b-a}\right)dx\,\frac{\mu_0 i}{2x}$$

$$B = \frac{N\mu_0 i}{2(b-a)}\ell n\left(\frac{b}{a}\right) \qquad (l_n = \log_e)$$

Q.4 In an ac circuit, an inductor, a capacitor and a resistor are connected in series with $X_L = R = X_C$. Impedance of this circuit is :

Options
1. $R\sqrt{2}$
2. $2R^2$
3. R
4. Zero

Ans: R

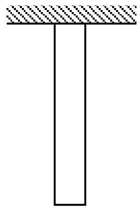
Sol: $Z = \sqrt{(X_L - X_C)^2 + R^2} = R \qquad \because X_L = X_C$

Q.5 A uniform heavy rod of weight 10 kg ms^{-2}, cross-sectional area 100 cm^2 and length 20 cm is hanging from a fixed support. Young modulus of the material of the rod is 2×10^{11} Nm^{-2}. Neglecting the lateral contraction, find the elongation of rod due to its own weight :

Options
1. 4×10^{-8} m
2. 5×10^{-8} m
3. 5×10^{-10} m
4. 2×10^{-9} m

Ans: 5×10^{-10} m

Sol:



We know, $\Delta\ell = \frac{WL}{2AY}$

$\Delta\ell = \frac{10\times1}{2\times5}\times100\times10^{-4}\times2\times10^{11}$

$\Delta\ell = \frac{1}{2}\times10^{-19} = 5\times10^{-10}$ m

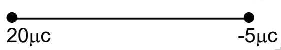
Q.6 Two particles A and B having charges 20 μC and −5 μC respectively are held fixed with a separation of 5 cm. At what position a third charged particle should be placed so that it does not experience a net electric force ?

Options

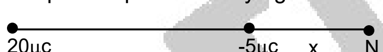
1. At midpoint between two charges
2. At 5 cm from 20 μC on the left side of system
3. At 5 cm from − 5 μC on the right side
4. At 1.25 cm from a − 5 μC between two charges

Ans: At 5 cm from −5μC on the right side

Sol:



Null point is possible only right side of -5μC



$E_N = +\frac{k(-5\mu C)}{x^2} + \frac{k(20\mu C)}{(5+x)^2} = 0 \quad \left(\because k = \frac{1}{4\pi\varepsilon_0}\right)$

x = 5 cm

Q.7 Match **List - I** with **List - II**.

	List - I		List - II
(a)	Torque	(i)	MLT^{-1}
(b)	Impulse	(ii)	MT^{-2}
(c)	Tension	(iii)	ML^2T^{-2}
(d)	Surface Tension	(iv)	MLT^{-2}

Choose the **most appropriate** answer from the option given below :

Options

1. (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)
2. (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)
3. (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)
4. (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

Ans: (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Sol: torque = $\tau \rightarrow ML^2T^{-2}$ (III)
Impulse $I \Rightarrow MLT^{-1}$ (I)
Tension force $\Rightarrow MLT^{-2}$ (IV)
Surface tension $\Rightarrow MT^{-2}$ (II)

Q.8 Which of the following equations is dimensionally incorrect ?
Where t = time, h = height, s = surface tension, θ = angle, ρ = density, a, r = radius, g = acceleration due to gravity, v = volume, p = pressure, W = work done, Γ = torque, ϵ = permittivity, E = electric field, J = current density, L = length.

Options

1. $v = \frac{\pi\, pa^4}{8\,\eta\, L}$
2. $W = \Gamma\theta$
3. $J = \epsilon \frac{\partial E}{\partial t}$
4. $h = \frac{2s\cos\theta}{\rho r g}$

Ans: $v = \frac{\pi\rho a^4}{8\eta L}$

Sol: (i) $\frac{\pi\rho a^4}{8\eta L} = \frac{dv}{dt}$ = volumetric flow rate

(ii) $h\rho g = \frac{2s}{r}\cos\theta$

(iii) RHS $\Rightarrow \varepsilon \times \frac{1}{4\pi\varepsilon_0}\frac{a}{r^2} \times \frac{1}{\varepsilon} = \frac{q}{t} \times \frac{1}{r^2} = \frac{I}{L^2} = IL^{-2}$

LHS

$R = \frac{I}{A} = IL^{-2}$

(iv) $W = \tau\theta$

Q.9 A reversible engine has an efficiency of $\frac{1}{4}$. If the temperature of the sink is reduced by 58°C, its efficiency becomes double. Calculate the temperature of the sink :

Options

1. 180.4°C
2. 382°C
3. 280°C
4. 174°C

Ans: 174°C

Sol: T_2 = sink temperature

$$\eta = 1 - \frac{T_2}{T_1}$$

$$\frac{1}{4} = 1 - \frac{T_2}{T_1}$$

$$\frac{T_2}{T_1} = \frac{3}{4} ------(i)$$

$$\frac{1}{2} = 1 - \frac{T_2 - 58}{T_1}$$

$$\frac{T_2}{T_1} = \frac{58}{T_1} = \frac{1}{2}$$

$$\frac{3}{4} = \frac{58}{T_1} + \frac{1}{2}$$

$$\frac{1}{4} = \frac{58}{T_1} \Rightarrow T_1 = 232$$

$$T_2 = \frac{3}{4} \times 232$$

T_2 = 174°C

Q.10 A small square loop of side 'a' and one turn is placed inside a larger square loop of side b and one turn (b > > a). The two loops are coplanar with their centres coinciding. If a current I is passed in the square loop of side 'b', then the coefficient of mutual inductance between the two loops is :

Options

1. $\frac{\mu_0}{4\pi} \frac{8\sqrt{2}}{a}$

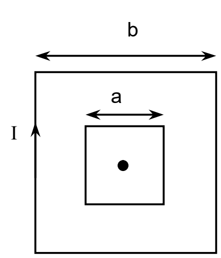
2. $\frac{\mu_0}{4\pi} 8\sqrt{2} \frac{a^2}{b}$

3. $\frac{\mu_0}{4\pi} \frac{8\sqrt{2}}{b}$

4. $\frac{\mu_0}{4\pi} 8\sqrt{2} \frac{b^2}{a}$

Ans: $\frac{\mu_0}{4\pi} 8\sqrt{2} \frac{a^2}{b}$

Sol:



$$B = \left[\frac{\mu_0}{4\pi}\,\frac{I}{b/2} \times 2\sin 45\right] \times 4$$

$$\phi = 2\sqrt{2}\,\frac{\mu_0}{\pi}\,\frac{I}{b} \times a^2$$

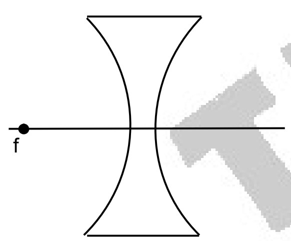
$$\therefore M = \frac{\phi}{I} = \frac{2\sqrt{2}\mu_0 a^2}{\pi b} = \frac{\mu_0}{4\pi} 8\sqrt{2}\,\frac{a^2}{b}$$

Q.11 An object is placed at the focus of concave lens having focal length f. What is the magnification and distance of the image from the optical centre of the lens ?

Options
1. Very high, ∞
2. $\frac{1}{2}, \frac{f}{2}$
3. $\frac{1}{4}, \frac{f}{4}$
4. 1, ∞

Ans: $\frac{1}{2}, \frac{f}{2}$

Sol:



U = –f

$$\frac{1}{V} - \frac{1}{U} = \frac{1}{-f} \Rightarrow \frac{1}{V} = -\frac{2}{f}$$

$$V = \frac{-f}{2}$$

$$m = \frac{V}{U} = \frac{1}{2}$$

Distance = $\frac{f}{2}$

Q.12 For an ideal gas the instantaneous change in pressure 'p' with volume 'v' is given by the equation $\frac{dp}{dv} = -ap$. If $p = p_0$ at $v = 0$ is the given boundary condition, then the maximum temperature one mole of gas can attain is :
(Here R is the gas constant)

Options

1. $\frac{ap_0}{eR}$
2. infinity
3. $0°C$
4. $\frac{p_0}{aeR}$

Ans: $\frac{p_0}{aeR}$

Sol: $\int_{p_0}^{p} \frac{dp}{P} = -a\int_{0}^{v} dv$

$\ell n\left(\frac{p}{p_0}\right) = -av$

$p = p_0 e^{-av}$

For temperature maximum p – v product should be maximum

$T = \frac{pv}{nR} = \frac{p_0 ve^{-av}}{R}$

$\frac{dT}{dv} = 0 \Rightarrow \frac{p_0}{R}\left\{e^{-av} + ve^{-av}(-a)\right\}$

$\frac{p_0 e^{-av}}{R}\{1 - av\} = 0$

$v = \frac{1}{a}, \infty$

$T = \frac{p_0 1}{Rae} = \frac{p_0}{Rae}$

At $v = \infty$

$T = 0$

Q.13 A moving proton and electron have the same de-Broglie wavelength. If K and P denote the K.E. and momentum respectively. Then choose the correct option :

Options

1. $K_p < K_e$ and $P_p = P_e$
2. $K_p > K_e$ and $P_p = P_e$
3. $K_p = K_e$ and $P_p = P_e$
4. $K_p < K_e$ and $P_p < P_e$

Ans: $K_p < K_e$ and $P_p = P_e$

Sol: $\lambda_p = \frac{h}{P_p} \quad \lambda_e = \frac{h}{P_e}$

$\because \lambda_p = \lambda_e$

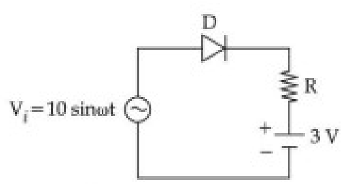
$\Rightarrow P_p = P_e$

$(K)_p = \frac{P_p^2}{2m_p}$

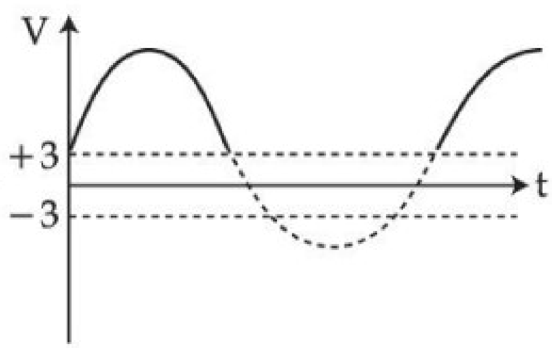
$(K)_e = \frac{P_e^2}{2m_e}$

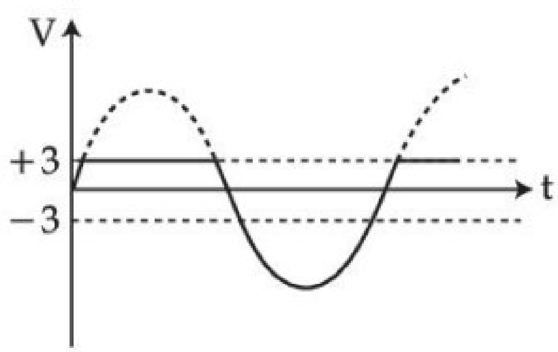
$K_p < K_e$ as $m_p > m_e$

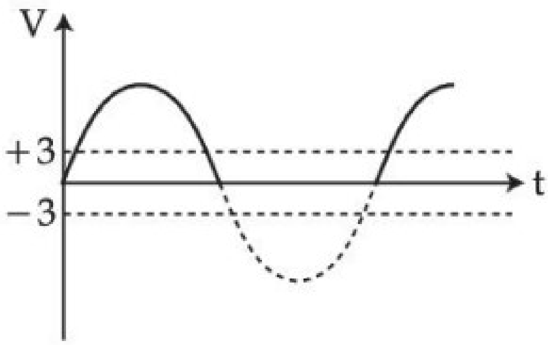
Q.14 Choose the correct waveform that can represent the voltage across R of the following circuit, assuming the diode is ideal one :

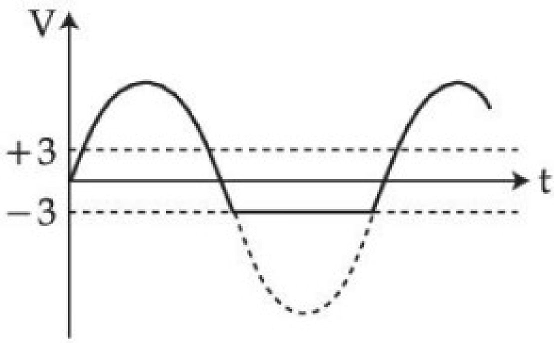


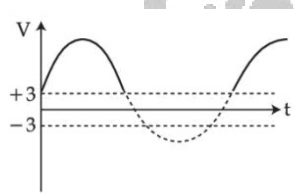
Options

1. 

