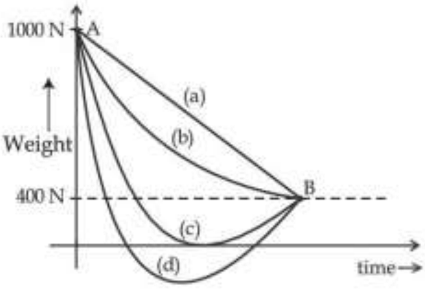
SOLUTIONS & ANSWERS FOR JEE MAINS-2021
20th July Shift 1

[PHYSICS, CHEMISTRY & MATHEMATICS]

PART – A – PHYSICS

Section A

Q.1 A person whose mass is 100 kg travels from Earth to Mars in a spaceship. Neglect all other objects in sky and take acceleration due to gravity on the surface of the Earth and Mars as 10 m/s^2 and 4 m/s^2 respectively. Identify from the below figures, the curve that fits best for the weight of the passenger as a function of time.



Options
1. (d)
2. (a)
3. (b)
4. (c)

Ans: (c)

Sol: $\vec{g}$ (at any point) = $\vec{g}$ (earth) + $\vec{g}$ (mars)

Since distance is large $|\vec{g}|$ = | g (earth) | = 10

As we move away from earth it decreases to zero at a point where $\vec{g}$ (earth) + $\vec{g}$ (mars) = 0. Then it increases at mars surface to $|\vec{g}| = |\vec{g}$ (mass)| = 4

Aliter → at neutral point, g = 0

Q.2 The radiation corresponding to $3 \to 2$ transition of a hydrogen atom falls on a gold surface to generate photoelectrons. These electrons are passed through a magnetic field of 5×10^{-4} T. Assume that the radius of the largest circular path followed by these electrons is 7 mm, the work function of the metal is : (Mass of electron $= 9.1 \times 10^{-31}$ kg)

Options
1. 1.36 eV
2. 1.88 eV
3. 0.16 eV
4. 0.82 eV

Ans: 0.82 eV

Sol: $E_p = 13.6\left[\frac{1}{R_1^2} - \frac{1}{R_2^2}\right]eV = 13.6\left[\frac{1}{4} - \frac{1}{9}\right] = 1.89\,eV$

For gold plate, $\phi = E_p - KE_{max}$

Where $KE_{max} = \frac{1}{2}mV^2$

V = ?

$$V = \frac{BqR}{m} = \frac{5\times10^{-4}\times1.6\times10^{-19}\times7\times10^{-3}}{9.1\times10^{-31}} = 6.15\times10^{5}$$

$$KE_{max} = \frac{1}{2}mV^2 = \frac{1}{2}\times\frac{9.1\times10^{-31}\times\left(6.15\times10^{5}\right)^2}{1.6\times10^{-19}}\ \text{eV} = 1.075\ \text{eV}$$

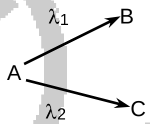
ϕ = 1.89 – 1.075 = 0.82 eV

Q.3 A radioactive material decays by simultaneous emissions of two particles with half lives of 1400 years and 700 years respectively. What will be the time after which one third of the material remains ? (Take ln 3 = 1.1)

Options
1. 340 years
2. 740 years
3. 700 years
4. 1110 years

Ans: 740 years

Sol: Given, $\lambda_1 = \frac{\ell n2}{700}$ / year



$\lambda_2 = \frac{\ell n2}{1400}$ / year

$\lambda_{total} = \lambda_1 + \lambda_2 = \frac{\ell n2}{700} + \frac{\ell n2}{1400}$

$= \ell n2\left[\frac{1}{700} + \frac{1}{1400}\right] = \frac{3\ell n2}{1400}$ / year

$\frac{N_0}{3} = N_0 e^{-\lambda_{total}t}$

$\ell n\frac{1}{3} = -\lambda_{total}t$

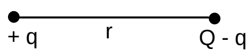
$1.1 = \frac{3\times0.693t}{1400} \Rightarrow t = 740\ \text{years}$

Q.4 A certain charge Q is divided into two parts q and $(Q-q)$. How should the charges Q and q be divided so that q and $(Q-q)$ placed at a certain distance apart experience maximum electrostatic repulsion ?

Options
1. $Q = 2q$
2. $Q = 3q$
3. $Q = 4q$
4. $Q = \frac{q}{2}$

Ans: Q = 2q

Sol:



$$F = \frac{Kq_1q_2}{r^2} = \frac{K(q)(Q-q)}{r^2}$$

$$\frac{dF}{dq} = 0$$ for maximum electrostatic repulsion

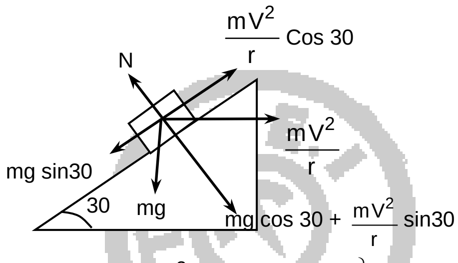
$\Rightarrow Q - 2q = 0 \Rightarrow Q = 2q$

Q.5 The normal reaction 'N' for a vehicle of 800 kg mass, negotiating a turn on a 30° banked road at maximum possible speed without skidding is __________ $\times 10^3$ kg m/s^2.
[Given $\cos 30° = 0.87$, $\mu_s = 0.2$]

Options
1. 10.2
2. 6.96
3. 7.2
4. 12.4

Ans: 10.2

Sol:



$$\left.\begin{aligned} N &= mg\cos 30 + \frac{mV^2}{R}\sin 30 \\ N - mg\cos 30 &= \frac{mV^2}{R}\sin 30 \;----(1) \end{aligned}\right\} \text{Perpendicular to the inclined plane}$$

$$mg\sin 30 + \mu_s N = \frac{mV^2}{r}\cos 30 \;--(2) \rightarrow \text{along inclined plane}$$

$$\frac{(1)}{(2)} \Rightarrow \frac{N - mg\cos 30}{mg\sin 30 + \mu_s N} = \tan 30$$

$\Rightarrow N = 10.2 \times 10^3$ kg m/s^2

Q.6 The value of tension in a long thin metal wire has been changed from T_1 to T_2. The lengths of the metal wire at two different values of tension T_1 and T_2 are l_1 and l_2 respectively. The actual length of the metal wire is :

Options
1. $\sqrt{T_1 T_2 l_1 l_2}$
2. $\dfrac{T_1 l_1 - T_2 l_2}{T_1 - T_2}$
3. $\dfrac{l_1 + l_2}{2}$
4. $\dfrac{T_1 l_2 - T_2 l_1}{T_1 - T_2}$

Ans: $\dfrac{T_1\ell_2 - T_2\ell_1}{T_1 - T_2}$

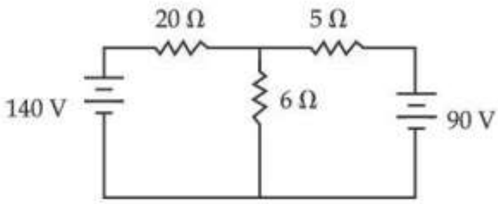
Sol: $Y = \frac{FL}{A\Delta L} \Rightarrow \frac{T_0\ell_0}{A(\ell_1 - \ell_0)} = \frac{T_2\ell_0}{A(\ell_2 - \ell_0)}$

$\Rightarrow T_2\ell_1 - T_2\ell_0 = T_1\ell_2 - T_1\ell_0$

$(T_1 - T_2)\ \ell_0 = T_1\ell_2 - T_2\ell_1$

$\Rightarrow \ell_0 = \frac{T_1\ell_2 - T_2\ell_1}{T_1 - T_2}$

Q.7

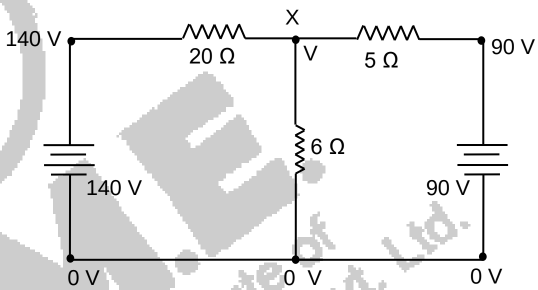


The value of current in the 6 Ω resistance is :

Options
1. 10 A
2. 8 A
3. 6 A
4. 4 A

Ans: 10 A

Sol: Applying KCL at point X



$\frac{V-0}{6} + \frac{V-90}{5} + \frac{V-140}{20} = 0$

$\Rightarrow 10V + 12V - 1080 + 3V - 420 = 0$

$\Rightarrow V = 60\ V$

$\therefore$ current in 6 Ω = $\frac{V-0}{6} = 10\ A$

Q.8

If $\vec{A}$ and $\vec{B}$ are two vectors satisfying the relation $\vec{A} \cdot \vec{B} = |\vec{A} \times \vec{B}|$. Then the value of $|\vec{A} - \vec{B}|$ will be :

Options
1. $\sqrt{A^2 + B^2 - \sqrt{2}AB}$
2. $\sqrt{A^2 + B^2 + 2AB}$
3. $\sqrt{A^2 + B^2 + \sqrt{2}AB}$
4. $\sqrt{A^2 + B^2}$

Ans: $\sqrt{A^2 + B^2 - \sqrt{2}AB}$

Sol: $\vec{A} \cdot \vec{B} = |\vec{A} \times \vec{B}|$

$AB\cos\theta = AB\sin\theta$

$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos 45} = \sqrt{A^2 + B^2 - \sqrt{2}AB}$

Q.9 The amount of heat needed to raise the temperature of 4 moles of a rigid diatomic gas from 0°C to 50°C when no work is done is ________. (R is the universal gas constant)

Options
1. 750 R
2. 175 R
3. 500 R
4. 250 R

Ans: 500 R

Sol: n = 4, ΔT= 50 K, $C_V = \frac{5R}{2}$, ΔW =0

$\Delta Q = \Delta U + \Delta W$

$\therefore \Delta Q = \Delta U = n\, e_V\, \Delta T = 4 \times \frac{5R}{2} \times 50 = 500$ R

Q.10 AC voltage $V(t) = 20 \sin\omega t$ of frequency 50 Hz is applied to a parallel plate capacitor. The separation between the plates is 2 mm and the area is 1 m^2. The amplitude of the oscillating displacement current for the applied AC voltage is ________.

[Take $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m]

Options
1. 55.58 μA
2. 83.37 μA
3. 21.14 μA
4. 27.79 μA

Ans: 27.79 μA

Sol: $C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 \times 1}{2 \times 10^{-3}}$ F

$X_C = \frac{1}{Cw} = \frac{2 \times 10^{-3}}{2 \times 50\pi \times \varepsilon_0} = \frac{2 \times 10^{-3}}{25 \times 4\pi\varepsilon_0}\,\Omega$

$= \frac{2 \times 10^{-3}}{25} \times 9 \times 10^9 = \frac{18}{25} \times 10^6 \Omega$

$i_0 = \frac{V_0}{X_C} = \frac{20 \times 25}{18} \times 10^{-6} = 27.47\ \mu A$

Aliter

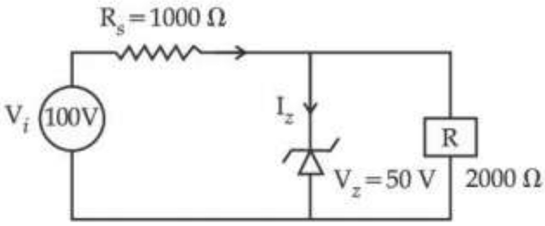
$I_d = \varepsilon_0 \frac{d\phi_e}{dt}$

$\phi_e = \frac{V(t)A}{d} = \frac{20 \sin 100\pi t \times 1}{2 \times 10^{-3}} = 10^4 \sin 100\,\pi t$

$I_d = \varepsilon_0 \frac{d}{dt}\left(10^4 \sin 100\pi t\right) = 8.85 \times 10^{-12} \times 10^4 \times 100\,\pi \cos 100\,\pi t$

$= 27.79 \cos 100\,\pi t$

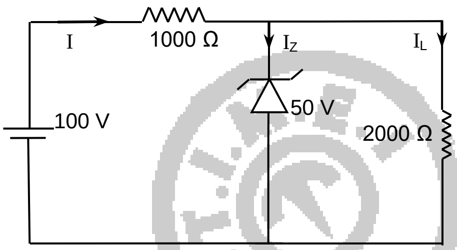
Q.11 For the circuit shown below, calculate the value of I_z :



Options
1. 0.15 A
2. 25 mA
3. 0.1 A
4. 0.05 A

Ans: 25 mA

Sol:



$$I = \frac{100-50}{1000} = 50\,mA$$

$$I_L = \frac{50}{2000} = 0.025\,mA$$

$I = I_Z + I_L \Rightarrow I_Z = I - I_L$, 50 mA – 25 mA = 25 mA

Q.12 A butterfly is flying with a velocity $4\sqrt{2}$ m/s in North-East direction. Wind is slowly blowing at 1 m/s from North to South. The resultant displacement of the butterfly in 3 seconds is :

Options
1. 3 m
2. 20 m
3. $12\sqrt{2}$ m
4. 15 m

Ans: 15 m

Sol:

$$\vec{V}_{BW} = 4\sqrt{2}\cos 45\hat{i} + 4\sqrt{2}\sin 45\hat{j}$$

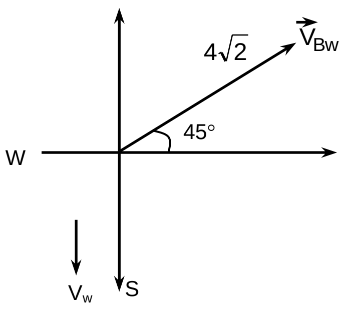
$$= 4\hat{i} + 4\hat{j}$$

$$\vec{V}_W = -\hat{j}$$

$$\vec{V}_B = \vec{V}_{BW} + \vec{V}_W = 4\hat{i} + 3\hat{j}$$

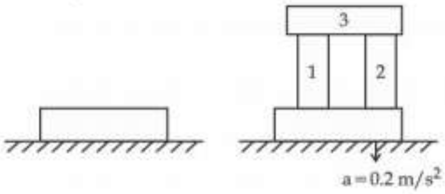
$$\vec{S}_B = \vec{V}_B \times t = (4\hat{i} + 3\hat{j}) \times 3 = 12\hat{i} + 9\hat{j}$$

$$|\vec{S}_B| = \sqrt{(12)^2 + (9)^2} = 15\,m$$



Q.13 A steel block of 10 kg rests on a horizontal floor as shown. When three iron cylinders are placed on it as shown, the block and cylinders go down with an acceleration 0.2 m/s^2. The normal reaction R′ by the floor if mass of the iron cylinders are equal and of 20 kg each, is ___________ N.

[Take $g = 10 \text{ m/s}^2$ and $\mu_s = 0.2$]



Options
1. 716
2. 686
3. 714
4. 684

Ans: 686

Sol: $\sum Fy = \mu a$

mg – N = μa

70g – N = 70 × 0.2

N = 70 [g – 0.2] = 70 × 9.8 = 686 N

Q.14 The entropy of any system is given by

$$S = \alpha^2 \beta \ln\left[\frac{\mu k R}{J\beta^2} + 3\right]$$

where α and β are the constants. μ, J, k and R are no. of moles, mechanical equivalent of heat, Boltzmann constant and gas constant respectively. [Take $S = \frac{dQ}{T}$]

Choose the incorrect option from the following :

Options
1. α and k have the same dimensions.
2. S and α have different dimensions.
3. S, β, k and μR have the same dimensions.
4. α and J have the same dimensions.

Ans: α and J have the same dimensions

Sol: $[S] = \left[\frac{Q}{\Delta T}\right] = ML^2T^{-2}K^{-1}$

$[K] = [S] = ML^2T^{-2}K^{-1}$

$[R] = \left[\frac{\text{Energy}}{nT}\right] = ML^2T^{-2}mol^{-1}K^{-1}$

$[J] = M^0L^0T^0$

$[\mu KR] = [J\beta^2]$

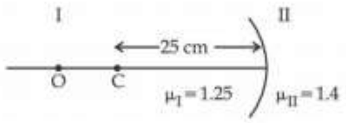
$\therefore [\beta]^2 = [\mu KR] = (mol)\ \frac{ML^2T^2}{K} \times \frac{ML^2T^{-2}}{molK}$

$[\beta] = ML^2T^{-2}K^{-1}$

$[\alpha^2] = \left[\frac{S}{\beta}\right] = \frac{ML^2T^{-2}}{K \times ML^2T^{-2}K^{-1}}$

$[\alpha] = M^0L^0T^0$

Q.15 Region I and II are separated by a spherical surface of radius 25 cm. An object is kept in region I at a distance of 40 cm from the surface. The distance of the image from the surface is :



Options
1. 9.52 cm
2. 18.23 cm
3. 55.44 cm
4. 37.58 cm

Ans: 37.58

Sol: $\dfrac{n_2}{V} - \dfrac{n_1}{U} = \dfrac{n_2 - n_1}{R}$

$\Rightarrow \dfrac{1.4}{V} - \dfrac{1.25}{-40} = \dfrac{1.4 - 1.25}{-25}$

$\Rightarrow$ V = -37.58 cm

Q.16 A deuteron and an alpha particle having equal kinetic energy enter perpendicularly into a magnetic field. Let r_d and r_α be their respective radii of circular path. The value of $\frac{r_d}{r_\alpha}$ is equal to :

Options
1. $\dfrac{1}{\sqrt{2}}$
2. 1
3. $\sqrt{2}$
4. 2

Ans: $\sqrt{2}$

Sol: $r = \dfrac{mV}{Bq} = \dfrac{\sqrt{2mk}}{Bq} \propto \dfrac{\sqrt{m}}{q}$

$m_2 = 2\,m_d$ and $q_\alpha = 2q_d$

$\therefore \dfrac{r_d}{r_\alpha} = \dfrac{\sqrt{m_d}}{q_d} \times \dfrac{2q_d}{\sqrt{2m_d}} = \sqrt{2}$

Q.17 Consider a mixture of gas molecule of types A, B and C having masses $m_A < m_B < m_C$. The ratio of their root mean square speeds at normal temperature and pressure is :

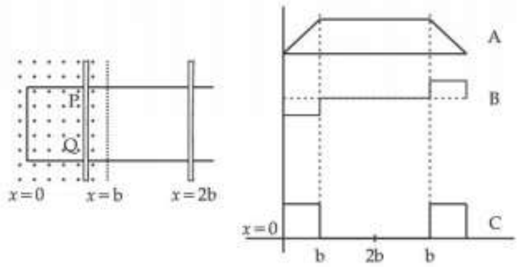
Options
1. $v_A = v_B = v_C = 0$
2. $v_A = v_B \neq v_C$
3. $\frac{1}{v_A} > \frac{1}{v_B} > \frac{1}{v_C}$
4. $\frac{1}{v_A} < \frac{1}{v_B} < \frac{1}{v_C}$

Ans: $\frac{1}{V_A} < \frac{1}{V_B} < \frac{1}{V_C}$

Sol: $V_{rms} = \sqrt{\frac{3RT}{M}}$

$m_a < m_b < m_c \Rightarrow V_A > V_B > V_C \Rightarrow \frac{1}{V_A} < \frac{1}{V_B} < \frac{1}{V_C}$

Q.18 The arm PQ of a rectangular conductor is moving from $x=0$ to $x=2b$ outwards and then inwards from $x=2b$ to $x=0$ as shown in the figure. A uniform magnetic field perpendicular to the plane is acting from $x=0$ to $x=b$. Identify the graph showing the variation of different quantities with distance.



Options
1. A-Flux, B-Power dissipated, C-EMF
2. A-Flux, B-EMF, C-Power dissipated
3. A-Power dissipated, B-Flux, C-EMF
4. A-EMF, B-Power dissipated, C-Flux

Ans: A-Flux, B-EMF, C-Power dissipated

Sol: As rod moves in filed, area increases upto x = b, then field is absent and again flux is generated on return journey from x = b to x = 0. Thus A represents flux

e = $\frac{-d\phi}{dt}$ $\Rightarrow$ curve B represents emf [slope of ϕ - t graph]

P = VI $\Rightarrow$ curve C represents power dissipated

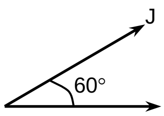
Q.19 A current of 5 A is passing through a non-linear magnesium wire of cross-section 0.04 m^2. At every point the direction of current density is at an angle of 60° with the unit vector of area of cross-section. The magnitude of electric field at every point of the conductor is :
(Resistivity of magnesium $\rho = 44 \times 10^{-8}\ \Omega m$)

Options

1. 11×10^{-2} V/m
2. 11×10^{-7} V/m
3. 11×10^{-5} V/m
4. 11×10^{-3} V/m

Ans: 11×10^{-5} V / m

Sol:



$$Di = JdA \cos\theta = \frac{JdA}{2}$$

$$i = \frac{EA}{2\rho} \Rightarrow E = \frac{2\rho i}{A} = \frac{2 \times 44 \times 10^{-8} \times 5}{4 \times 10^{-2}} = 11 \times 10^{-5} \text{ V/m}$$

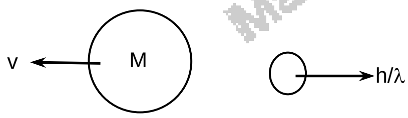
Q.20 A nucleus of mass M emits γ-ray photon of frequency 'ν'. The loss of internal energy by the nucleus is :
[Take 'c' as the speed of electromagnetic wave]

Options

1. $h\nu\left[1 + \frac{h\nu}{2Mc^2}\right]$
2. 0
3. $h\nu\left[1 - \frac{h\nu}{2Mc^2}\right]$
4. $h\nu$

Ans: $h\nu\left(1 + \frac{h\nu}{2Mc^2}\right)$

Sol:



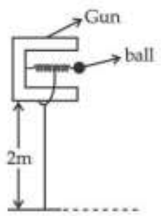
$$Mv = \frac{h}{\lambda} = \frac{h\nu}{c}$$

$$\text{Loss of energy} = \frac{1}{2}Mv^2 + h\nu = \frac{1}{2}\frac{P^2}{M} + h\nu$$

$$= \frac{1}{2M}\left(\frac{h\nu}{c}\right)^2 + h\nu = h\nu\left(1 + \frac{h\nu}{2Mc^2}\right)$$

Section B

Q.1 In a spring gun having spring constant 100 N/m a small ball 'B' of mass 100 g is put in its barrel (as shown in figure) by compressing the spring through 0.05 m. There should be a box placed at a distance 'd' on the ground so that the ball falls in it. If the ball leaves the gun horizontally at a height of 2 m above the ground. The value of d is __________ m. ($g = 10\ m/s^2$).