2. 

3. 

4. 

Ans: 

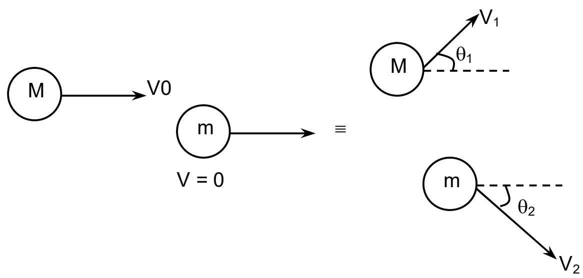
Sol: When $V_i > 3$ volt, $V_R > 0$
Because diode will be in forward biased state
When $V_i \leq 3$ volt; $V_R = 0$
Because diode will be in reverse biased state

Q.15 A body of mass M moving at speed V_0 collides elastically with a mass 'm' at rest. After the collision, the two masses move at angles θ_1 and θ_2 with respect to the initial direction of motion of the body of mass M. The largest possible value of the ratio M/m, for which the angles θ_1 and θ_2 will be equal, is :

Options
1. 2
2. 3
3. 1
4. 4

Ans: 3

Sol:



Given $\theta_1 = \theta_2 = \theta$

From momentum conservation in x-direction $MV_0 = MV_1 \cos\theta + mV_2 \cos\theta$

in y-direction $0 = MV_1 \sin\theta - mV_2 \sin\theta$

Solving above equations

$V_2 = \frac{MV_1}{m}, V_0 = 2V\cos\theta$

From energy conservation

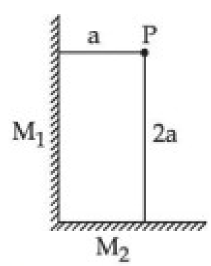
$\frac{1}{2}MV_0^2 = \frac{1}{2}MV_1^2 + \frac{1}{2}MV_2^2$

Substituting value of V_2 and V_0, we will get

$\frac{M}{m} + 1 = 4\cos^2\theta \le 4$

$\frac{M}{m} \le 3$

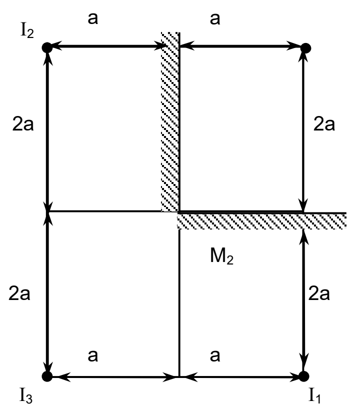
Q.16 Two plane mirrors M_1 and M_2 are at right angle to each other shown. A point source 'P' is placed at 'a' and '2a' meter away from M_1 and M_2 respectively. The shortest distance between the images thus formed is : (Take $\sqrt{5} = 2.3$)



Options
1. $4.6a$
2. $2.3a$
3. $3a$
4. $2\sqrt{10}\ a$

Ans: 4.6 a

Sol:



Shortest distance is 2a between I_1 and I_3
But answer given is for I_1 and I_2

$\sqrt{(4a)^2 + (2a)^2}$

$a\sqrt{20}$

4.47 a

Q.17 A helicopter is flying horizontally with a speed 'v' at an altitude 'h' has to drop a food packet for a man on the ground. What is the distance of helicopter from the man when the food packet is dropped ?

Options

1. $\sqrt{\dfrac{2ghv^2 + 1}{h^2}}$

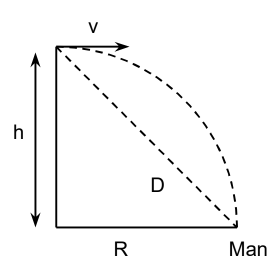
2. $\sqrt{2ghv^2 + h^2}$

3. $\sqrt{\dfrac{2gh}{v^2} + h^2}$

4. $\sqrt{\dfrac{2v^2h}{g} + h^2}$

Ans: $\sqrt{\dfrac{2v^2h}{g} + h^2}$

Sol:



$$R = \sqrt{\frac{2h}{g}}v$$

$$D = \sqrt{R^2 + h^2} = \sqrt{\left(\sqrt{\frac{2h}{g}}v\right)^2 + h^2}$$

$$D = \sqrt{\frac{2hv^2}{g} + h^2}$$

Q.18 The masses and radii of the earth and moon are (M_1, R_1) and (M_2, R_2) respectively. Their centres are at a distance 'r' apart. Find the minimum escape velocity for a particle of mass 'm' to be projected from the middle of these two masses :

Options

1. $V = \frac{1}{2}\sqrt{\frac{2G\,(M_1 + M_2)}{r}}$

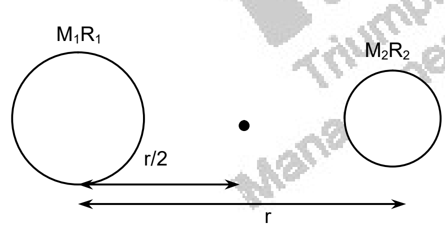
2. $V = \frac{1}{2}\sqrt{\frac{4G\,(M_1 + M_2)}{r}}$

3. $V = \frac{\sqrt{2G}\,(M_1 + M_2)}{r}$

4. $V = \sqrt{\frac{4G\,(M_1 + M_2)}{r}}$

Ans: $V = \sqrt{\frac{4G(M_1 + M_2)}{r}}$

Sol:

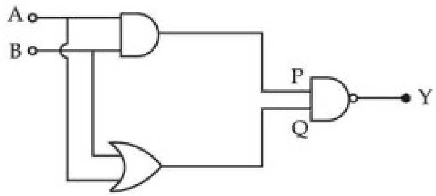


$$\frac{1}{2}mV^2 - \frac{GM_1m}{r/2} - \frac{GM_2m}{r/2} = 0$$

$$\frac{1}{2}mV^2 = \frac{2Gm}{r}(M_1 + M_2)$$

$$V = \sqrt{\frac{4G(M_1 + M_2)}{r}}$$

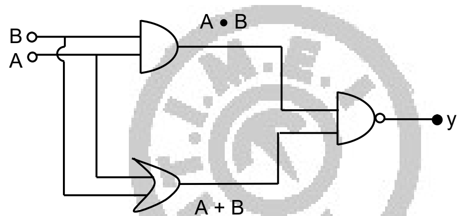
Q.19 In the following logic circuit the sequence of the inputs A, B are (0, 0), (0, 1), (1, 0) and (1, 1). The output Y for this sequence will be :



Options
1. 0, 0, 1, 1
2. 1, 1, 1, 0
3. 1, 0, 1, 0
4. 0, 1, 0, 1

Ans: 1, 1, 1, 0

Sol:



$Y = \overline{(A \cdot B) \cdot (A + B)}$

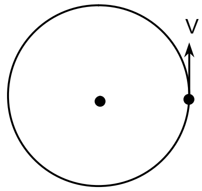
$Y)_{(0,\,0)} = 1$
$Y)_{(0,\,1)} = 1$
$Y)_{(1,\,0)} = 1$
$Y)_{(1,\,1)} = 0$

Q.20 Angular momentum of a single particle moving with constant speed along circular path :

Options
1. is zero
2. remains same in magnitude and direction
3. changes in magnitude but remains same in the direction
4. remains same in magnitude but changes in the direction

Ans: remains same in magnitude and direction

Sol:



$|\vec{L}|$ = mvr

And direction will be upward and remain constant

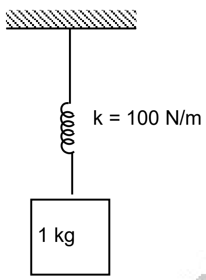
Section B

Q.1 A particle of mass 1 kg is hanging from a spring of force constant 100 Nm^{-1}. The mass is pulled slightly downward and released so that it executes free simple harmonic motion with time period T. The time when the kinetic energy and potential energy of the system will become equal, is $\frac{T}{x}$. The value of x is ______.

Given --
Answer :

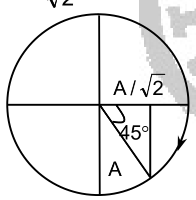
Ans: 8

Sol:



KE = PE

$y = \frac{A}{\sqrt{2}} - \sin\omega t$



$t = \frac{T}{8} = \frac{T}{x}$

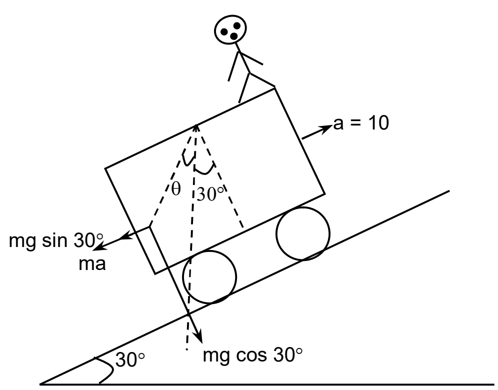
x = 8

Q.2 A car is moving on a plane inclined at 30° to the horizontal with an acceleration of 10 ms^{-2} parallel to the plane upward. A bob is suspended by a string from the roof of the car. The angle in degrees which the string makes with the vertical is ______.
(Take $g = 10\ ms^{-2}$)

Given **45**
Answer :

Ans: 30

Sol:



$$\tan(30+\theta)=\frac{mg\ \sin 30^\circ + ma}{mg\ \cos 30^\circ}$$

$$\tan(30+\theta)=\frac{5+10}{5\sqrt{3}}=\frac{1+2}{\sqrt{3}}$$

$$\frac{\tan\theta+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}\tan\theta}=\sqrt{3}$$

$$\sqrt{3}\tan\theta+1=3-\sqrt{3}\tan\theta$$

$$2\sqrt{3}\tan\theta=2$$

$$\tan\theta=\frac{1}{\sqrt{3}}$$

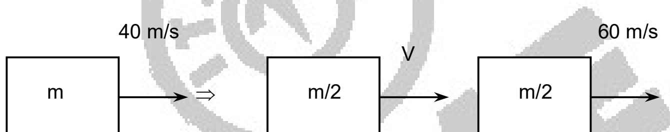
$\theta = 30°$

Q.3 A block moving horizontally on a smooth surface with a speed of 40 ms^{-1} splits into two equal parts. If one of the parts moves at 60 ms^{-1} in the same direction, then the fractional change in the kinetic energy will be $x : 4$ where $x =$__________.

Given **5**
Answer :

Ans: 1

Sol:



$P_i = P_f$

$$m\times 40=\frac{m}{2}\times v+\frac{m}{2}\times 60$$

$$40=\frac{v}{2}+30$$

$\Rightarrow v = 20$

$$(K.E.)_i=\frac{1}{2}m\times(40)^2=800\,m$$

$$(K.E.)_f=\frac{1}{2}\frac{m}{2}(20)^2+\frac{1}{2}\frac{m}{2}(60)^2=1000\,m$$

$$|\Delta K.E|=|1000m-800m|=200\,m$$

$$\frac{\Delta K.E}{(K.E)_i}=\frac{200\,m}{800\,m}=\frac{1}{4}=\frac{x}{4}$$

$x = 1$

Q.4 When a rubber ball is taken to a depth of __________ m in deep sea, its volume decreases by 0.5%.
(The bulk modulus of rubber $= 9.8\times 10^8\ Nm^{-2}$
Density of sea water $= 10^3\ kgm^{-3}$
$g = 9.8\ m/s^2$)

Given **--**
Answer :

Ans: 500

Sol: $B = -\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)} = -\frac{\rho g h}{\left(\frac{\Delta V}{V}\right)}$

$-\frac{B\frac{\Delta V}{V}}{\rho g} = h$

$\frac{9.8 \times 10^8 \times 0.5}{100 \times 10^3 \times 9.8} = h$

h = 500

Q.5 A wire having a linear mass density 9.0×10^{-4} kg/m is stretched between two rigid supports with a tension of 900 N. The wire resonates at a frequency of 500 Hz. The next higher frequency at which the same wire resonates is 550 Hz. The length of the wire is __________m.

Given --
Answer :

Ans: 10

Sol: $\mu = 9.0 \times 10^{-4} \frac{kg}{m}$

T = 900 N

$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{900}{9 \times 10^{-4}}} = 100\,m/s$

f_1 = 500 Hz

f = 550

$\frac{nV}{2\ell} = 500$ ------(i)

$\frac{(n+1)V}{2\ell} = 550$ ----(ii)

(ii) – (i) $\frac{V}{2\ell} = 50$

$\ell = \frac{1000}{2 \times 50} = 10$

Q.6 If the sum of the heights of transmitting and receiving antennas in the line of sight of communication is fixed at 160 m, then the maximum range of LOS communication is __________ km.

(Take radius of Earth = 6400 km)

Given --
Answer :

Ans: 64

Sol: $h_T = h_R = 160$ ------- (i)

$d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$d = \sqrt{2R}\left[\sqrt{h_T} + \sqrt{h_R}\right]$

$d = \sqrt{2R}\left[\sqrt{x} + \sqrt{160 - x}\right]$

$\frac{d(d)}{dx} = 0$

$\frac{1}{2\sqrt{x}} + \frac{1(-1)}{2\sqrt{160 - x}} = 0$

$$\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{160 - x}}$$

x = 80 m

$$d_{max} = \sqrt{2 \times 6400}\left[\sqrt{\frac{80}{1000}} + \sqrt{\frac{20}{1000}}\right] = \frac{80\sqrt{2} \times 2\sqrt{80}}{10\sqrt{10}} = 8 \times 2 \times \sqrt{2} \times 2\sqrt{2} = 64 \text{ km}$$

Q.7 The electric field in an electromagnetic wave is given by

$E = (50\ NC^{-1})\ \sin\omega(t - x/c)$

The energy contained in a cylinder of volume V is 5.5×10^{-12} J. The value of V is ________ cm^3.

(given $\epsilon_0 = 8.8 \times 10^{-12}\ C^2N^{-1}m^{-2}$)

Given --
Answer :

Ans: 500

Sol: $E = 50 \sin\left(\omega t - \frac{\omega}{c}x\right)$

Energy density = $\frac{1}{2}\epsilon_0 E_0^2$

Energy for volume V = $\frac{1}{2}\epsilon_0 E_0^2 V = 5.5 \times 10^{-12}$

$\frac{1}{2} 8.8 \times 10^{-12} \times 2500\ V = 5.5 \times 10^{-12}$

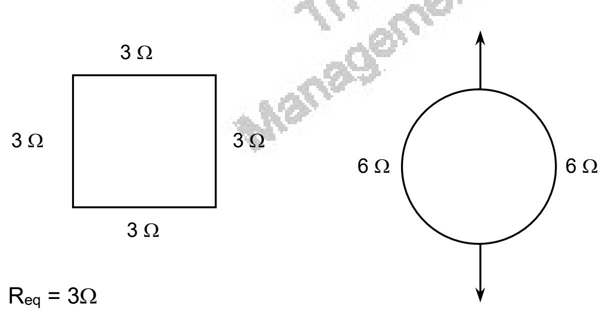
$V = \frac{5.5 \times 2}{2500 \times 8.8} = .0005\ m^3 = 0.0005 \times 10^6\ (cm)^3 = 500\ (cm)^3$

Q.8 A square shaped wire with resistance of each side 3 Ω is bent to form a complete circle. The resistance between two diametrically opposite points of the circle in unit of Ω will be ________.

Given **3**
Answer :

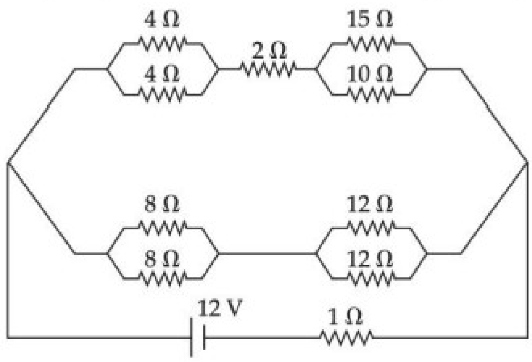
Ans: 3

Sol:



$R_{eq} = 3\Omega$

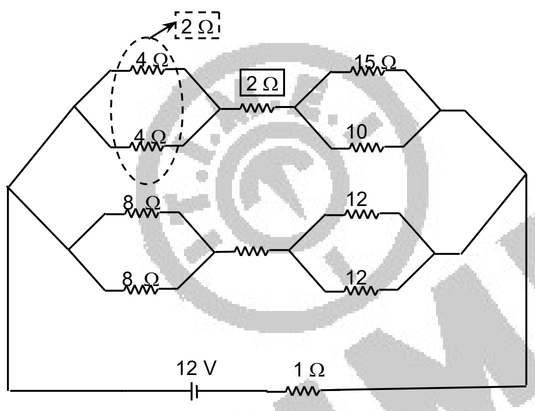
Q.9 The voltage drop across 15 Ω resistance in the given figure will be ________ V.



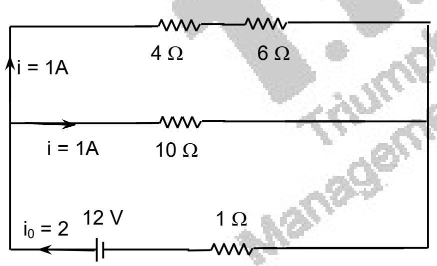
Given **6**
Answer :

Ans: 6

Sol:

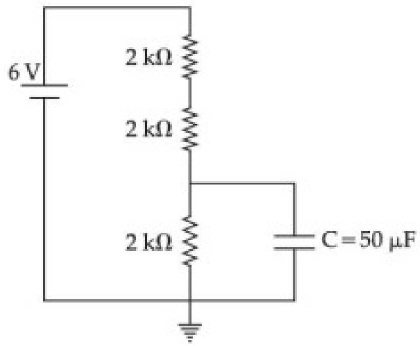


⇒ effective circuit diagram will be



Point drop across 6Ω = 1 × 6 = 6 = V_{AB}
⇒ Hence point drop across 15 Ω = 6 volt = V_{AB}

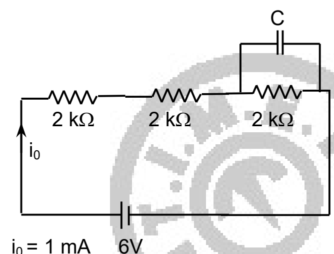
Q.10 A capacitor of 50 μF is connected in a circuit as shown in figure. The charge on the upper plate of the capacitor is __________ μC.



Given **2**
Answer :

Ans: 100

Sol:



Potential difference across each resistor = 2V
$q = CV = 50 \times 10^{-6} \times 2 = 100 \times 10^{-6} = 100\ \mu C$

PART – B – CHEMISTRY

Section A

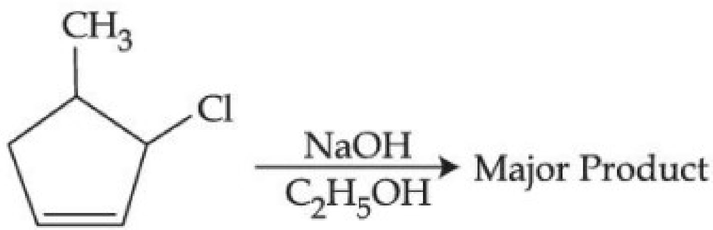
Q.1 Which among the following is not a polyester ?

Options
1. Dacron
2. PHBV
3. Glyptal
4. Novolac

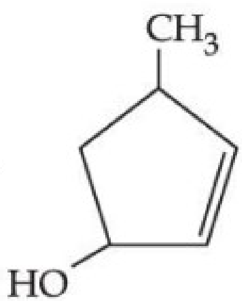
Ans: Novolac

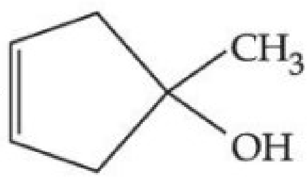
Sol: Novolac is not a polyester. It is a linear polymer of HCHO and phenol.

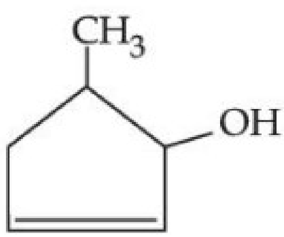
Q.2 The major product of the following reaction is :

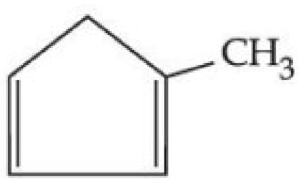


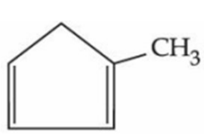
Options

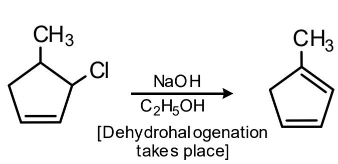
1. 

2. 

3. 

4. 

Ans: 

Sol: 

Q.3 Match **List - I** with **List - II** :

	List - I (**Metal Ion**)		**List - II** (**Group in Qualitative analysis**)
(a)	Mn^{2+}	(i)	Group - III
(b)	As^{3+}	(ii)	Group – IIA
(c)	Cu^{2+}	(iii)	Group - IV
(d)	Al^{3+}	(iv)	Group – IIB

Choose the **most appropriate** answer from the options given below :

Options
1. (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)
2. (a)-(i), (b)-(iv), (c)-(ii), (d)-(iii)
3. (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)
4. (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

Ans: (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

Sol:

Metal ion		Group in Qualitative analysis
Mn^{2+}	$\rightarrow$	Gp IV
As^{3+}	$\rightarrow$	Gp II B
Cu^{2+}	$\rightarrow$	Gp II A
Al^{3+}	$\rightarrow$	Gp III

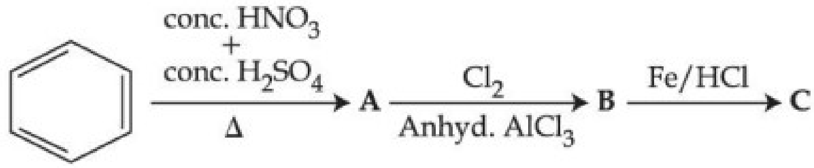
Q.4 In which one of the following sets all species show disproportionation reaction ?

Options
1. $Cr_2O_7^{2-}$, MnO_4^- , ClO_2^- and Cl_2
2. MnO_4^- , ClO_2^- , Cl_2 and Mn^{3+}
3. ClO_2^- , F_2, MnO_4^- and $Cr_2O_7^{2-}$
4. ClO_4^- , MnO_4^- , ClO_2^- and F_2

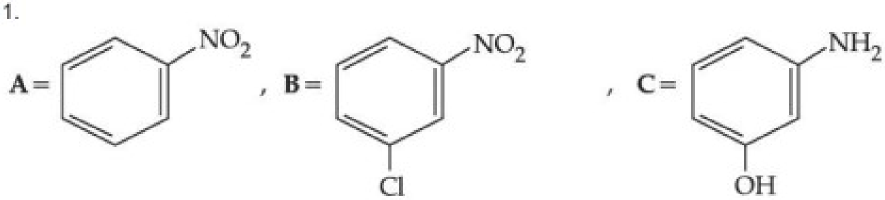
Ans: No answer

Sol: $Cr_2O_7^{2-}$, MnO_4^- and ClO_4^- doesn't undergo disproportionation reaction

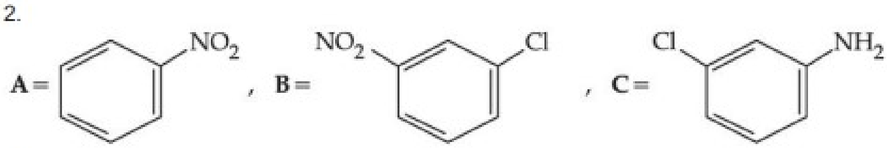
Q.5 Identify correct **A, B** and **C** in the reaction sequence given below :



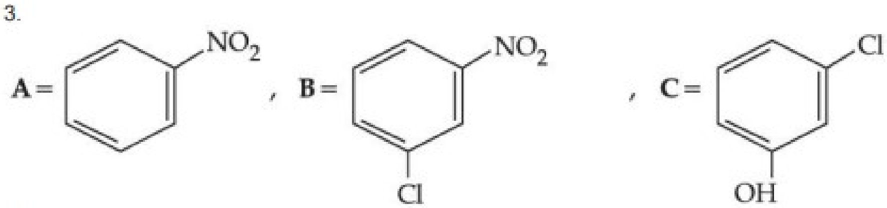
Options 1.