Given **6**
Answer :

Ans: 1.00

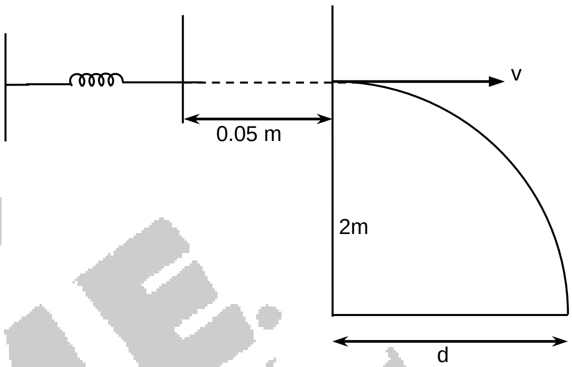
Sol: $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$\Rightarrow kx^2 = mv^2 \Rightarrow v = x\sqrt{\frac{k}{m}}$

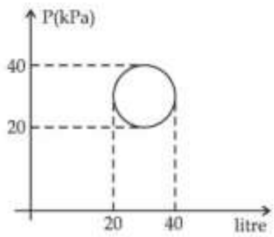
$= 0.05\sqrt{\frac{100}{0.1}} = 0.05 \times 10\sqrt{10} = 0.5\sqrt{10}$

$h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 2}{10}} = \frac{2}{\sqrt{10}}$

$d = vt = 0.5\sqrt{10} \times \frac{2}{\sqrt{10}} = 1m$



Q.2 In the reported figure, heat energy absorbed by a system in going through a cyclic process is __________ πJ.



Given **3**
Answer :

Ans: 100.00

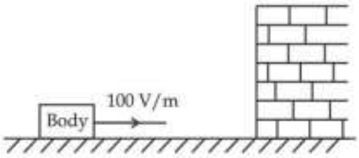
Sol: $\Delta Q = w + \Delta U$ where $w \rightarrow$ area enclosed by the curve

$\Rightarrow \Delta Q = \pi ab$

$= \pi\left[\frac{40-20}{2} \times 10^3\right] \times \left[\frac{40-20}{2} \times 10^{-3}\right]$

$= 100\ \pi J$

Q.3 A body having specific charge 8 $\mu C/g$ is resting on a frictionless plane at a distance 10 cm from the wall (as shown in the figure). It starts moving towards the wall when a uniform electric field of 100 V/m is applied horizontally towards the wall. If the collision of the body with the wall is perfectly elastic, then the time period of the motion will be ________ s.



Given **10**
Answer :

Ans: 1.00

Sol: $a = \frac{qE}{m} = \frac{8 \times 10^{-6}}{10^{-3}} \times 100 = 0.8\, m/s^2$

As the electric field is switched on ball strikes the wall and returns back

$S = ut_1 + \frac{1}{2} a\ t_1^2$

$0.1 = \frac{1}{2} \times 0.8 \times t_1^2 \Rightarrow t_1 = \frac{1}{2} s$

$\therefore$ Time period, $T = 2 \times \frac{1}{2} = 1s$

Q.4 The amplitude of wave disturbance propagating in the positive x-direction is given by $y = \frac{1}{(1+x)^2}$ at time $t=0$ and $y = \frac{1}{1+(x-2)^2}$ at $t=1$ s, where x and y are in metres. The shape of wave doesnot change during the propagation. The velocity of the wave will be ________ m/s.

Given **9**
Answer :

Ans: 2.00

Sol: At t = 0, $y = \frac{1}{1+x^2}$

At time t, $y = \frac{1}{1+(x-vt)^2}$

At time t = 1, $y = \frac{1}{1+(x-v)^2}$ ----(1)

At time t = 1, $y = \frac{1}{1+(x-2)^2}$ ----(2)

Comparing (1) and (2), v = 2 m/s

Q.5 In an LCR series circuit, an inductor 30 mH and a resistor 1 Ω are connected to an AC source of angular frequency 300 rad/s. The value of capacitance for which, the current leads the voltage by 45° is $\frac{1}{x} \times 10^{-3}$ F. Then the value of x is ________.

Given **8**
Answer :

Ans: 3.00

Sol: $\tan \phi = \frac{X_C - X_L}{R}$

$\tan 45 = \frac{X_C - X_L}{R}$

$X_C - X_L = R$

$$\frac{1}{\omega C} - \omega L = R$$

$$\frac{1}{\omega C} - 300 \times 0.03 = 1$$

$$\frac{1}{\omega C} = 10$$

$$C = \frac{1}{10\omega} = \frac{1}{10 \times 30} = \frac{1}{3} \times 10^{-3}$$

$\Rightarrow x = 3$

Q.6 The frequency of a car horn encountered a change from 400 Hz to 500 Hz, when the car approaches a vertical wall. If the speed of sound is 330 m/s. Then the speed of car is ________ km/h.

Given -- Answer :

Ans: 132.00

Sol: Frequency received by the wall, $\nu' = \left(\frac{V_S}{V_S - V}\right) \times \nu_0$

Reflected frequency received man, $\nu'' = \left(\frac{V_S + V}{V_S}\right) \times \nu'$

$$\nu'' = \left(\frac{V_S + V}{V_S}\right)\left(\frac{V_S}{V_S - V}\right)\nu_0$$

$$\Rightarrow \nu'' = \left(\frac{V_S + V}{V_S - V}\right)\nu_0$$

$$\Rightarrow 500 = \left(\frac{330 + V}{330 - V}\right)400$$

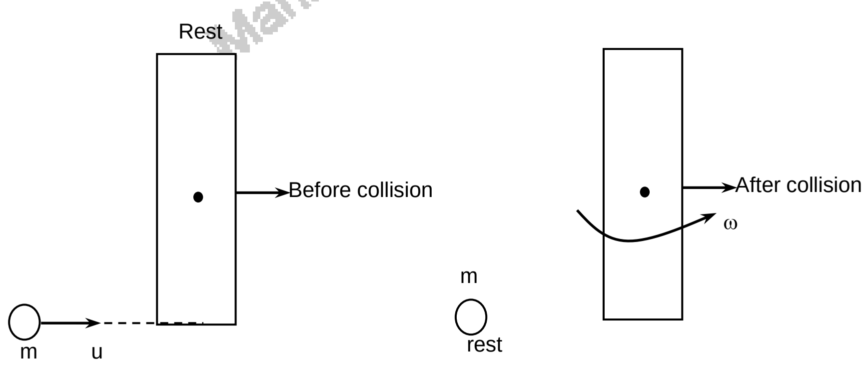
$$\Rightarrow V = \frac{330}{9} \times \frac{18}{5} = 132 \text{ km/hr}$$

Q.7 A rod of mass M and length L is lying on a horizontal frictionless surface. A particle of mass 'm' travelling along the surface hits at one end of the rod with a velocity 'u' in a direction perpendicular to the rod. The collision is completely elastic. After collision, particle comes to rest. The ratio of masses $\left(\frac{m}{M}\right)$ is $\frac{1}{x}$. The value of 'x' will be ________.

Given -- Answer :

Ans: 4.00

Sol:



According to law of conservation of angular momentum

$$mu\left(\frac{L}{2}\right)=\frac{ML^2}{12}(\omega)----(1)$$

mu = MV_1 ---------------------(2)

$$1 = \frac{V_1+\frac{\omega L}{2}}{u} --------(3)$$

Substituting V_1 from (2) and ωL from (1) in (3)

$$u=\frac{m}{M}u+\frac{6mu}{2M}$$

$$\frac{4m}{M}=1$$

$$\frac{m}{M}=\frac{1}{4}\Rightarrow x=4$$

Q.8 A circular disc reaches from top to bottom of an inclined plane of length 'L'. When it slips down the plane, it takes time 't_1'. When it rolls down the plane, it takes time t_2. The value of $\frac{t_2}{t_1}$ is $\sqrt{\frac{3}{x}}$. The value of x will be __________.

Given --
Answer :

Ans: 2.00

Sol: When the disc is sliding $a_1 = g\sin\theta$

$$S = ut_1 + \frac{1}{2}a_1t_1^2=\frac{1}{2}g\sin\theta\ t_1^2 ------(1)$$

When disc is rolling, $a = \dfrac{g\sin\theta}{1+\frac{K^2}{R^2}}=\dfrac{g\sin\theta}{1+\frac{1}{2}}$

$$=\frac{2}{3}g\sin\theta$$

$$\therefore S = ut_2 + \frac{1}{2}a_2t_2^2=\frac{1}{2}\times\frac{2}{3}\times g\sin\theta t_2^2 ------(2)$$

From (1) and (2), $\frac{t_2}{t_1}=\sqrt{\frac{3}{2}}\Rightarrow x=2$

Q.9 A carrier wave $V_c(t)=160\sin(2\pi\times10^6t)$ volts is made to vary between $V_{max}=200$ V and $V_{min}=120$ V by a message signal $V_m(t)=A_m\sin(2\pi\times10^3t)$ volts. The peak voltage A_m of the modulating signal is __________.

Given --
Answer :

Ans: 40.00

Sol: $A_{max} = A_m + A_C$
$V_{max} = V_m + V_C$
$200 = V_m + 160 \Rightarrow V_m = 40$

Q.10 An object viewed from a near point distance of 25 cm, using a microscopic lens with magnification '6', gives an unresolved image. A resolved image is observed at infinite distance with a total magnification double the earlier using an eyepiece along with the given lens and a tube of length 0.6 m, if the focal length of the eyepiece is equal to __________ cm.

Given --
Answer :

Ans: 25.00

Sol: Magnification of microscopic lens, m = 1 + $\frac{D}{f_0}$

$6 = 1 + \frac{25}{f_0} \Rightarrow f_0 = 5\,\text{cm}$

Magnification of compound microscope when image formed infinity,

m = $\frac{\ell}{f_0} \times \frac{D}{f_e}$

$12 = \frac{60}{5} \times \frac{25}{f_e}$

$\Rightarrow f_e$ = 25 cm

PART – B – CHEMISTRY

Section A

Q.1 Green Chemistry in day-to-day life is in the use of :

Options 1. Large amount of water alone for washing clothes

2. Chlorine for bleaching of paper

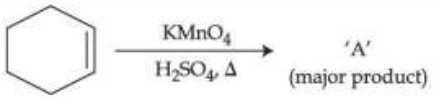
3. Tetrachloroethene for laundry

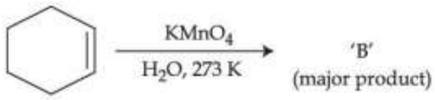
4. Liquified CO_2 for dry cleaning of clothes

Ans: Liquid CO_2 for dry cleaning clothes

Sol: Liquid CO_2 is used for dry cleaning of clothes

Q.2





For above chemical reactions, identify the correct statement from the following.

Options 1. Compound 'A' is dicarboxylic acid and compound 'B' is diol.

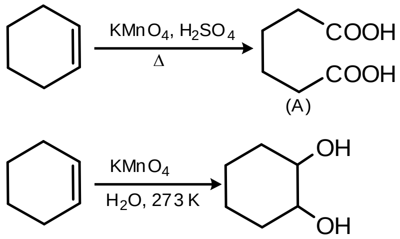
2. Compound 'A' is diol and compound 'B' is dicarboxylic acid.