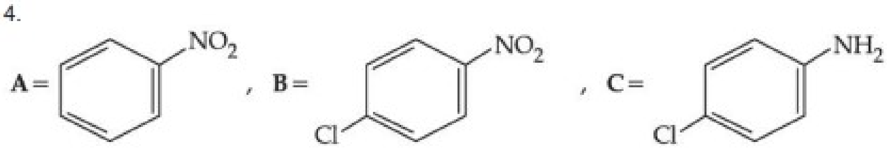
2.



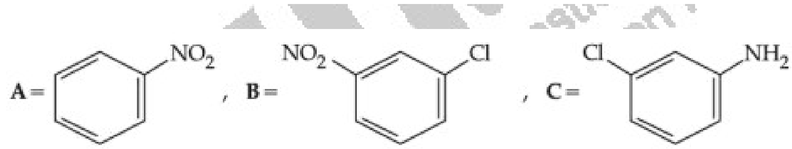
3.



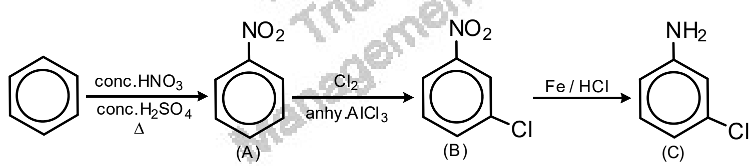
4.



Ans:



Sol:



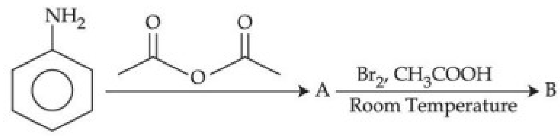
Q.6 Which of the following is NOT an example of fibrous protein ?

Options
1. Collagen
2. Myosin
3. Albumin
4. Keratin

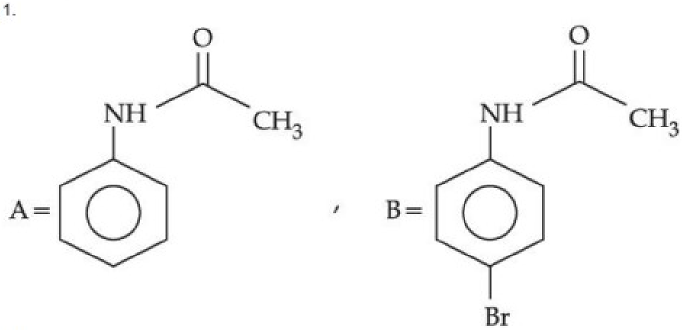
Ans: Albumin

Sol: Albumin is not a fibrous protein

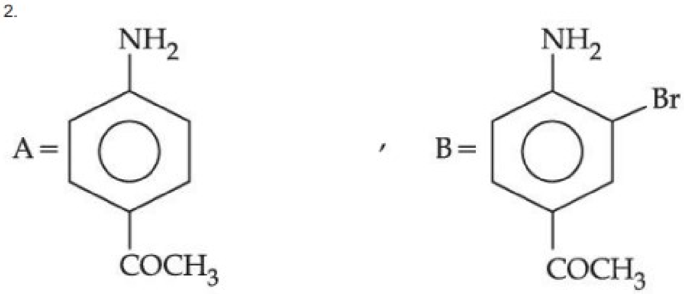
Q.7 The major products A and B formed in the following reaction sequence are :



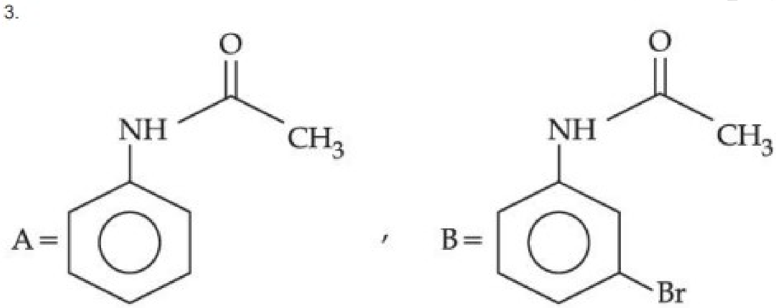
Options 1.



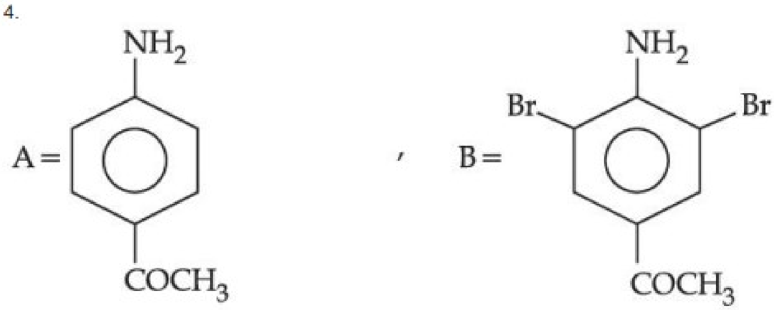
2.

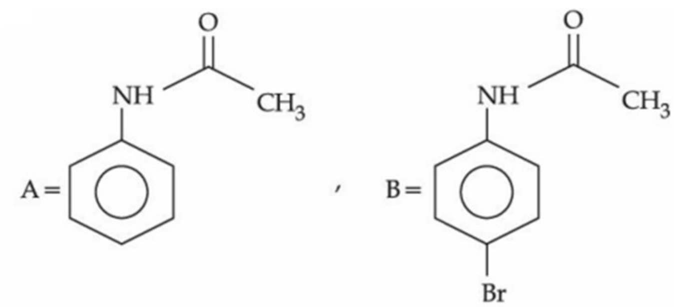


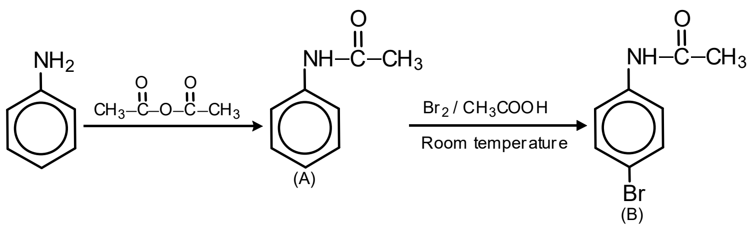
3.



4.



Ans: 

Sol: 

Q.8 Which one of the following correctly represents the order of stability of oxides, X_2O; (X = halogen) ?

Options
1. Cl > I > Br
2. Br > I > Cl
3. Br > Cl > I
4. I > Cl > Br

Ans: I > Cl > Br

Sol: The stability of oxides of halogens decreases in the order I > Cl > Br

Q.9 Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Lithium salts are hydrated.

Reason (R) : Lithium has higher polarising power than other alkali metal group members.

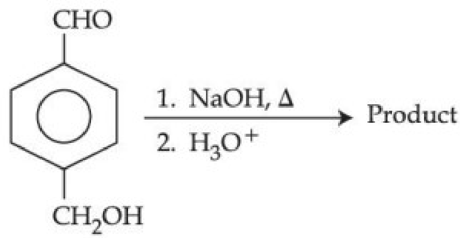
In the light of the above statements, choose the **most appropriate** answer from the options given below :

Options
1. **(A)** is correct but **(R)** is not correct.
2. **(A)** is not correct but **(R)** is correct.
3. Both **(A)** and **(R)** are correct but **(R)** is NOT the correct explanation of **(A)**.
4. Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**.

Ans: Both (A) and (R) are correct but (R) is NOT the correct explanation of (A)

Sol: Li^+ has maximum degree of hydration and for this reason lithium salts are mostly hydrated

Q.10 For the reaction given below :



The compound which is **not** formed as a product in the reaction is a :

Options 1. compound with both alcohol and acid functional groups

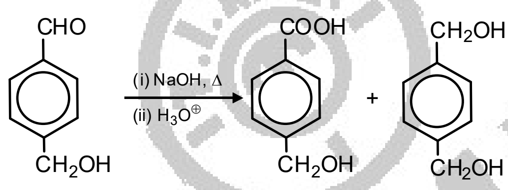
2. monocarboxylic acid

3. diol

4. dicarboxylic acid

Ans: dicarboxylic acid

Sol:



The reaction is Cannizzaro reaction

Q.11 Which one of the following statements is **incorrect** ?

Options 1. At around 2000 K, the dissociation of dihydrogen into its atoms is nearly 8.1%.

2. Dihydrogen is produced on reacting zinc with HCl as well as $NaOH_{(aq)}$.

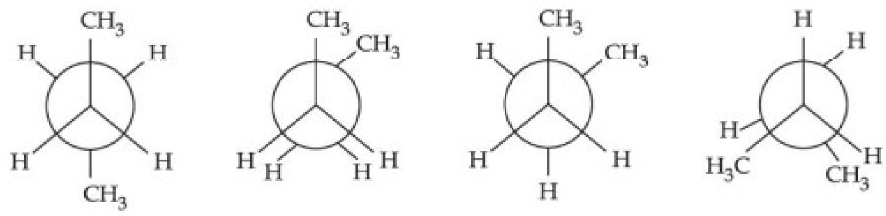
3. Bond dissociation enthalpy of H_2 is highest among diatomic gaseous molecules which contain a single bond.

4. Atomic hydrogen is produced when H_2 molecules at a high temperature are irradiated with UV radiation.

Ans: At around 2000 K, the dissociation of dihydrogen into its atoms is nearly 8.1%

Sol: The dissociation of hydrogen into its atoms is only ~0.081% around 2000 K which increases to 95.5% at 5000 K.