3. Both compound 'A' and compound 'B' are dicarboxylic acids.

4. Both compound 'A' and compound 'B' are diols.

Ans: Compound 'A' is dicarboxylic acid and compound 'B' is diol

Sol:



Q.3 The correct order of intensity of colors of the compounds is :

Options
1. $[Ni(CN)_4]^{2-} > [NiCl_4]^{2-} > [Ni(H_2O)_6]^{2+}$
2. $[NiCl_4]^{2-} > [Ni(H_2O)_6]^{2+} > [Ni(CN)_4]^{2-}$
3. $[NiCl_4]^{2-} > [Ni(CN)_4]^{2-} > [Ni(H_2O)_6]^{2+}$
4. $[Ni(H_2O)_6]^{2+} > [NiCl_4]^{2-} > [Ni(CN)_4]^{2-}$

Ans: $[NiCl_4]^{2-} > [NiH_2O)_6]^{2+} > [Ni(CN)_4]^{2-}$

Sol: $[NiCl_4]^{2-} \rightarrow$ Bright blue colour

$[Ni(H_2O)_6]^{2+} \rightarrow$ has two unpaired electrons which result in green colour. Due to d-d transition red light is absorbed and complimentary light emitted is green

$[Ni(CN)_4]^{2-} \rightarrow$ In presence of strong field cyanide ligand, the unpaired electrons in 3d pairs up, no d-d transition possible and the complex is colourless

Q.4 According to the valence bond theory the hybridization of central metal atom is dsp^2 for which one of the following compounds ?

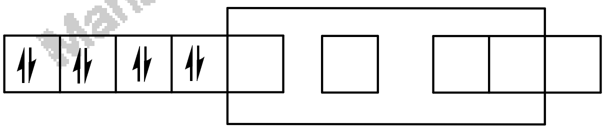
Options
1. $K_2[Ni(CN)_4]$
2. $NiCl_2.6H_2O$
3. $Na_2[NiCl_4]$
4. $[Ni(CO)_4]$

Ans: $K_2[Ni(CN)_4]$

Sol: In $K_2[Ni(CN)_4]$,

$Ni^{2+} \rightarrow 3d^8\ 4s^0$

CN^- strong ligand,



dsp^2 hybridisation

Q.5 The conditions given below are in the context of observing Tyndall effect in colloidal solutions :

(A) The diameter of the colloidal particles is comparable to the wavelength of light used.

(B) The diameter of the colloidal particles is much smaller than the wavelength of light used.

(C) The diameter of the colloidal particles is much larger than the wavelength of light used.

(D) The refractive indices of the dispersed phase and the dispersion medium are comparable.

(E) The dispersed phase has a very different refractive index from the dispersion medium.

Choose the most appropriate conditions from the options given below.

Options
1. (A) and (D) only
2. (B) and (E) only
3. (A) and (E) only
4. (C) and (D) only

Ans: (A) and (E) only

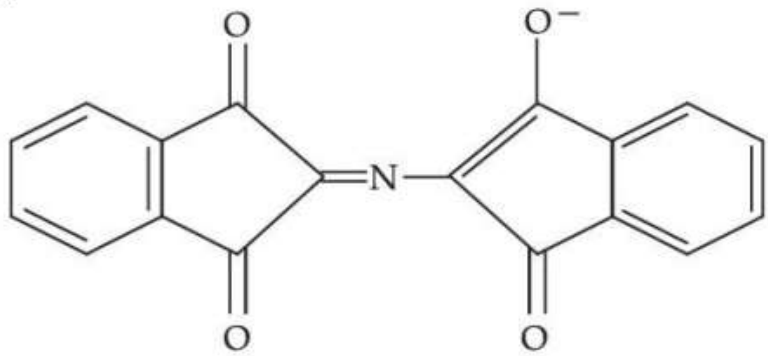
Sol: Tyndall effect is observed only when the following conditions are satisfied

(i) The diameter of he dispersed phase particles is not much smaller as compared to the wavelength of light used

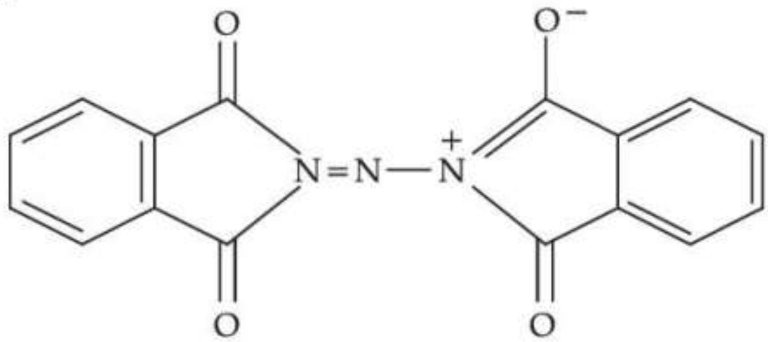
(ii) There is a large different in the refractive indices of the dispersed phase and dispersed medium

Q.6 The correct structure of Rhumann's Purple, the compound formed in the reaction of ninhydrin with proteins is :

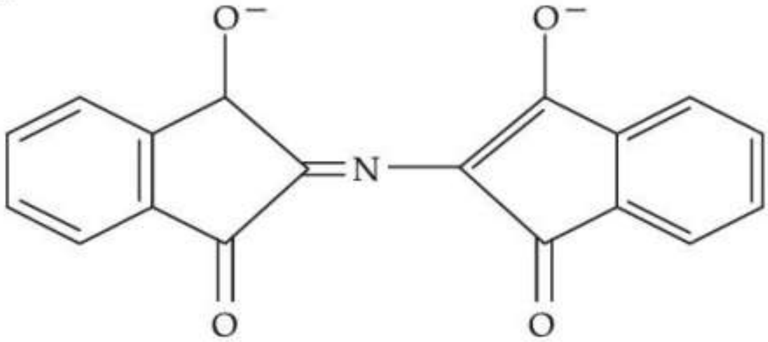
Options 1.



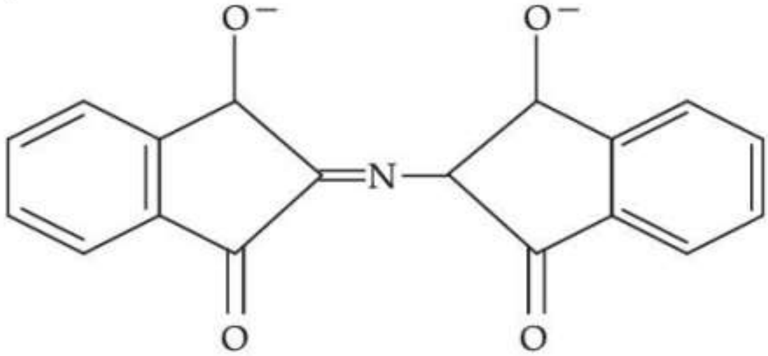
2.



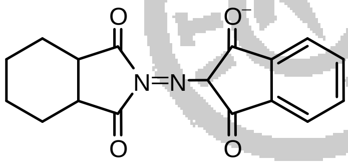
3.



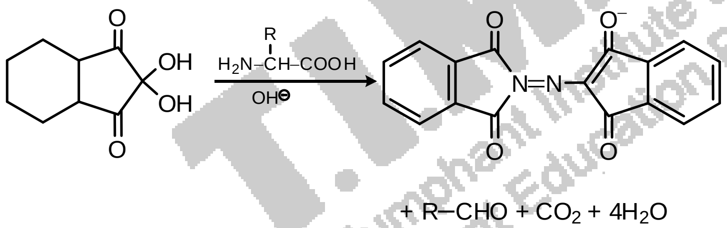
4.



Ans:



Sol:



Q.7 An inorganic Compound 'X' on treatment with concentrated H_2SO_4 produces brown fumes and gives dark brown ring with $FeSO_4$ in presence of concentrated H_2SO_4. Also Compound 'X' gives precipitate 'Y', when its solution in dilute HCl is treated with H_2S gas. The precipitate 'Y' on treatment with concentrated HNO_3 followed by excess of NH_4OH further gives deep blue coloured solution, Compound 'X' is :

Options
1. $Pb(NO_3)_2$
2. $Cu(NO_3)_2$
3. $Pb(NO_2)_2$
4. $Co(NO_3)_2$

Ans: $Cu(NO_3)_2$

Sol: $Cu^{2+} + H_2S \xrightarrow{H^+} \underset{(Y)}{CuS}\downarrow$ (black ppt)

$$\underset{(Y)}{CuS} \xrightarrow{conc.HNO_3} \underset{Soluble}{Cu(NO_3)_2} + NO_2 + S + H_2O$$

$$\underset{Soluble}{Cu(NO_3)_2} \xrightarrow[NH_4OH]{excess} \underset{(X)}{[Cu(NH_3)_4]^{2+}}\text{[Deep blue]}$$

Q.8 Identify the incorrect statement from the following :

Options 1. β-Glycosidic linkage makes cellulose polymer

2. Amylose is a branched chain polymer of glucose

3. Glycogen is called as animal starch

4. Starch is a polymer of α-D glucose

Ans: Amylose is branched chain polymer of glucose

Sol: Amylose is a linear chain polymer of glucose

Q.9 The species given below that does NOT show disproportionation reaction is :

Options 1. BrO_3^-

2. BrO_2^-

3. BrO_4^-

4. BrO^-

Ans: BrO_4^-

Sol: In BrO_4^-, Br is in maximum oxidation state (+7). So it cannot undergo further oxidation.

Q.10 Chemical nature of the nitrogen oxide compound obtained from a reaction of concentrated nitric acid and P_4O_{10} (in 4 : 1 ratio) is :

Options 1. neutral

2. acidic

3. amphoteric

4. basic

Ans: acidic

Sol: $4HNO_3 + P_4O_{10} \longrightarrow \underset{(acidic)}{2N_2O_5} + 4HPO_3$

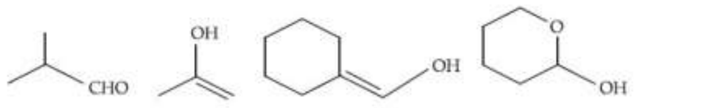
Q.11 The metal that can be purified economically by fractional distillation method is :

Options
1. Ni
2. Zn
3. Cu
4. Fe

Ans: Zn

Sol: Zn is a low boiling metal. Hence can be purified by fractional distillation method

Q.12



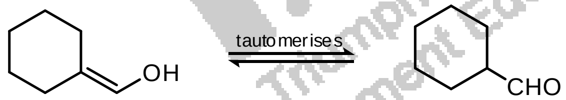
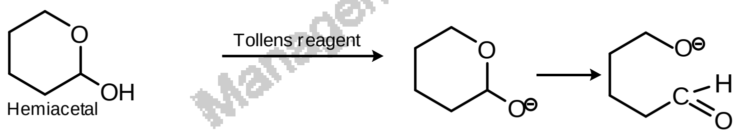
(I) (II) (III) (IV)

Which among the above compound/s does/do not form Silver mirror when treated with Tollen's reagent ?

Options
1. Only (II)
2. (III) and (IV) only
3. (I), (III) and (IV) only
4. Only (IV)

Ans: Only (II)

Sol:

	Compound			Tollens test
(i)	$CH_3-CH(CH_3)-CHO$	$\Rightarrow$		positive
(ii)	$CH_3-C(OH)=CH_2$	$\xrightarrow{\text{tautomerises}}$ (⇌)	$CH_3-C(=O)-CH_3$	negative
(iii)				positive
(iv)				positive

Q.13 Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : The dihedral angles in H_2O_2 in gaseous phase is 90.2° and in solid phase is 111.5°.

Reason R : The change in dihedral angle in solid and gaseous phase is due to the difference in the intermolecular forces.

Choose the most appropriate answer from the options given below for **A** and **R**.

Options
1. Both **A** and **R** are correct and **R** is the correct explanation of **A**.
2. Both **A** and **R** are correct but **R** is not the correct explanation of **A**.
3. **A** is correct but **R** is not correct.
4. **A** is not correct but **R** is correct.

Ans: A is not correct but R is correct

Sol: The dihedral angles in H_2O_2 in gaseous phase is 111.5° and in solid phase is 90.2°

Q.14 Compound A is converted to B on reaction with $CHCl_3$ and KOH. The compound B is toxic and can be decomposed by C. A, B and C respectively are :

Options
1. primary amine, isonitrile compound, conc. HCl
2. secondary amine, nitrile compound, conc. NaOH
3. secondary amine, isonitrile compound, conc. NaOH
4. primary amine, nitrile compound, conc. HCl

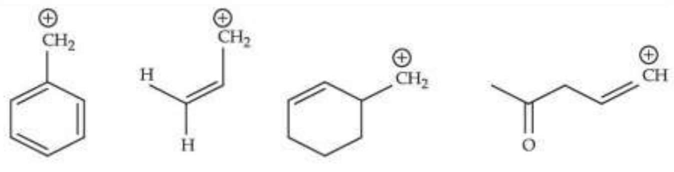
Ans: Primary amine, isonitrile compound, conc.HCl

Sol: Only 1° amines gives carbylamine reaction
To destroy the offensive odour of carbylamine the reaction mixture is treated with conc.HCl

$$\underset{\substack{1^\circ \text{amine} \\ (A)}}{R-NH_2} + CHCl_3 + 3KOH \longrightarrow \underset{\substack{\text{Isonitrile} \\ (B)}}{R-NC} + 3KCl + 3H_2O$$

$$RNC + \underset{(C)}{\text{conc.HCl}} \xrightarrow{H_2O} R-NH_2 + HCOOH$$

Q.15



(A) (B) (C) (D)

Among the given species the Resonance stabilised carbocations are :

Options
1. (A) and (B) only
2. (A), (B) and (C) only
3. (A), (B) and (D) only
4. (C) and (D) only

Ans: (A) and (B) only

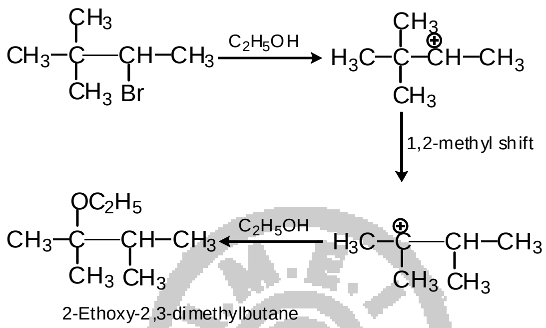
Sol: (A) and (B) only are involved in resonance

Q.16 In the given reaction 3-Bromo-2,2-dimethyl butane $\xrightarrow{C_2H_5OH}$ 'A' (Major Product)
Product A is :

Options
1. 2-Ethoxy-3,3-dimethyl butane.
2. 2-Ethoxy-2,3-dimethyl butane.
3. 1-Ethoxy-3,3-dimethyl butane.
4. 2-Hydroxy-3,3-dimethyl butane.

Ans: 2-Ethoxy-2,3-dimethylbutane

Sol:



Q.17 A s-block element (M) reacts with oxygen to form an oxide of the formula MO_2. The oxide is pale yellow in colour and paramagnetic. The element (M) is :

Options
1. K
2. Na
3. Mg
4. Ca

Ans: K

Sol: $K + O_2 \longrightarrow O_2$

Potassium reacts with oxygen to form superoxide (KO_2) which is pale yellow in colour and paramagnetic

Q.18 Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R**.
Assertion A : Sharp glass edge becomes smooth on heating it upto its melting point.
Reason R : The viscosity of glass decreases on melting.
Choose the most appropriate answer from the options given below.

Options
1. **A** is true but **R** is false.
2. Both **A** and **R** are true but **R** is NOT the correct explanation of **A**.
3. **A** is false but **R** is true.
4. Both **A** and **R** are true and **R** is the correct explanation of **A**.

Ans: Both A and R are true but R is NOT the correct explanation of A

Sol: Sharp glass edge becomes smooth on heating it upto its melting point. On heating glass melts and surface of the liquid tends to take rounded shape at the edges which has minimum surface area

Q.19 The set in which compounds have different nature is :

Options
1. $B(OH)_3$ and $Al(OH)_3$
2. $B(OH)_3$ and H_3PO_3
3. $Be(OH)_2$ and $Al(OH)_3$
4. NaOH and $Ca(OH)_2$

Ans: $B(OH)_3$ and $Al(OH)_3$

Sol: $B(OH)_3$ is H_3BO_3 which is acidic in nature
$Al(OH)_3$ is amphoteric in nature

Q.20 Orlon fibres are made up of :

Options
1. Polyesters
2. Cellulose
3. Polyacrylonitrile
4. Polyamide

Ans: Polyacrylonitrile

Sol: $\underset{\text{Acrylonitrile}}{nCH_2 = CH - CN} \longrightarrow \underset{\text{PAN or Orlon}}{\left[CH_2 - \underset{|}{\overset{CN}{CH}} \right]_n}$

Section B

Q.1 An average person needs about 10000 kJ energy per day. The amount of glucose (molar mass = 180.0 g mol^{-1}) needed to meet this energy requirement is _________ g. (Nearest integer)
(Use : $\Delta_c H$(glucose) = -2700 kJ mol^{-1})

Given --
Answer :

Ans: 667

Sol: $\Delta_c H$(glucose)= -2700 kJ mol^{-1}

No. Of mole of glucose require for production of 1000 kJ heat $= \frac{10000}{2700}$ mole

Mass of glucose $= \frac{10000}{2700} \times 180 = 666.66$

Q.2 250 mL of 0.5 M NaOH was added to 500 mL of 1 M HCl. The number of unreacted HCl molecules in the solution after complete reaction is _________ $\times 10^{21}$. (Nearest integer)
($N_A = 6.022 \times 10^{23}$)

Given --
Answer :

Ans: 226

Sol: $\underset{1\text{ mol}}{HCl} + \underset{1\text{ mol}}{NaOH} \longrightarrow \underset{1\text{ mol}}{NaCl} + H_2O$

No. of moles of NaOH = $M_1V_1 = 0.5 \times 250 \times 10^{-3} = 0.125$ mol

No. of moles of HCl = $M_2V_2 = 500 \times 10^{-3} = 0.5$ mol

0.125 mol NaOH react with 0.125 mol HCl to form 0.125 mole NaCl

Remaining no. of moles of HCl, n = 0.5 − 0.125 = 0.375

No. of moles of HCl unreacted = $n \times N_A = 0.375 \times 6.022 \times 10^{23}$

$= 2.258 \times 10^{23}$

$= 226 \times 10^{21}$

Q.3 The inactivation rate of a viral preparation is proportional to the amount of virus. In the first minute after preparation, 10% of the virus is inactivated. The rate constant for viral inactivation is ________ $\times 10^{-3}$ min^{-1}. (Nearest integer)

[Use : ln 10 = 2.303 ; $\log_{10} 3 = 0.477$;

property of logarithm : $\log x^y = y \log x$]

Given --
Answer :

Ans: 106

Sol: $k = \frac{2.303}{t}\log\frac{[R]_0}{[R]} = \frac{2.303}{1}\log\frac{100}{90}$

$= 2.303(\log 10 - \log 9) = 2.303 \times (1 - 0.954)$

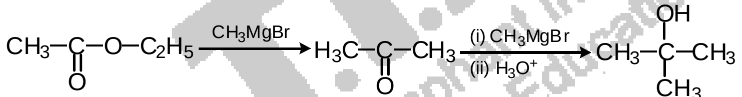
$= 2.303 \times 0.046 = 0.105938 = 105.938 \times 10^{-3}\ mol^{-1}$

Q.4 To synthesise 1.0 mole of 2-methylpropan-2-ol from Ethylethanoate ________ equivalents of CH_3MgBr reagent will be required. (Integer value)

Given --
Answer :

Ans: 2

Sol:



(Ethyl ethanoate) 2-Methylpropan-2-ol

Q.5 The Azimuthal quantum number for the valence electrons of Ga^+ ion is ________. (Atomic number of Ga = 31)

Given **4**
Answer :

Ans: 0

Sol: $\underset{31}{Ga} \rightarrow 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2\ 4p^1$

$Ga^+ \rightarrow 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2$

For 4s orbital, n = 4, $\ell = 0$

(Valence electron)

Q.6 At 20°C, the vapour pressure of benzene is 70 torr and that of methyl benzene is 20 torr. The mole fraction of benzene in the vapor phase at 20°C above an equimolar mixture of benzene and methyl benzene is ________ $\times 10^{-2}$. (Nearest integer)

Given **7.8**
Answer :

Ans: 78

Sol: $P_{Total} = P^\circ_{benzene} X_{benzene} + P^\circ_{toluene} X_{toluene}$

$$= 70 \times \frac{1}{2} + 20 \times \frac{1}{2} = 35 + 10 = 45 \text{ torr}$$

$$X'_A \text{ (In vapour phase)} = \frac{P_A}{P_T}$$

$$= \frac{P^\circ_A X_A}{P_T} = \frac{70 \times 0.5}{45} = 0.777 = 77.7 \times 10^{-2}$$

Q.7 $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$

In an equilibrium mixture, the partial pressures are

$P_{SO_3} = 43$ kPa ; $P_{O_2} = 530$ Pa and

$P_{SO_2} = 45$ kPa. The equilibrium constant K_p = ________ $\times 10^{-2}$. (Nearest integer)

Given **56.2**
Answer :

Ans: 172

Sol: $$K_p = \frac{(p_{SO_3})^2}{(p_{SO_2})^2 (p_{O_2})}$$

$$= \frac{(43)^2}{(45)^2 \times (530 \times 10^{-3})} = 1.722 = 172.2 \times 10^{-2}$$

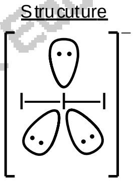
Q.8 The number of lone pairs of electrons on the central I atom in I_3^- is ________.

Given **5**
Answer :

Ans: 3

Sol: $I_3^- \rightarrow I^- + I_2$

Strucuture



Total e^- pairs $= \frac{7+2+1}{2} = \frac{10}{2} = 5$

2-Bond pairs and 3 lone pairs

Q.9 The spin-only magnetic moment value for the complex $[Co(CN)_6]^{4-}$ is ________ BM.
[At. no. of Co = 27]

Given --
Answer :

Ans: 2

Sol: $[Co(CN)_6]^{4-} \Rightarrow Co^{2+} \rightarrow 3d^7 4s^0$

CN^- strong ligand : No. of unpaired electrons = 1

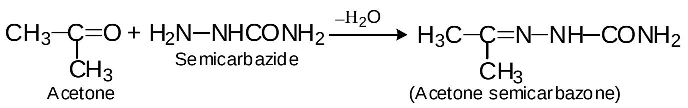
$\mu = \sqrt{n(n+2)} = \sqrt{1(1+2)} = \sqrt{3} = 1.73 BM = 2BM$

Q.10 The number of nitrogen atoms in a semicarbazone molecule of acetone is ________.