Q.12 Arrange the following conformational isomers of n-butane in order of their increasing potential energy :



Options
1. II < IV < III < I
2. I < IV < III < II
3. II < III < IV < I
4. I < III < IV < II

Ans: I < III < IV < II

Sol: Higher the potential energy, lesser the stability of the conformational isomer. The stability among the conformational isomers are fully eclipsed (II) < partially eclipsed (IV), Gauche (III) < Staggered or Anti (I). Hence the potential energy will be in the reverse order.
i.e., I < III < IV < II

Q.13 Spin only magnetic moment in BM of $[Fe(CO)_4(C_2O_4)]^+$ is :

Options
1. 5.92
2. 0
3. 1.73
4. 1

Ans: 1.73

Sol: In $[Fe(CO)_4(C_2O_4)]^+$, Fe is in +3 oxidation state. According to CFT
Configuration of Fe^{3+} in this complex = $t_2g^5\ eg^0$
Number of unpaired electrons (n) = 1
$\therefore \mu = \sqrt{n(n+2)} = \sqrt{1(1+2)} = 1.73\,BM$

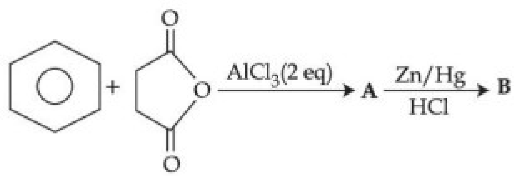
Q.14 The deposition of X and Y on ground surfaces is referred as wet and dry depositions, respectively. X and Y are :

Options
1. X = Ammonium salts , Y = CO_2
2. X = CO_2 , Y = SO_2
3. X = SO_2 , Y = Ammonium salts
4. X = Ammonium salts , Y = SO_2

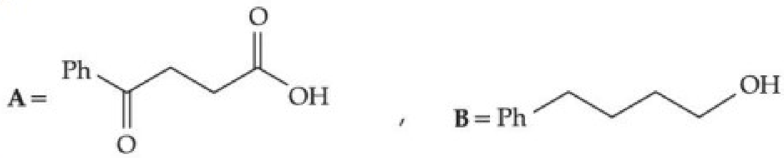
Ans: X = Ammonium salts, Y = SO_2

Sol: Ammonium salts in rain drops results in wet deposition. Oxides of nitrogen and sulphur which are acidic in nature can be blown by wind along with solid particles in the atmosphere and finally settle down on the ground as dry deposition

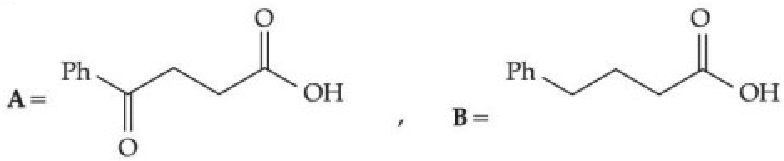
Q.15 The structures of **A** and **B** formed in the following reaction are : [Ph = $-C_6H_5$]



Options 1.



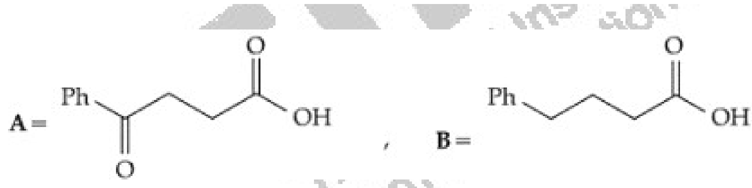
2.



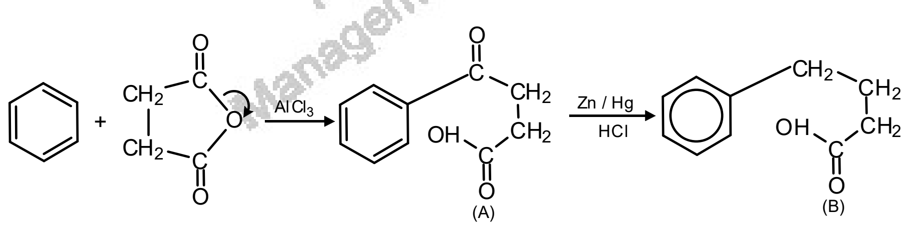
3.

4.

Ans:



Sol:



Q.16 The **incorrect** expression among the following is :

Options 1.

For isothermal process $w_{reversible} = -\ nRT \ln \frac{V_f}{V_i}$

2. $\ln K = \frac{\Delta H^\circ - T\Delta S^\circ}{RT}$

3. $K = e^{-\Delta G^\circ / RT}$

4. $\frac{\Delta G_{System}}{\Delta S_{Total}} = -\ T$ (at constant P)

Ans: $\ln K = \frac{\Delta H^\circ - T\Delta S^\circ}{RT}$

Sol: $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

$\Delta G^\circ = -RT \ln K$

$-RT \ln K = \Delta H^\circ - T\Delta S^\circ$

$-\ln K = \frac{\Delta H^\circ - T\Delta S^\circ}{RT}$

Q.17 The number of S=O bonds present in sulphurous acid, peroxodisulphuric acid and pyrosulphuric acid, respectively are :

Options 1. 2, 3 and 4

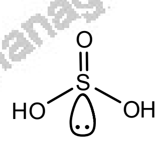
2. 2, 4 and 3

3. 1, 4 and 3

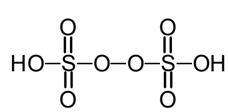
4. 1, 4 and 4

Ans: 1, 4 and 4

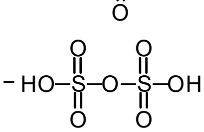
Sol: Sulphurous acid (H_2SO_3) –



Peroxodisulphuric acid ($H_2S_2O_8$) –



Pyro sulphuric acid ($H_2S_2O_7$) –



Q.18 Match **List - I** with **List - II** :

	List - I (Parameter)		List - II (Unit)
(a)	Cell constant	(i)	$S\ cm^2\ mol^{-1}$
(b)	Molar conductivity	(ii)	Dimensionless
(c)	Conductivity	(iii)	m^{-1}
(d)	Degree of dissociation of electrolyte	(iv)	$\Omega^{-1}m^{-1}$

Choose the **most appropriate** answer from the options given below :

Options
1. (a)-(i), (b)-(iv), (c)-(iii), (d)-(ii)
2. (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)
3. (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)
4. (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

Ans: (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Sol: Cell constant – m^{-1}
Molar conductivity – $S\ cm^2\ mol^{-1}$
Conductivity – $\Omega^{-1}\ m^{-1}$
Degree of dissociation of electrolyte – dimensionless

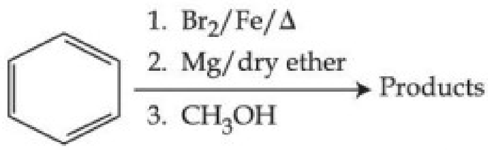
Q.19 The Eu^{2+} ion is a strong reducing agent in spite of its ground state electronic configuration (outermost) : [Atomic number of Eu = 63]

Options
1. $4f^6 6s^2$
2. $4f^6$
3. $4f^7 6s^2$
4. $4f^7$

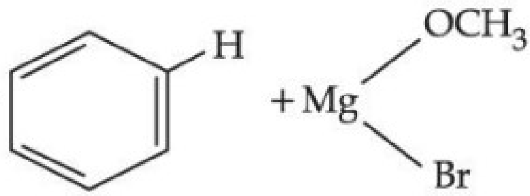
Ans: $4f^7$

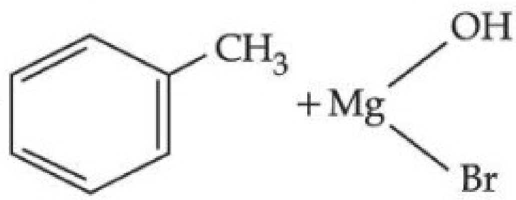
Sol: Eu (Z = 63) → [Xe] $4f^7\ 5d^0\ 6s^2$
Eu^{2+} → [Xe] $4f^7$

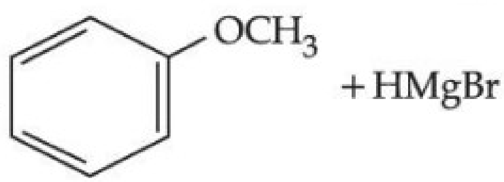
Q.20 For the following sequence of reactions, the correct products are :

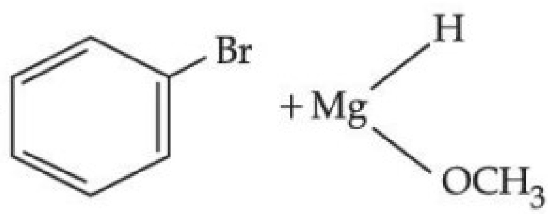


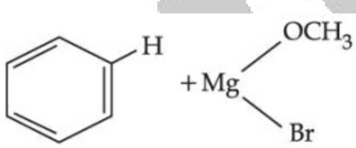
Options

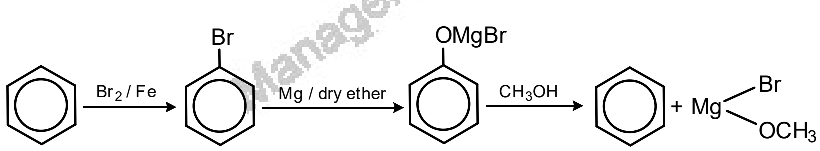
1. 

2. 

3. 

4. 

Ans: 

Sol: 

Section B

Q.1 Sodium oxide reacts with water to produce sodium hydroxide. 20.0 g of sodium oxide is dissolved in 500 mL of water. Neglecting the change in volume, the concentration of the resulting NaOH solution is _________ $\times 10^{-1}$ M. (Nearest integer)
[Atomic mass : Na = 23.0, O = 16.0, H = 1.0]

Given --
Answer :

Ans: 13

Sol: $\underset{1\,mol}{Na_2O} + \underset{1\,mol}{H_2O} \longrightarrow \underset{2\,mol}{2\,NaOH}$

No. of moles of $Na_2O = \frac{20}{62} = 0.322$

Molarity of $Na_2O = \frac{0.322}{0.5} = 0.644$

$\therefore$ 1 M Na_2O produces 2 M NaOH

$\therefore$ 0.644 M Na_2O produces 1.288 M NaOH

i.e., 12.88×10^{-1} M

Q.2 The value of magnetic quantum number of the outermost electron of Zn^+ ion is __________. (Integer answer)

Given Answer : **0**

Ans: 0

Sol: Zn (Z = 30) $\rightarrow 1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,4s^2\,3d^{10}$

$Zn^{2+} \rightarrow 1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^{10}\,4s^1$

The outermost electron is in 4s orbital

Hence the value of magnetic quantum number, $m_\ell = 0$

Q.3 For the reaction $A \rightarrow B$, the rate constant k (in s^{-1}) is given by

$$\log_{10} k = 20.35 - \frac{(2.47 \times 10^3)}{T}$$

The energy of activation in kJ mol^{-1} is __________. (Nearest integer)
[Given : $R = 8.314\ J\ K^{-1}\ mol^{-1}$]

Given Answer : --

Ans: 47

Sol: $\log k = \log A - \frac{E_a}{2.303\,RT}$

$\log k = 20.35 - \frac{(2.47 \times 10^3)}{T}$

Comparing the two equations,

$\frac{E_a}{2.303\,RT} = \frac{2.47 \times 10^3}{T}$

$E_a = 2.47 \times 10^3 \times 2.303 \times 8.314 = 47.29 \times 10^3\ J\ mol^{-1} = 47.29\ kJ\ mol^{-1}$

Q.4 According to molecular orbital theory, the number of unpaired electron(s) in O_2^{2-} is __________.

Given Answer : --

Ans: 0

Sol: According to MOT,

Molecular orbital configuration of

$O_2^-(18\ e^-s) \rightarrow \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \sigma 2p_z^2\ \pi 2p_x^2 = \pi 2p_y^2\ \pi^* 2p_x^2 = \pi^* 2p_y^2$

$\therefore$ No. unpaired electrons

Q.5 The transformation occurring in Duma's method is given below

$$C_2H_7N + \left(2x + \frac{y}{2}\right) CuO \rightarrow x\,CO_2 + \frac{y}{2}H_2O + \frac{z}{2}N_2 + \left(2x + \frac{y}{2}\right) Cu$$

The value of y is __________. (Integer answer)

Given Answer : **7**

Ans: 7

Sol: According to Duma's method

$$C_xH_yN_z + \left(2x + y/2\right)CuO \rightarrow xCO_2 + y/2\,H_2O + z/2\,N_2 + \left(2x + y/2\right)Cu$$

In the given problem,

$C_xH_yN_z$ is given as C_7H_7N

x = 2 y = 7 z = 1

Q.6 The empirical formula for a compound with a cubic close packed arrangement of anions and with cations occupying all the octahedral sites in A_xB. The value of x is __________. (Integer answer)

Given Answer : **3**

Ans: 1

Sol: B occupies ccp structure

$\therefore$ No. of B atoms per unit cell = 4

A occupies all octahedral sites

$\therefore$ No. of A atoms per unit cell = 4

$\therefore$ Formula = AB

Q.7 1.22 g of an organic acid is separately dissolved in 100 g of benzene ($K_b = 2.6$ K kg mol^{-1}) and 100 g of acetone ($K_b = 1.7$ K kg mol^{-1}). The acid is known to dimerize in benzene but remain as a monomer in acetone. The boiling point of the solution in acetone increases by 0.17°C. The increase in boiling point of solution in benzene in °C is $x \times 10^{-2}$. The value of x is __________. (Nearest integer)

[Atomic mass : C = 12.0, H = 1.0, O = 16.0]

Given Answer : **--**

Ans: 13

Sol: $\Delta T_f = \dfrac{i.K_f \times W_B \times 1000}{M_B \times W_A}$

Since the same amount of the same solute is dissolved in the two solvents, $\dfrac{W_B}{M_B}$ remains constant.