Given **2**
Answer :

Ans: 3

Sol:



PART – C – MATHEMATICS

Section A

Q.1 The probability of selecting integers $a \in [-5, 30]$ such that $x^2+2(a+4)x-5a+64>0$, for all $x \in \mathbf{R}$, is :

Options

1. $\frac{1}{4}$
2. $\frac{7}{36}$
3. $\frac{1}{6}$
4. $\frac{2}{9}$

Ans: $\frac{2}{9}$

Sol: $x^2+2(a+4)x-5a+64>0$ for all $x \in R$

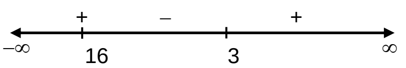
$b^2-4ac<0 \Rightarrow [2(a+4)]^2-4(64-5a)<0$

$4(a^2+8a+16)-4(64-5a)<0$

$(a^2+8a+16)-64+5a<0$

$a^2+13a-48<0$

$(a+16)(a-3)<0$



$\therefore a \in (-16,3)$

$\Rightarrow a=-15,-14,-13,.........\ .2$

Given $a \in [-5,30]$

$\Rightarrow a=-5,-4,........,30$

$F=\{-5,-4,-3,.....,2\}$

$n(F)=8, n(T)=36$

$P=\frac{8}{36}=\frac{2}{9}$

`

Q.2 Let $y = y(x)$ be the solution of the differential equation

$$x\tan\left(\frac{y}{x}\right)dy = \left(y\tan\left(\frac{y}{x}\right) - x\right)dx,\ -1 \le x \le 1,\ y\left(\frac{1}{2}\right) = \frac{\pi}{6}.$$

Then the area of the region bounded by the curves $x = 0$, $x = \frac{1}{\sqrt{2}}$ and $y = y(x)$ in the upper half plane is :

Options

1. $\frac{1}{8}(\pi - 1)$
2. $\frac{1}{12}(\pi - 3)$
3. $\frac{1}{6}(\pi - 1)$
4. $\frac{1}{4}(\pi - 2)$

Ans: $\frac{1}{8}(\pi - 1)$

Sol: $x\tan\left(\frac{y}{x}\right)dy = \left[y\tan\left(\frac{y}{x}\right) - x\right]dx$

$$\frac{dy}{dx} = \frac{y\tan\left(\frac{y}{x}\right) - x}{x\tan\left(\frac{y}{x}\right)} = \frac{y}{x} - \cot\left(\frac{y}{x}\right)$$

$$V + x\frac{dV}{dx} = V - \cot V \qquad (\because y = Vx)$$

$$\Rightarrow x\frac{dV}{dx} = -\cot V$$

$$\Rightarrow \int \tan V dV = -\int \frac{dx}{x}$$

$$\log(\sec V) = -\log x + \log C$$

$$\Rightarrow \log(x.\sec V) = \log C$$

$$\Rightarrow \frac{x}{\cos V} = C$$

$$\Rightarrow x = C\cos V$$

$$\Rightarrow x = C\cos\left(\frac{y}{x}\right)$$

$$\Rightarrow \cos\left(\frac{y}{x}\right) = kx \quad \left(\text{where } k = \frac{1}{2}\right)$$

$$\Rightarrow \frac{y}{x} = \cos^{-1}(kx)$$

$$\Rightarrow y = x\cos^{-1}(kx)$$

$$y\left(\frac{1}{2}\right) = \frac{\pi}{6} \Rightarrow \frac{\pi}{6} = \frac{1}{2}\cos^{-1}\left(k.\frac{1}{2}\right)$$

$$\Rightarrow \cos^{-1}\left(\frac{k}{2}\right) = \frac{\pi}{3} \Rightarrow \frac{k}{2} = \frac{1}{2} \Rightarrow k = 1$$

$$\therefore y = x\cos^{-1}x$$

$$A = \int_0^{1/\sqrt{2}} x\cos^{-1}x\,dx = \int_0^{1/\sqrt{2}} \cos^{-1}x.x.dx = \left[\cos^{-1}x.\frac{x^2}{2}\right]_0^{1/\sqrt{2}} - \int_0^{1/\sqrt{2}} \frac{-1}{\sqrt{1-x^2}}.\frac{x^2}{2}dx$$

$$= \frac{\pi}{4}.\frac{\left(\frac{1}{\sqrt{2}}\right)^2}{2} + \frac{1}{2}\int_0^{1/\sqrt{2}} \frac{x^2}{\sqrt{1-x^2}}dx$$

$$= \frac{\pi}{16} + \frac{1}{2}\int_0^{1/\sqrt{2}} \frac{1-(1-x^2)}{\sqrt{1-x^2}}dx = \frac{\pi}{16} + \frac{1}{2}\int_0^{1/\sqrt{2}} \left(\frac{1}{\sqrt{1-x^2}} - \sqrt{1-x^2}\right)dx$$

$$= \frac{\pi}{16} + \frac{1}{2}[\sin^{-1}x]_0^{1/\sqrt{2}} - \frac{1}{2}\int_0^{1/\sqrt{2}} \sqrt{1-x^2}\,dx = \frac{\pi}{16} + \frac{1}{2}\left(\frac{\pi}{4}\right) - \frac{1}{2}\int_0^{1/\sqrt{2}} \sqrt{1-x^2}\,dx$$

$$= \frac{\pi}{16} + \frac{\pi}{8} - \frac{1}{2}\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right]_0^{1/\sqrt{2}} = \frac{3\pi}{16} - \frac{1}{2}\left[\frac{1}{2\sqrt{2}}.\frac{1}{\sqrt{2}} + \frac{1}{2}.\frac{\pi}{4}\right]$$

$$= \frac{3\pi}{16} - \frac{1}{8} - \frac{\pi}{16} = \frac{2\pi}{16} - \frac{1}{8} = \frac{\pi}{8} - \frac{1}{8} = \frac{\pi-1}{8}$$

Q.3 Let a be a positive real number such that

$\int_0^a e^{x-[x]}\,dx = 10e - 9$

where $[x]$ is the greatest integer less than or equal to x. Then a is equal to :

Options
1. $10 + \log_e 3$
2. $10 + \log_e 2$
3. $10 + \log_e(1+e)$
4. $10 - \log_e(1+e)$

Ans: $10 + \log_e 2$

Sol: $\int_0^a e^{x-[x]}dx = $ **?**

Let a=N+f where $N \in N$ and $0<f<1$

$$\int_0^a e^{x-[x]}dx = \int_0^N e^{x-[x]}dx + \int_N^{N+f} e^{x-[x]}dx$$

$$= N\int_0^1 e^{x-[x]}dx + \int_N^{N+F} e^{x-N}dx = N\int_0^1 e^x dx + \int_N^{N+f} e^{x-N}dx$$

$$= N(e-1) + e^{-N}[e^{N+f} - e^N] = N(e-1) + e^f - 1 = 10e - 9$$

$$\Rightarrow Ne + (e^f - N - 1) = 10e - 9 \Rightarrow N = 10 \text{ and } e^f - 10 - 1 = -9$$

$$\Rightarrow e^f = 2 \Rightarrow f = \log_e 2 \Rightarrow a = 10 + \log_e 2$$

Q.4 The number of real roots of the equation

$$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{4} \text{ is :}$$

Options
1. 0
2. 2
3. 4
4. 1

Ans: 0

Sol: $\tan^{-1}\left(\sqrt{x^2+x}\right)+\sin^{-1}\left(\sqrt{x^2+x+1}\right)=\frac{\pi}{4}$

$x^2+x\ge 0$ and $x^2+x+1\le 1$

$\Rightarrow x^2+x=0\Rightarrow x=0$ or $x=-1$

If $x=0$, LHS $=\tan^{-1}\sqrt{10}+\sin^{-1}\sqrt{1}\ne\frac{\pi}{4}$

If $x=-1$, LHS $=\tan^{-1}\left(\sqrt{1+-1}\right)+\sin^{-1}\left(\sqrt{1-1+1}\right)=\tan^{-1}0+\sin^{-1}1\ne\frac{\pi}{4}$

$\therefore$ No solution

Q.5 Let $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c}=|\vec{c}|$, $|\vec{c}-\vec{a}|=2\sqrt{2}$ and the angle between $(\vec{a}\times\vec{b})$ and $\vec{c}$ is $\frac{\pi}{6}$, then the value of $|(\vec{a}\times\vec{b})\times\vec{c}|$ is :

Options
1. 4
2. 3
3. $\frac{2}{3}$
4. $\frac{3}{2}$

Ans: $\frac{3}{2}$

Sol: $|(\vec{a}\times\vec{b})\times\vec{c}|=|\vec{a}\times\vec{b}||\vec{c}|\sin\frac{\pi}{6}=\frac{1}{2}|\vec{a}\times\vec{b}||\vec{c}| \quad -(1)$

$(\vec{c}-\vec{a})^2=(2\sqrt{2})^2=8$

$|\vec{c}|^2-2\vec{c}.\vec{a}+|\vec{a}|^2=8$

$\Rightarrow|\vec{c}|^2-2|\vec{c}|+(\sqrt{4+1+4})^2=8$

$\Rightarrow|\vec{c}|^2-2|\vec{c}|+9-8=0$

$\Rightarrow|\vec{c}|^2-2|\vec{c}|+1=0\Rightarrow|\vec{c}|=1$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -2\\ 1 & 1 & 0\end{vmatrix}=2\hat{i}-2\hat{j}+\hat{k}$$

$$\Rightarrow\left|\vec{a}\times\vec{b}\right|=\sqrt{4+4+1}=3$$

$$\therefore (1)\Rightarrow\frac{1}{2}\times3\times1=\frac{3}{2}$$

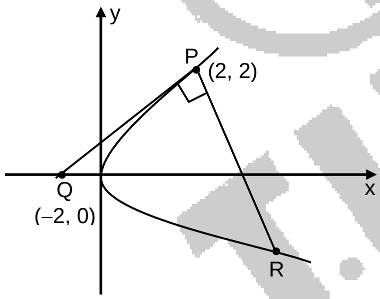
Q.6 Let the tangent to the parabola $S: y^2=2x$ at the point $P(2, 2)$ meet the x-axis at Q and normal at it meet the parabola S at the point R. Then the area (in sq. units) of the triangle PQR is equal to :

Options

1. $\frac{35}{2}$
2. $\frac{15}{2}$
3. 25
4. $\frac{25}{2}$

Ans: $\frac{25}{2}$

Sol:



Area of $\Delta PQR=\frac{1}{2}\times PQ\times PR$

PQ: $y(2)=1(x+2)$

$\Rightarrow 2y=x+2$

$y=0\Rightarrow x=-2$

$\left(\text{Note : the of the tangent}=\frac{1}{2}\right)$

$PQ = \sqrt{(2-0)^2 + (2+2)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$PR : y - 2 = \frac{-1}{\left(\frac{1}{2}\right)}(x-2)$

$\Rightarrow y - 2 = -2(x-2)$

$\Rightarrow y - 2 = -2x + 4$

$\Rightarrow y = -2x + 6$ cut $y^2 = 2x$ at R.

$\Rightarrow (-2x+6)^2 = 2x$

$\Rightarrow 4x^2 - 24x + 36 - 2x = 0$

$\Rightarrow 4x^2 - 26x + 36 = 0$

$\Rightarrow 2x^2 - 13x + 18 = 0$

$\Rightarrow x = 2 \text{ or } \frac{9}{2}$

Substituting in y=-2x+6,

$y = -2\left(\frac{9}{2}\right) + 6 = -3$

$\therefore R\left(\frac{9}{2}, -3\right)$

$PR = \sqrt{\left(\frac{9}{2} - 2\right)^2 + (-3-2)^2}$

$= \sqrt{\left(\frac{5}{2}\right)^2 + 25}$

$= 5\sqrt{\frac{1}{4} + 1} = \frac{5\sqrt{5}}{2}$

$\therefore \text{Area} = \frac{1}{2} \times 2\sqrt{5} \times \frac{5\sqrt{5}}{2} = \frac{25}{2}$

Q.7 If α and β are the distinct roots of the equation $x^2 + (3)^{1/4}x + 3^{1/2} = 0$, then the value of $\alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1)$ is equal to :

Options

1. 52×3^{24}
2. 56×3^{24}
3. 56×3^{25}
4. 28×3^{25}

Ans: 52×3^{24}

Sol: $\alpha, \beta = \frac{-3^{1/4} \pm \sqrt{3^{1/2} - 4.3^{1/2}}}{2} = \frac{-3^{1/4} \pm 3^{1/4}\sqrt{-3}}{2} = 3^{1/4}\left[\frac{-1 \pm \sqrt{3}i}{2}\right]$

$= 3^{1/4}\left[\frac{-1}{2} + \frac{\sqrt{3}i}{2}\right] or\, 3^{1/4}\left[\frac{-1}{2} - \frac{\sqrt{3}i}{2}\right]$

$\therefore \alpha = 3^{1/4}.\omega \, and \, \beta = 3^{1/4}\omega$

$\alpha^{96}(\alpha^{12} - 1) = 3^{24}(3^3 - 1) = 26(3^{24})$

$\beta^{96}(\beta^{12} - 1) = 3^{24}(3^3 - 1) = 26(3^{24})$

$\therefore Sum = 52 \times 3^{24}$

Q.8 Let $A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}$, $a \in \mathbf{R}$ be written as $P+Q$ where P is a symmetric matrix and Q is skew symmetric matrix. If $\det(Q)=9$, then the modulus of the sum of all possible values of determinant of P is equal to :

Options

1. 45
2. 18
3. 36
4. 24

Ans: 36

Sol: $\begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix} = A = P+Q$

$$P = \frac{1}{2}(A+A') = \frac{1}{2}\left\{\begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix} + \begin{bmatrix} 2 & a \\ 3 & 0 \end{bmatrix}\right\} = \frac{1}{2}\begin{bmatrix} 4 & a+3 \\ a+3 & 0 \end{bmatrix}$$

$$\Rightarrow P = \begin{bmatrix} 2 & \frac{(a+3)}{2} \\ \frac{(a+3)}{2} & 0 \end{bmatrix}$$

$$Q = \frac{1}{2}(A-A') = \frac{1}{2}\begin{bmatrix} 0 & 3-a \\ a-3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{(3-a)}{2} \\ \frac{(a-3)}{2} & 0 \end{bmatrix}$$

$$Q = 0 - \frac{(3-a)}{2}\frac{(a-3)}{2} = \frac{(a-3)^2}{4} = 9 \Rightarrow (a-3) = \pm 6$$

$$\Rightarrow a = 9 \text{ or } -3$$

$$\text{If } a = 9, P = \begin{bmatrix} 2 & 6 \\ 6 & 0 \end{bmatrix} \Rightarrow |P| = 0 - 36 = -36$$

$$\text{If } a = -3, P = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow |P| = 0$$

Sum $= -36$

Ans $= |-36| = 36$

Q.9 If z and ω are two complex numbers such that $|z\omega| = 1$ and $\arg(z) - \arg(\omega) = \frac{3\pi}{2}$, then $\arg\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$ is :

(Here $\arg(z)$ denotes the principal argument of complex number z)

Options

1. $-\frac{\pi}{4}$
2. $\frac{3\pi}{4}$
3. $-\frac{3\pi}{4}$
4. $\frac{\pi}{4}$

Ans: $-\frac{3\pi}{4}$

Sol: $\text{Let}\, \omega = r(\cos\theta + i\sin\theta)$

$$\Rightarrow z = \frac{1}{r}\left[\cos\left(\theta + \frac{3\pi}{2}\right) + i\sin\left(\theta + \frac{3\pi}{2}\right)\right]$$

$$= \frac{1}{r}(\cos\theta - i\sin\theta)$$

$$\bar{z}.\omega = \frac{1}{r}(\cos\theta + i\sin\theta).r(\cos\theta + i\sin\theta)$$

$$= (\sin\theta\cos\theta - \sin\theta\cos\theta) + i^2(\sin^2\theta + \cos^2\theta) = i$$

$$\therefore \arg\left(\frac{1-2\,\bar{z}.\omega}{1+3\bar{z}.\omega}\right) = \arg\left(\frac{1-2i}{1+3i}\right) \qquad -(1)$$

$$\frac{1-2i}{1+3i} = \frac{(1-2i)(1-3i)}{10} = \frac{1-3i-2i+6i^2}{10} = \frac{-5-5i}{10} = -\frac{1}{2} - \frac{1}{2}i$$

$$\therefore (1) \Rightarrow -\pi + \tan^{-1}\left|\frac{-1/2}{-1/2}\right| = -\pi + \frac{\pi}{4} = -\frac{3\pi}{4}$$

Q.10 Let $y = y(x)$ be the solution of the differential equation

$$e^x\sqrt{1-y^2}\, dx + \left(\frac{y}{x}\right) dy = 0, y(1) = -1.$$

Then the value of $(y(3))^2$ is equal to :

Options
1. $1-4e^6$
2. $1-4e^3$
3. $1+4e^6$
4. $1+4e^3$

Ans: $1 - 4e^6$

Sol: $e^x\sqrt{1-y^2}dx + \frac{y}{x}dy = 0$

$$\Rightarrow -\frac{y}{x}.\frac{dy}{dx} = e^x\sqrt{1-y^2}$$

$$\int \frac{-ydy}{\sqrt{1-y^2}} = \int xe^x dx$$

$$\Rightarrow \sqrt{1-y^2} = xe^x - e^x + C = (x-1)e^x + C$$

Given $y(1) = -1$

$$\Rightarrow \sqrt{1-1} = (1-1)e^1 + C$$

$$\Rightarrow C = 0$$

$$\therefore \sqrt{1-y^2} = (x-1)e^x$$

$$\Rightarrow 1-y^2 = \left[(x-1)e^x\right]^2$$

$$\Rightarrow y^2 = 1-\left[(x-1)e^x\right]^2$$

$$[y(3)]^2 = 1-\left[(3-1)e^3\right]^2$$

$$= 1-(2e^3)^2$$

$$= 1-4e^6$$

Q.11 Let $A=[a_{ij}]$ be a 3×3 matrix, where

$$a_{ij}=\begin{cases}1 & , \text{ if } i=j\\ -x & , \text{ if } |i-j|=1\\ 2x+1, & \text{otherwise.}\end{cases}$$

Let a function $f:\mathbf{R}\to\mathbf{R}$ be defined as $f(x)=\det(A)$. Then the sum of maximum and minimum values of f on $\mathbf{R}$ is equal to :

Options

1. $\frac{20}{27}$
2. $-\frac{20}{27}$
3. $-\frac{88}{27}$
4. $\frac{88}{27}$

Ans: $-\frac{88}{27}$

Sol: $A=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}=\begin{bmatrix}1 & -x & 2x+1\\ -x & 1 & -x\\ 2x+1 & -x & 1\end{bmatrix}$

$$|A| = 1(1-x^2)+x(-x+x(2x+1))+(2x+1)(x^2-2x-1)$$

$$= 1-x^2-x^2+x^2(2x+1)+(2x+1)(x^2-2x-1)$$

$$= 1-2x^2+2x^3+x^2+2x^3-4x^2-2x+x^2-2x-1$$

$$= 4x^3-4x^2-4x$$

$$\therefore y = 4x^3-4x^2-4x = 4(x^3-x^2-x)$$

$$\frac{dy}{dx} = 4(3x^2-2x-1) = 0$$

$$\Rightarrow x = \frac{2\pm\sqrt{4+12}}{6} = \frac{2\pm4}{6} = 1 or -\frac{1}{3}$$

$$\frac{d^2y}{dx^2} = 4(6x-2)$$

$$if x = 1, \frac{d^2y}{dx^2} > 0 min$$

$$if x = -\frac{1}{3}, \frac{d^2y}{dx^2} < 0 max$$

$$\therefore y\left[\left(-\frac{1}{3}\right)^3-\left(-\frac{1}{3}\right)^2-\left(-\frac{1}{3}\right)\right]_{max}$$

$$= 4\left[-\frac{1}{27}-\frac{1}{9}+\frac{1}{3}\right] = \frac{4}{27}(-1-3+9) = \frac{20}{27}$$

$$\therefore y_{min} = -4$$

$$ymin + y_{max} = -\frac{88}{27}$$

Q.12 Let $[x]$ denote the greatest integer $\leq x$, where $x \in \mathbf{R}$. If the domain of the real valued function

$$f(x) = \sqrt{\frac{|[x]| - 2}{|[x]| - 3}}$$ is $(-\infty, a) \cup [b, c) \cup [4, \infty)$, $a < b < c$, then the value of $a + b + c$ is :

Options
1. 8
2. -2
3. -3
4. 1

Ans: -2

Sol: $f(x) = \sqrt{\dfrac{|[x]| - 2}{|[x]| - 3}}$

$\Rightarrow |[x]| \leq 2$ or $|[x]| > 3$

$\Rightarrow -2 \leq [x] \leq 2$ or $[x] < -3$ or $[x] > 3$

$\Rightarrow [x] = -2, -1, 0, 1, 2 \Rightarrow x \in [-2, 3)$

or

$[x] = -4, -5, -6, \ldots \Rightarrow x \in (-\infty, -3)$

or

$[x] = 4, 5, 6, \ldots \Rightarrow x \in [4, \infty)$

$\therefore x \in (-\infty, -3) \cup [-2, 3) \cup [4, \infty)$

$\Rightarrow a = -3, b = -2, c = 3$

$a + b + c = -2$

Q.13 Words with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the letter M appears at the fourth position in any such word is :

Options
1. $\dfrac{1}{9}$
2. $\dfrac{1}{66}$
3. $\dfrac{1}{11}$
4. $\dfrac{2}{11}$

Ans: $\dfrac{1}{11}$

Sol: **EXAMINATION**

_ _ _ M _ _ _ _ _ _ _

$$n(F) = \frac{10!}{2!\ 2!\ 2!}$$

$$n(T) = \frac{11!}{2!\ 2!\ 2!}$$

$$P = \frac{n(F)}{n(T)} = \frac{1}{11}$$

Q.14 Let a function $f: \mathbf{R} \to \mathbf{R}$ be defined as

$$f(x) = \begin{cases} \sin x - e^x & \text{if } x \le 0 \\ a + [-x] & \text{if } 0 < x < 1 \\ 2x - b & \text{if } x \geqslant 1 \end{cases}$$

where $[x]$ is the greatest integer less than or equal to x. If f is continuous on $\mathbf{R}$, then $(a+b)$ is equal to :

Options
1. 3
2. 4
3. 5
4. 2

Ans: 2

Sol: $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x) \Rightarrow \sin 0 - e^0 = a + \lim\limits_{x\to 0^+}[-x]$

$\Rightarrow 0 - 1 = a - 1 \Rightarrow a = 0$

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^+} f(x) \Rightarrow a + \lim\limits_{x\to 1^-}[x] = 2(1) - b$

$\Rightarrow a + 0 = 2 - b \Rightarrow a + b = 2$

Q.15 The mean of 6 distinct observations is 6.5 and their variance is 10.25. If 4 out of 6 observations are 2, 4, 5 and 7, then the remaining two observations are :

Options
1. 3, 18
2. 1, 20
3. 8, 13
4. 10, 11

Ans: 10, 11

Sol: $\bar{x} = \dfrac{2+4+5+7+a+b}{6} = 3 + \dfrac{a+b}{6} = \dfrac{13}{2}$

$\Rightarrow \dfrac{a+b}{6} = \dfrac{13}{2} - 3 = \dfrac{7}{2} \Rightarrow \dfrac{a+b}{3} = 7 \Rightarrow a+b = 21 \quad -(1)$