Hence,

$$\left(\frac{\Delta T_f \times W_A}{i \times K_b}\right)_{benzene} = \left(\frac{\Delta T_f \times W_A}{i \times K_b}\right)_{acetone}$$

$$\frac{\Delta T_f \times 100}{0.5 \times 2.6} = \frac{0.17 \times 100}{1 \times 1.7}$$

$$\therefore (\Delta T_f)_{benzene} = \frac{0.5 \times 2.6 \times 0.17 \times 100}{100 \times 1.7} = 0.13°C = 13 \times 10^{-2}\ °C$$

Q.8 In the electrolytic refining of blister copper, the total number of main impurities, from the following, removed as anode mud is __________.

Pb, Sb, Se, Te, Ru, Ag, Au and Pt

Given **5**
Answer :

Ans: 6

Sol: During the electrolytic refining of copper, the anode mud contains antimony. selenium, tellurium, gold, platinum and silver.

Q.9 The pH of a solution obtained by mixing 50 mL of 1 M HCl and 30 mL of 1 M NaOH is $x \times 10^{-4}$. The value of x is __________. (Nearest integer)
[log 2.5 = 0.3979]

Given **6021**
Answer :

Ans: 6021

Sol: $(M \times V)_{HCl} = 1 \times 50 = 50$

$(M \times V)_{NaOH} = 1 \times 30 = 30$

$$M = \frac{M_1V_1 - M_2V_2}{V} = \frac{50-30}{80} = \frac{20}{80} = 0.25$$

i.e., $[H^+] = 2.5 \times 10^{-1}$

$\therefore pH = -\log[2.5 \times 10^{-1}]$

$= \log 10 - \log 2.5$

$= 1 - 0.3979$

$= 0.6021$

$= 6021 \times 10^{-4}$

Q.10 CH_4 is adsorbed on 1 g charcoal at 0°C following the Freundlich adsorption isotherm. 10.0 mL of CH_4 is adsorbed at 100 mm of Hg, whereas 15.0 mL is adsorbed at 200 mm of Hg. The volume of CH_4 adsorbed at 300 mm of Hg is 10^x mL. The value of x is __________ $\times 10^{-2}$. (Nearest integer)
[Use $\log_{10}2 = 0.3010$, $\log_{10}3 = 0.4771$]

Given **--**
Answer :

Ans: 128

Sol: According to Freundlich adsorption isotherm,

$$\frac{x}{m} = kP^{1/n}$$

$$10/1 = k.(100)^{1/n} \text{ ---- (1)}$$

$$15/1 = k.(200)^{1/n} \text{ ---- (2)}$$

$(2) \div (1) \Rightarrow$

$$\frac{15}{10} = \left(\frac{200}{100}\right)^{1/n}$$

$$\frac{3}{2} = 2^{1/n}$$

$$\log\left(3/2\right) = \frac{1}{n}\log 2$$

$\log 3 - \log 2 = \frac{1}{n}\log 2$

$\frac{1}{n} = \frac{\log 3 - \log 2}{\log 2} = \frac{0.4771 - 0.3010}{0.3010} = 0.585$

Volume of CH_4 adsorbed at 300 mm of Hg

$\frac{V}{1} = k.300^{\frac{1}{n}}$ --- (3)

$(3) \div (1) \Rightarrow$

$\frac{V}{10} = \left(\frac{300}{100}\right)^{1/n}$

$\left(\frac{V}{10}\right) = 3^{1/n} = 3^{0.585}$

$\log\left(\frac{V}{10}\right) = 0.585\ \log 3$

$\log\left(\frac{V}{10}\right) = 0.2794$

$\left(\frac{V}{10}\right) = 10^{0.2791}$

$V = 10 \times 10^{0.2971}$

$V = 10^{1.2971}$ mL = 10^x mL

$\therefore$ x = 1.2791

= 127.91×10^{-2}

PART – C – MATHEMATICS

Section A

Q.1 The sum of the roots of the equation,

$x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$, is :

Options

1. $\log_2 13$
2. $\log_2 11$
3. $\log_2 14$
4. $\log_2 12$

Ans: $\log_2 11$

Sol: $x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$

Using properties of logarithm

$\Rightarrow \log_2(2^{x+1}) - \log_2(3 + 2^x)^2 + \log_2(10 - 2^{-x}) = 0$

$\Rightarrow \log_2\left(\frac{2^{x+1}(10 - 2^{-x})}{(3 + 2^x)^2}\right) = 0$

$\Rightarrow \frac{2(10.2^x - 1)}{(3 + 2^x)^2} = 1$

Arranging

$(2^x)^2 - 14(2^x) + 11 = 0$

Roots are $2^{\alpha}.\&2^{\beta}$

$\therefore 2^{\alpha}.2^{\beta}=11$

$\alpha+\beta=\log_2(11)$

Q.2 The distance of the point $(-1, 2, -2)$ from the line of intersection of the planes $2x+3y+2z=0$ and $x-2y+z=0$ is :

Options

1. $\dfrac{\sqrt{34}}{2}$
2. $\dfrac{\sqrt{42}}{2}$
3. $\dfrac{5}{2}$
4. $\dfrac{1}{\sqrt{2}}$

Ans: $\dfrac{\sqrt{34}}{2}$

Sol: $P_1 : 2x+3y+2z=0$

Normal vector, $\vec{n}_1 = 2\hat{i}+3\hat{j}+2\hat{k}$

$P_2 : x-2y+z=0$

Normal vector, $\vec{n}_2 = \hat{i}-2\hat{j}+\hat{k}$

Direction vector of line L which is line of intersection of P_1 & P_2

$\vec{r}=\vec{n}_1\times\vec{n}_2 = 7\hat{i}+0\hat{j}-7\hat{k}$

DR's of L are (1, 0, –1)

$\Rightarrow$ Equation of L : $\dfrac{x}{1}=\dfrac{y}{0}=\dfrac{z}{-1}=\lambda$

DR's of $\overline{PQ}=(\lambda+1,-2,2-\lambda)$

$\overline{PQ}.\vec{r}=0$

$\Rightarrow(\lambda+1)(1)+(-2)(0)+(2-\lambda)(-1)=0$

$\Rightarrow\lambda=\dfrac{1}{2}\Rightarrow Q=\left(\dfrac{1}{2},0,-\dfrac{1}{2}\right)$

Using distance formula

$\Rightarrow$PQ = $\dfrac{\sqrt{34}}{2}$

Q.3 Let $f : \mathbf{N} \to \mathbf{N}$ be a function such that $f(m+n) = f(m) + f(n)$ for every m, n $\in$ **N**. If $f(6) = 18$, then $f(2) \cdot f(3)$ is equal to :

Options
1. 18
2. 6
3. 36
4. 54

Ans: 54

Sol: $f(m+n) = f(m) + f(n)$ ----- (1)
Put m = 1, n = 1 in (1)
f(2) = f(1) + f(1) = 2f(1)
Put m = 2, n = 1
f(3) = f(2) + f(1) = 3f(1)
Put m = 3, n = 3
f(6) = 2f(3) $\Rightarrow$ f(3) = 9
$\Rightarrow f(1) = 3, f(2) = 6$
$f(2).f(3) = 6 \times 9 = 54$

Q.4 If $\alpha + \beta + \gamma = 2\pi$, then the system of equations

$x + (\cos\gamma)y + (\cos\beta)z = 0$

$(\cos\gamma)x + y + (\cos\alpha)z = 0$

$(\cos\beta)x + (\cos\alpha)y + z = 0$

has :

Options
1. infinitely many solutions
2. a unique solution
3. exactly two solutions
4. no solution

Ans: infinitely many solutions

Sol: $\alpha + \beta + \lambda = 2\pi$

$$\Rightarrow \begin{vmatrix} 1 & \cos\gamma & \cos\beta \\ \cos\gamma & 1 & \cos\alpha \\ \cos\beta & \cos\alpha & 1 \end{vmatrix} = 0$$

$\Rightarrow 1 + 2\cos\alpha.\cos\beta.\cos\gamma - \cos^2\alpha - \cos^2\beta - \cos^2\gamma = 0$

$\Rightarrow \sin^2\gamma - \cos^2\alpha - \cos^2\beta + (\cos(\alpha+\beta) + \cos(\alpha-\beta))\cos\gamma = 0$

$\Rightarrow \sin^2\gamma - \cos^2\alpha - \cos^2\beta + \cos^2\gamma + \cos(\alpha-\beta)\cos\gamma = 0$

$\Rightarrow \sin^2\alpha - \cos^2\beta + \cos(\alpha-\beta)\cos(\alpha+\beta) = 0$

$\Rightarrow \sin^2\alpha - \cos^2\beta + \cos^2\alpha - \sin^2\beta = 0$

Q.5 Let f be any continuous function on [0, 2] and twice differentiable on (0, 2). If $f(0)=0$, $f(1)=1$ and $f(2)=2$, then :

Options

1. $f'(x)=0$ for some $x \in [0, 2]$
2. $f''(x) > 0$ for all $x \in (0, 2)$
3. $f''(x)=0$ for some $x \in (0, 2)$
4. $f''(x)=0$ for all $x \in (0, 2)$

Ans: $f''(x)=0$ for some $x \in (0,2)$

Sol: $f(0)=0$, $f(1)=1$ and $f(2)=2$

Let g(x) = f(x) – x has three roots

By Rolle's theorem g'(x) = f'(x) – 1 has at least two roots

g''(x) = f''(x) = 0 has at least one roots

Q.6 The locus of mid-points of the line segments joining (−3, −5) and the points on the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ is :

Options

1. $9x^2+4y^2+18x+8y+145=0$
2. $36x^2+16y^2+72x+32y+145=0$
3. $36x^2+16y^2+108x+80y+145=0$
4. $36x^2+16y^2+90x+56y+145=0$

Ans: $36x^2 + 16y^2 + 108x + 80y + 145 = 0$

Sol: General points on $\frac{x^2}{4}+\frac{y^2}{9}=1$ is $A(2\cos\theta, 3\sin\theta)$

Given B (–3, –5)

Midpoint C is $\left(\frac{2\cos\theta-3}{2}, \frac{3\sin\theta-5}{2}\right)$

h = $\frac{2\cos\theta-3}{2}$; k = $\frac{3\sin\theta-5}{2}$

Using the identity $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \left(\frac{2h+3}{2}\right)^2 + \left(\frac{2k+5}{3}\right)^2 = 1$

$\Rightarrow 36x^2 + 16y^2 + 108x + 80y + 145 = 0$

Q.7 Negation of the statement $(p \vee r) \Rightarrow (q \vee r)$ is :

Options
1. $p \wedge \sim q \wedge \sim r$
2. $\sim p \wedge q \wedge r$
3. $\sim p \wedge q \wedge \sim r$
4. $p \wedge q \wedge r$

Ans: $p \wedge \sim q \wedge \sim r$

Sol: We know that

$\sim(A \Rightarrow B) = A \wedge \sim B$

$\therefore \sim((p \vee r) \Rightarrow (q \vee r)) = (p \vee r) \wedge (\sim q \wedge \sim r)$

$= ((p \vee r) \wedge (\sim r)) \wedge (\sim q)$

$= p \wedge (\sim r) \wedge (\sim q)$

Q.8 If $\frac{dy}{dx} = \frac{2^x y + 2^y \cdot 2^x}{2^x + 2^{x+y} \log_e 2}$, $y(0) = 0$, then for $y = 1$, the value of x lies in the interval :

Options
1. $(2, 3)$
2. $(1, 2)$
3. $\left(\frac{1}{2}, 1\right]$
4. $\left(0, \frac{1}{2}\right]$

Ans: (1, 2)

Sol: $\frac{dy}{dx} = \frac{2^x\left(y + 2^y\right)}{2^x\left(1 + 2^y \log_e 2\right)}$

$\Rightarrow \int \frac{\left(1 + 2^y\right)\log_e 2}{\left(y + 2^y\right)} dy = \int dx$

$\Rightarrow \log_e |y + 2^y| = x + C$

$x = 0;\ y = 0 \Rightarrow C = 0$

$\Rightarrow x = \log_e |y + 2^y|$

$\Rightarrow$ at $y = 1$, $x = \log_e 3$

$\because 3 \in \left(e, e^2\right) \Rightarrow x \in (1, 2)$

Q.9

If $[x]$ is the greatest integer $\le x$, then $\pi^2 \int_0^2 \left(\sin\frac{\pi x}{2}\right)(x-[x])^{[x]}\,dx$ is equal to :

Options
1. $4(\pi-1)$
2. $2(\pi-1)$
3. $2(\pi+1)$
4. $4(\pi+1)$

Ans: $4(\pi-1)$

Sol: $\pi^2\left[\int_0^1 \sin\frac{\pi x}{2}dx + \int_1^2 \sin\frac{\pi x}{2}(x-1)dx\right]$

$= \pi^2\left[-\frac{2}{\pi}\left(\cos\frac{\pi x}{2}\right) + \left((x-1)\left(-\frac{2}{\pi}\cos\frac{\pi x}{2}\right)\right)_1^2 - \int_1^2 -\frac{2}{\pi}\cos\frac{\pi x}{2}dx\right]$

$= \pi^2\left[0+\frac{2}{\pi}+\frac{2}{\pi}+\frac{2}{\pi}.\frac{2}{\pi}\left(\sin\frac{\pi x}{2}\right)_1^2\right] = 4(\pi-1)$

Q.10

Let $a_1, a_2, a_3, \ldots$ be an A.P. If $\frac{a_1+a_2+\ldots+a_{10}}{a_1+a_2+\ldots+a_p} = \frac{100}{p^2}$, $p \ne 10$, then $\frac{a_{11}}{a_{10}}$ is equal to :

Options
1. $\frac{21}{19}$
2. $\frac{19}{21}$
3. $\frac{100}{121}$
4. $\frac{121}{100}$

Ans: $\frac{21}{19}$

Sol: $$\frac{\frac{10}{2}(2a+9d)}{\frac{p}{2}(2a+(p-1)d)} = \frac{100}{p^2}$$

$$(2a+9d)p = 10(2a+(p-1)d)$$

$$9dp = 20a - 2pa + 10d(p-1)$$

$$9p = (20-2p)\frac{a}{d} + 10d(p-1)$$

$$\frac{a}{d} = \frac{(10-p)}{2(10-p)} = \frac{1}{2}$$

$$\therefore \frac{a_{11}}{a_{10}} = \frac{a+10d}{a+9d} = \frac{\frac{1}{2}+10}{\frac{1}{2}+9} = \frac{21}{19}$$

Q.11 The mean and variance of 7 observations are 8 and 16 respetively. If two observations are 6 and 8, then the variance of the remaining 5 observations is :

Options

1. $\frac{536}{25}$

2. $\frac{112}{5}$

3. $\frac{92}{5}$

4. $\frac{134}{5}$

Ans: $\frac{536}{25}$

Sol: $$\frac{x_1 + x_2 + ... + x_5 + 14}{7} = 18$$

$$\Rightarrow \sum_{i=1}^{5} x_i = 42$$

And $$\frac{x_1^2 + x_2^2 + ...x_5^2 + 8^2 + 6^2}{7} - 64 = 16$$

$$\Rightarrow \sum_{i=1}^{5} x_i^2 = 560 - 100 = 460$$

So variance of $x_1, x_2, ...x_3 = \frac{460}{5} - \left(\frac{42}{5}\right)^2 = \frac{2300 - 1764}{25} = \frac{536}{25}$

Q.12

If $\alpha = \lim_{x\to\pi/4} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$ and $\beta = \lim_{x\to 0} (\cos x)^{\cot x}$ are the roots of the equation, $ax^2 + bx - 4 = 0$, then the ordered pair (a, b) is :

Options

1. $(-1, 3)$
2. $(-1, -3)$
3. $(1, -3)$
4. $(1, 3)$

Ans: (1, 3)

Sol: $\alpha = \lim_{x\to\frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$; $\frac{0}{0}$ form

Using L hospital's rule

$\alpha = \lim_{x\to\frac{\pi}{4}} \frac{3\tan^2 x \sec^2 x - \sec^2 x}{-\sin\left(x + \frac{\pi}{4}\right)}$

$\Rightarrow \alpha = -4$

$\beta = \lim_{x\to 0} (\cos x)^{\cot x} \Rightarrow \log\beta = \lim_{x\to 0} (\cot x)\log\cos x = \lim_{x\to 0} \frac{\log \cos x}{\tan x} = 0$

$\beta = 1$

$\alpha = -4$; $\beta = 1$

If $ax^2 + bx - 4 = 0$ and $a + b - 4 = 0$

$\Rightarrow$a = 1 and b = 3

Q.13

If z is a complex number such that $\frac{z-i}{z-1}$ is purely imaginary, then the minimum value of $|z - (3 + 3i)|$ is :

Options

1. $6\sqrt{2}$
2. $2\sqrt{2}$
3. $2\sqrt{2} - 1$
4. $3\sqrt{2}$

Ans: $2\sqrt{2}$

Sol: $\frac{z-i}{z-1}$ is purely Imaginary number

Let z = x+ iy

$\therefore \frac{x + i(y-1)}{(x-1) + iy} \times \frac{(x-1) - iy}{(x-1) - iy}$

$\Rightarrow \dfrac{x(x-1)+y(y-1)+i(-y-x+1)}{(x-1)^2+y^2}$ is purely Imaginary number

$\Rightarrow x(x-1)+y(y-1)=0$

$\Rightarrow \left(x-\dfrac{1}{2}\right)^2+\left(y-\dfrac{1}{2}\right)^2=\dfrac{1}{2}$

$\therefore |z-(3+3i)|_{min}$ = distance from the point to the centre − radius = $\dfrac{5}{\sqrt{2}}-\dfrac{1}{\sqrt{2}}=2\sqrt{2}$

Q.14 The domain of the function

$f(x)=\sin^{-1}\left(\dfrac{3x^2+x-1}{(x-1)^2}\right)+\cos^{-1}\left(\dfrac{x-1}{x+1}\right)$ is :

Options

1. $\left[\dfrac{1}{4},\dfrac{1}{2}\right]\cup\{0\}$
2. $\left[0,\dfrac{1}{2}\right]$
3. $[-2,0]\cup\left[\dfrac{1}{4},\dfrac{1}{2}\right]$
4. $\left[0,\dfrac{1}{4}\right]$

Ans: $\left[\dfrac{1}{4},\dfrac{1}{2}\right]\cup\{0\}$

Sol: $f(x)=\sin^{-1}\left(\dfrac{3x^2+x-1}{(x-1)^2}\right)+\cos^{-1}\left(\dfrac{x-1}{x+1}\right)$

$-1\le\dfrac{x-1}{x+1}\le1\Rightarrow 0\le x<\infty$ ...(1)

$-1\le\dfrac{3x^2+x-1}{(x-1)^2}\le1\Rightarrow x\in\left[-\dfrac{1}{4},\dfrac{1}{2}\right]\cup\{0\}$ ..(2)

From (1) and (2)

$\Rightarrow$ Domain = $\left[\dfrac{1}{4},\dfrac{1}{2}\right]\cup\{0\}$

Q.15 Let $\vec{a},\vec{b},\vec{c}$ be three vectors mutually perpendicular to each other and have same magnitude. If a vector $\vec{r}$ satisfies

$\vec{a}\times\{(\vec{r}-\vec{b})\times\vec{a}\}+\vec{b}\times\{(\vec{r}-\vec{c})\times\vec{b}\}+\vec{c}\times\{(\vec{r}-\vec{a})\times\vec{c}\}=\vec{0}$, then $\vec{r}$ is equal to :

Options

1. $\dfrac{1}{2}\left(\vec{a}+\vec{b}+\vec{c}\right)$
2. $\dfrac{1}{3}\left(\vec{a}+\vec{b}+\vec{c}\right)$
3. $\dfrac{1}{2}\left(\vec{a}+\vec{b}+2\vec{c}\right)$
4. $\dfrac{1}{3}\left(2\vec{a}+\vec{b}-\vec{c}\right)$

Ans: $\frac{1}{2}\left(\vec{a}+\vec{b}+\vec{c}\right)$

Sol: Suppose $r = x\vec{a} + y\vec{b} + z\vec{c}$ and $|\vec{a}| = |\vec{b}| = |\vec{c}| = k$

$$\vec{a}\times\{(\vec{r}-\vec{b})\times\vec{a}\}+\vec{b}\times\{(\vec{r}-\vec{c})\times\vec{b}\}+\vec{c}\times\{(\vec{r}-\vec{a})\times\vec{c}\}=\vec{0}$$

$$\Rightarrow k^2(\vec{r}-\vec{b})-k^2\times\vec{a}+k^2(\vec{r}-\vec{c})-k^2y\vec{b}+k^2(\vec{r}-\vec{a})-k^2z\vec{c}$$

$$\Rightarrow 2\vec{r}-(\vec{a}+\vec{b}+\vec{c})=\vec{0}$$

$$\Rightarrow \vec{r}=\frac{\vec{a}+\vec{b}+\vec{c}}{2}$$

Q.16

An angle of intersection of the curves, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $x^2+y^2=ab$, $a > b$, is :

Options

1. $\tan^{-1}\left(2\sqrt{ab}\right)$
2. $\tan^{-1}\left(\frac{a+b}{\sqrt{ab}}\right)$
3. $\tan^{-1}\left(\frac{a-b}{2\sqrt{ab}}\right)$
4. $\tan^{-1}\left(\frac{a-b}{\sqrt{ab}}\right)$

Ans: $\tan^{-1}\left(\frac{a-b}{\sqrt{ab}}\right)$

Sol: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, x^2+y^2=ab$

For the first curve,

$$\frac{2x_1}{a^2}+\frac{2y_1y'}{b^2}=0$$

$$\Rightarrow y_1'=\frac{-x_1}{a^2}\frac{b^2}{y_1} \qquad \ldots(1)$$

For the second curve,

$$2x_1+2y_1y'=0$$

$$\Rightarrow y_2'=\frac{-x_1}{y_1} \qquad \ldots(2)$$

Here $(x_1\ y_1)$ is point of intersection of both curves

$$\therefore \tan\theta=\left|\frac{y_1'-y_2'}{1+y_1'y_2'}\right|=\left|\frac{\frac{-x_1b^2}{a^2y_1}+\frac{x_1}{y_1}}{1+\frac{x_1^2b^2}{a^2y_1^2}{}^{2}}\right|$$

$$\tan\theta = \left|\frac{\frac{-x_1b^2}{a^2y_1} + \frac{x_1}{y_1}}{1 + \frac{x_1^2b^2}{a^2y_1^2}^{2}}\right| = \left|\frac{-b^2x_1y_1 + a^2x_1y_1}{a^2y_1^2 + b^2x_1^2}\right|$$

$$\tan\theta = \left|\frac{a-b}{\sqrt{ab}}\right|$$

Q.17

The number of solutions of the equation $32^{\tan^2 x} + 32^{\sec^2 x} = 81,\ 0 \le x \le \frac{\pi}{4}$ is :

Options
1. 3
2. 0
3. 1
4. 2

Ans: 1

Sol: $(32)^{\tan^2 x} + (32)^{\sec^2 x} = 81$

$\Rightarrow (32)^{\tan^2 x} + (32)^{1+\tan^2 x} = 81$

$\Rightarrow (32)^{\tan^2 x}[1+32] = 81$

$\Rightarrow (32)^{\tan^2 x} = \frac{81}{33}$

In interval $\left[0.\frac{\pi}{4}\right]$ only one solution

Q.18 Let $S = \{1, 2, 3, 4, 5, 6\}$. Then the probability that a randomly chosen onto function g from S to S satisfies $g(3) = 2g(1)$ is :

Options
1. $\frac{1}{5}$
2. $\frac{1}{30}$
3. $\frac{1}{15}$
4. $\frac{1}{10}$

Ans: $\frac{1}{10}$

Sol: Number of onto
functions in this condition = 3 × 4!
Total number of onto functions = 6!
Required probability = $\frac{3 \times 4!}{6!} = \frac{1}{10}$

Q.19 Let A be the set of all points (α, β) such that the area of triangle formed by the points (5, 6), (3, 2) and (α, β) is 12 square units. Then the least possible length of a line segment joining the origin to a point in A, is :

Options

1. $\frac{4}{\sqrt{5}}$
2. $\frac{8}{\sqrt{5}}$
3. $\frac{12}{\sqrt{5}}$
4. $\frac{16}{\sqrt{5}}$

Ans: $\frac{8}{\sqrt{5}}$

Sol: Area = $\left|\frac{1}{2}\begin{vmatrix} 3 & 2 & 1 \\ 5 & 6 & 1 \\ \alpha & \beta & 1 \end{vmatrix}\right| = 12$

$d = \left|\frac{0-0-16}{\sqrt{5}}\right| = \frac{16}{\sqrt{5}}$

Perpendicular distance from (0, 0) is

$\left|\frac{0-0+8}{\sqrt{5}}\right| = \frac{8}{\sqrt{5}}$

Q.20

If $y\frac{dy}{dx}=x\left[\frac{y^2}{x^2}+\frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$, $x>0$, $\phi>0$, and $y(1)=-1$, then $\phi\left(\frac{y^2}{4}\right)$ is equal to :

Options
1. $4\phi(2)$
2. $\phi(1)$
3. $4\phi(1)$
4. $2\phi(1)$

Ans: $4\phi(1)$

Sol: Let, y = vx

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$\therefore vx\left(v+x\frac{dv}{dx}\right)=x\left(v^2+\frac{\phi(v^2)}{\phi'(v^2)}\right)$$

$$v^2+xv\frac{dv}{dx}=v^2+\frac{\phi(v^2)}{\phi'(v^2)}$$

$$\int\frac{v\phi'(v^2)}{\phi(v^2)}dv=\int\frac{dx}{x}$$

Let $\phi(v^2)=p$

$$\therefore \phi'(v^2)2vdv=dp$$

$$\Rightarrow\int\frac{dy}{2p}=\int\frac{dx}{x}$$

$$\frac{1}{2}\ln\phi(t^2)=\ln x+\ln c$$

$$\phi(t^2)=x^2k$$

$$\phi\left(\frac{y^2}{x^2}\right)=kx^2,\phi(1)=k$$

$$\phi\left(\frac{y^2}{4}\right)=4\phi(1)$$

Q.1 If the coefficient of a^7b^8 in the expansion of $(a+2b+4ab)^{10}$ is $K\cdot 2^{16}$, then K is equal to __________.

Given -- Answer :

Ans: 315

Sol: $\frac{10!}{p!q!r!}a^{p}(2b)^{q}.(4ab)^{r}$

$\frac{10!}{p!q!r!}a^{p+r}.b^{q+r}.2^{q}.4^{r}$

$p+q+r=10$ ...(1)

$p+r=7$ ...(2)

$q+r=8$ ...(3)

(1) + (3) – (1)$\Rightarrow r=5$, $p=2, q=3$

So coefficients = $\frac{10!}{2!3!5!}2^{3}.2^{10}=\frac{10\times9\times8\times7\times6\times5}{2\times3\times2\times5!}\times2^{13}=315\times2^{16}$

$\Rightarrow$ k =315

Q.2

If the line $y=mx$ bisects the area enclosed by the lines $x=0$, $y=0$, $x=\frac{3}{2}$ and the curve $y=1+4x-x^2$, then 12 m is equal to _________.

Given --
Answer :

Ans: 26

Sol:

Total area = $\int_0^{3/2}(1+4x-x^2)dx = \left. x+2x^2-\frac{x^3}{3}\right|_0^{3/2} = \frac{1}{2}.\frac{3}{2}.\frac{3}{2}m$

$\Rightarrow 3m=\frac{13}{2}\Rightarrow 12m=26$

Q.3

If $S=\frac{7}{5}+\frac{9}{5^2}+\frac{13}{5^3}+\frac{19}{5^4}+...$, then 160 S is equal to __________.

Given --
Answer :

Ans: 305

Sol: $S=\frac{7}{5}+\frac{9}{5^2}+\frac{13}{5^3}+\frac{19}{5^4}+....$ ----- (1)

$\frac{1}{5}S=\frac{7}{5^2}+\frac{9}{5^3}+\frac{13}{5^4}+...$ ----- (2)

(2) – (1)

$\frac{4}{5}S=\frac{7}{5}+\frac{2}{5^2}+\frac{4}{5^3}+\frac{6}{5^4}+....$

$S=\frac{7}{4}+\frac{1}{10}\left(1+\frac{2}{5}+\frac{3}{5^2}+.....\right)$

$S=\frac{7}{4}+\frac{1}{10}\left(1-\frac{1}{5}\right)^{-2}=\frac{61}{32}$

$\Rightarrow 160S = 5\times61=305$

Q.4 Let B be the centre of the circle $x^2+y^2-2x+4y+1=0$. Let the tangents at two points P and Q on the circle intersect at the point A(3, 1). Then $8\cdot\left(\frac{\text{area } \Delta APQ}{\text{area } \Delta BPQ}\right)$ is equal to ________.

Given **1**
Answer :

Ans: 18

Sol: $\tan\theta = \frac{3}{2}$

$$\frac{\text{Area } \Delta APQ}{\text{Area } \Delta BPQ} = \frac{AR}{RB} = \frac{3\sin\theta}{2\cos\theta} = \frac{9}{4}$$

$$8\left(\frac{\text{Area } \Delta APQ}{\text{Area } \Delta BPQ}\right) = 18$$

Q.5 The number of elements in the set

$\left\{A = \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in \{-1, 0, 1\} \text{ and } (I-A)^3 = I - A^3\right\}$, where I is 2×2 identity matrix, is ________.

Given **--**
Answer :

Ans: 8

Sol: $(I-A)^3 = I^3 - A - 3A(I-A) = I - A^3$

$\Rightarrow 3A(I-A) = 0$ or $A^2 = A$

$\Rightarrow \begin{bmatrix} a^2 & ab+bd \\ 0 & d^2 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$

$\Rightarrow a^2 = a, b(a+d-1) = 0, d^2 = d$

If $b \neq 0, a + d = 1 \Rightarrow$ 4 ways

If b = 0 a = 0, 1 & d = 0, 1 $\Rightarrow$ 4 ways

$\Rightarrow$Total 8 matrices

Q.6 A tangent line L is drawn at the point (2, −4) on the parabola $y^2=8x$. If the line L is also tangent to the circle $x^2+y^2=a$, then 'a' is equal to ________.

Given **--**
Answer :

Ans: 2

Sol: Tangent of $y^2 = 4\times 2x$ is $y = mx + \frac{2}{m}$

Substituting the point (2, –4)

$-4 = 2m + \frac{2}{m}$

$\Rightarrow m + \frac{1}{m} = -2 \Rightarrow m = -1$

∴ Tangent is y = –x – 2

$\Rightarrow$x + y + 2 = 0 …(1)

(1) is also tangent to $x^2 + y^2 = a$

So, $\frac{2}{\sqrt{2}} = \sqrt{a} \Rightarrow \sqrt{a} = \sqrt{2}$

$\Rightarrow$a = 2

Q.7 Suppose the line $\frac{x-2}{\alpha} = \frac{y-2}{-5} = \frac{z+2}{2}$ lies on the plane $x + 3y - 2z + \beta = 0$. Then $(\alpha + \beta)$ is equal to ________.

Given **7**
Answer :

Ans: 7

Sol: Substituting (2, 2, –2)

$2 + 3 \times 2 - 2(-2) + \beta = 0$

$\Rightarrow \beta = -12$

Also a× 1 –5×3+ 2 × –2 = 0

$\Rightarrow \alpha - 15 - 4 = 0 \Rightarrow \alpha = 19$

$\therefore \alpha + \beta = 19 - 12 = 7$

Q.8 If $\int \frac{\sin x}{\sin^3 x + \cos^3 x} dx = \alpha \log_e |1 + \tan x| + \beta \log_e |1 - \tan x + \tan^2 x| + \gamma \tan^{-1}\left(\frac{2\tan x - 1}{\sqrt{3}}\right) + C$, when C is constant of integration, then the value of $18(\alpha + \beta + \gamma^2)$ is ________.

Given --
Answer :

Ans: 3

Sol: $I = \int \frac{\tan x . \sec^2 x}{(\tan x + 1)(1 + \tan^2 x - \tan x)} dx$ (dividing by $\cos^3 x$)

Let tanx = t $\Rightarrow \sec^2 x dx = dt$

$I = \int \frac{t}{(t+1)(t^2 - t + 1)} dt$

$= \int \left(\frac{A}{t+1} + \frac{B(2t-1)}{t^2 - t + 1} + \frac{C}{t^2 - t + 1} \right) dx$

$\Rightarrow A(t^2 - t + 1) + B(2t-1)(t^2 - t + 1) + C(t+1) = t$

$\Rightarrow t^2(A + 2B) + (-A + B + C) + A - B + C = 1$

$\therefore A + 2B = 0$ ..(1)

$-A + B + C = 1$ ...(2)

$A - B + C = 0$ ...(3)

(2) + (3) $C = \frac{1}{2} \Rightarrow A - B = -\frac{1}{2}$

A + 2B = 0

From (1) and (2)

3B = $\frac{1}{2} \Rightarrow$B = $\frac{1}{6}$

A = $-\frac{1}{3}$

$I = -\frac{1}{3}\int \frac{dt}{1+t} + \frac{1}{6}\int \frac{2t-1}{t^2 - t + 1} dt + \frac{1}{2}\int \frac{dt}{t^2 - t + 1}$

$$= -\frac{1}{3}\ln|(1+\tan x)| + \frac{1}{6}\ln|\tan^2 x - \tan x + 1| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x - 1}{\sqrt{3}}\right) + C$$

$$\alpha = -\frac{1}{3}, \beta = \frac{1}{6}, \gamma = \frac{1}{\sqrt{3}}$$

$$18(\alpha + \beta + \gamma^2) = 18\left(-\frac{1}{3} + \frac{1}{6} + \frac{1}{3}\right) = 3$$

Q.9 The number of 4-digit numbers which are neither multiple of 7 nor multiple of 3 is __________.

Given Answer : **5143**

Ans: 5143

Sol: 4 – digit numbers divisible by 3
1002, 1005, ……, 9999.
1002 + (n – 1)3 = 9999
$\Rightarrow$n = 3000
4 – digit numbers divisible by 7
1001, 1008, ….., 9996
1001 + (n – 1)7 = 9996
$\Rightarrow$n = 1286
A $\cap$B = 1008, 1029,….., 9996
9996 = 1008 + (n – 1)21
$\Rightarrow$n = 429
So, no divisible by either 3 or 7 = 3000 + 1286 – 429 = 3857
total 4-digits numbers = 9000
required numbers = 9000 – 3857 = 5143

Q.10 Let $f(x)$ be a cubic polynomial with $f(1) = -10$, $f(-1) = 6$, and has a local minima at $x = 1$, and $f'(x)$ has a local minima at $x = -1$. Then $f(3)$ is equal to __________.

Given Answer : **22**

Ans: 22

Sol: $f'(x) = a(x - 1)(x + 3)$
$f''(x) = 6a(x + 1)$
$f'(x) = 3a(x + 1)^2 + b$
$f'(1) = 0 \Rightarrow b = -12a$
$f(x) = a(x + 1)^3 - 12ax + c = (x + 1)^3 - 12x - 6$
$f(3) = 22$