$\sigma^2 = \sqrt{\dfrac{\Sigma x_i^2}{n} - (\bar{x})^2} = \sqrt{\dfrac{2^2+4^2+5^2+7^2+a^2+b^2}{6} - \left(\dfrac{13}{2}\right)^2} = 10.25$

$\Rightarrow \dfrac{94+a^2+b^2}{6} - 42.25 = 10.25 \Rightarrow \dfrac{94+a^2+b^2}{6} = 52.5$

$\Rightarrow a^2 + b^2 = 221 \quad -(2)$

From(1)and(2), $a = 10, b = 11$

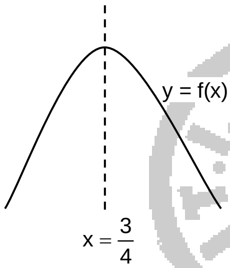
Q.16 Let 'a' be a real number such that the function $f(x) = ax^2 + 6x - 15,\ x \in \mathbf{R}$ is increasing in $\left(-\infty, \frac{3}{4}\right)$ and decreasing in $\left(\frac{3}{4}, \infty\right)$. Then the function $g(x) = ax^2 - 6x + 15,\ x \in \mathbf{R}$ has a :

Options

1. local maximum at $x = \frac{3}{4}$
2. local maximum at $x = -\frac{3}{4}$
3. local minimum at $x = \frac{3}{4}$
4. local minimum at $x = -\frac{3}{4}$

Ans: local maximum at $x = -\frac{3}{4}$

Sol:

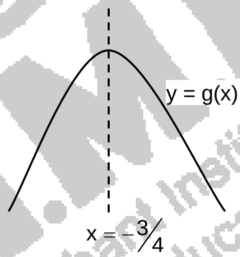


$$\frac{3}{4} = -\frac{b}{2a} = -\frac{6}{2a}$$

$$\Rightarrow \frac{3}{4} = -\frac{3}{a} \Rightarrow a = -4$$

$$g(x) = -4x^2 - 6x + 15$$

$$-\frac{b}{2a} = -\frac{(-6)}{2(-4)} = \frac{6}{-8} = -\frac{3}{4}$$



Q.17 The value of the integral $\int_{-1}^{1} \log_e\left(\sqrt{1-x} + \sqrt{1+x}\right) dx$ is equal to :

Options

1. $2\log_e 2 + \frac{\pi}{2} - \frac{1}{2}$
2. $\frac{1}{2}\log_e 2 + \frac{\pi}{4} - \frac{3}{2}$
3. $\log_e 2 + \frac{\pi}{2} - 1$
4. $2\log_e 2 + \frac{\pi}{4} - 1$

Ans: $\log_e 2 + \frac{\pi}{2} - 1$

Sol: $f(-x) = f(x)$

$$\therefore \int_{-1}^{1} \log(\sqrt{1-x} + \sqrt{1+x})dx = 2\int_{0}^{1} \log(\sqrt{1-x} + \sqrt{1+x})dx = I$$

$$x = \cos\theta \Rightarrow dx = -\sin\theta d\theta$$

$$I = 2\int_{\pi/2}^{0} \log(\sqrt{1-\cos\theta} + \sqrt{1+\cos\theta})(-\sin\theta)d\theta$$

$$= 2\int_{0}^{\pi/2} \sin\theta.\log\left(\sqrt{2}\sin\frac{\theta}{2} + \sqrt{2}\cos\frac{\theta}{2}\right)d\theta$$

$$= 2\left[\int_{0}^{\pi/2} \sin\theta.\log\sqrt{2}d\theta + \int_{0}^{\pi/2} \sin\theta.\log\left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right)d\theta\right]$$

$$= -2\log(\sqrt{2})[\cos\theta]_0^{\pi/2} + 2\int_{0}^{\pi/2} \sin\theta.\log\left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right)d\theta$$

$$= -\log 2(0-1) + \int_{0}^{\pi/2} \sin\theta.\log\left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right)^2 d\theta$$

$$= \log 2 + \int_{0}^{\pi/2} \sin\theta.\log(1+\sin\theta)d\theta$$

$$= \log 2 + \int_{0}^{\pi/2} \log(1+\sin\theta)\sin\theta.d\theta$$

$$= \log 2 + [\log(1+\sin\theta)(-\cos\theta)]_0^{\pi/2} - \int_{0}^{\pi/2} \frac{1}{1+\sin\theta}.\cos\theta(-\cos\theta)d\theta$$

$$= \log 2 + 0 + \int_{0}^{\pi/2} \frac{\cos^2\theta}{1+\sin\theta}.d\theta = \log 2 + \int_{0}^{\pi/2} (1-\sin\theta)d\theta$$

$$= \log 2 + [\theta + \cos\theta]_0^{\pi/2} = \log 2 + \left(\frac{\pi}{2} + 0\right) - (0+1)$$

$$= \log 2 + \frac{\pi}{2} - 1$$

Q.18 The Boolean expression $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$ is equivalent to :

Options
1. $\sim q \Rightarrow p$
2. $p \Rightarrow \sim q$
3. $p \Rightarrow q$
4. $q \Rightarrow p$

Ans: $p \Rightarrow q$

Sol: $(p\wedge \sim q) \Rightarrow (q\vee \sim p) = (p.q')' + q + p'$ $[\because x \Rightarrow y = x' + y \text{ in boolean algebra}]$

$$= p' + (q')' + q + p'$$

$$= (p' + p') + (q + q) = p' + q \quad [\because x + x \equiv x \vee x = x]$$

$$= p \Rightarrow q$$

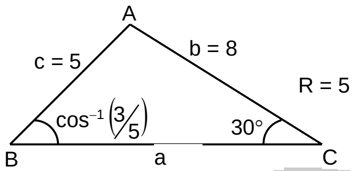
Q.19 If in a triangle ABC, AB = 5 units, $\angle B = \cos^{-1}\left(\frac{3}{5}\right)$ and radius of circumcircle of ΔABC is 5 units, then the area (in sq. units) of ΔABC is :

Options

1. $10 + 6\sqrt{2}$
2. $6 + 8\sqrt{3}$
3. $4 + 2\sqrt{3}$
4. $8 + 2\sqrt{2}$

Ans: $6+8\sqrt{3}$

Sol:



$$b = 2R\sin B = 2\times 5\times \sin\left[\cos^{-1}\left(\frac{3}{5}\right)\right] = 10.\sqrt{1-\left(\frac{3}{5}\right)^2}$$

$$= 10\times\frac{4}{5} = 8$$

$$\sin C = \frac{c}{2R} = \frac{5}{2\times 5} = \frac{1}{2} \Rightarrow C = 30^\circ$$

$$A = 180 - \left[30+\cos^{-1}\left(\frac{3}{5}\right)\right]$$

$$\text{Area} = \frac{1}{2}bc\sin A = \frac{1}{2}\times 8\times 5\times \sin\left(30+\cos^{-1}\left(\frac{3}{5}\right)\right)$$

$$= 20\sin\left(30+\cos^{-1}\left(\frac{3}{5}\right)\right)$$

$$= 20\left[\sin 30.\cos\left(\cos^{-1}\left(\frac{3}{5}\right)\right)+\cos 30.\sin\left(\cos^{-1}\left(\frac{3}{5}\right)\right)\right]$$

$$= 20\left[\frac{1}{2}.\frac{3}{5}+\frac{\sqrt{3}}{2}\sqrt{1-\left(\frac{3}{5}\right)^2}\right] = 20\left(\frac{3}{10}+\frac{\sqrt{3}}{2}.\frac{4}{5}\right)$$

$$= 6+8\sqrt{3}$$

Q.20 The coefficient of x^{256} in the expansion of $(1-x)^{101}(x^2+x+1)^{100}$ is :

Options

1. $-{}^{100}C_{15}$
2. ${}^{100}C_{15}$
3. ${}^{100}C_{16}$
4. $-{}^{100}C_{16}$

Ans: ${}^{100}C_{15}$

Sol: $(1-x)^{101}.(1+x+x^2)^{100} = (1-x)\left[(1-x)(1+x+x^2)\right]^{100}$

$= (1-x)(1-x^3)^{100}$

$= (1-x)\left[1-{}^{100}C_1x^3+{}^{100}C_2(x^3)^2-{}^{100}C_3(x^3)^3+{}^{100}C_4(x^3)^4-.....\right]$

The term containing x^{256} is $(-x)\times -{}^{100}C_{85}(x^3)^{85}$

$\therefore$ Coefficient of x^{256} is ${}^{100}C_{85} = {}^{100}C_{15}$

Section B

Q.1 There are 15 players in a cricket team, out of which 6 are bowlers, 7 are batsmen and 2 are wicketkeepers. The number of ways, a team of 11 players be selected from them so as to include at least 4 bowlers, 5 batsmen and 1 wicketkeeper, is _________.

Given 3
Answer :

Ans: 777

Sol:

6 bo, 7 ba, 2w

⇓

5 bo, 5 ba, 2w

or

5 bo, 5 ba, 1w

or

4 bo, 6 ba, 1w

Number of selection = ${}^6C_4.{}^7C_5.{}^2C_2+{}^6C_5.{}^7C_5.{}^2C_1+{}^6C_4.{}^7C_6.{}^2C_1$

$= \frac{6\cdot 5}{1\cdot 2}.\frac{7\cdot 6}{1\cdot 2}+6.\frac{7.6}{1\cdot 2}.2+\frac{6\cdot 5}{1\cdot 2}.7\cdot 2 = 777$

Q.2 Let a, b, c, d be in arithmetic progression with common difference λ. If

$$\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix} = 2,$$

then value of λ^2 is equal to _________.

Given 4
Answer :

Ans: 1

Sol: $C_2 \to C_2 - C_3$

$$\begin{vmatrix} x-2\lambda & \lambda & x+a \\ x-1 & \lambda & x+b \\ x+2\lambda & \lambda & x+c \end{vmatrix} = \lambda\begin{vmatrix} 1 & x+a & x-2\lambda \\ 1 & x+b & x-1 \\ 1 & x+c & x+2\lambda \end{vmatrix}$$

$C_3 \to C_3 - C_2$, $C_2 \to C_2 - C_1$

$$= \lambda\begin{vmatrix} 1 & x+a & x-2\lambda \\ 0 & \lambda & -1+2\lambda \\ 0 & \lambda & 2\lambda+1 \end{vmatrix}$$

$= \lambda[2\lambda^2+\lambda+\lambda-2\lambda^2] = 2\lambda^2 = 2$

$\Rightarrow \lambda^2 = 1$

Q.3 Let $y = mx + c$, $m > 0$ be the focal chord of $y^2 = -64x$, which is tangent to $(x+10)^2 + y^2 = 4$. Then, the value of $4\sqrt{2}(m+c)$ is equal to ________.

Given **5**
Answer :

Ans: 34.00

Sol: $y^2 = -4(16)x \Rightarrow a = 16$

$F(-16,0)$

$y = mx + c$ passes through $(-16,0)$

$\Rightarrow 0 = m(-16) + C \Rightarrow 16m = C$

Substituting $y = mx + 16m$ in $(x+10)^2 + y^2 = 4$,

$(x+10)^2 + (mx+16m)^2 = 4$

$\Rightarrow x^2 + 20x + 100 + m^2x^2 + 32m^2x + 256m^2 - 4 = 0$

$\Rightarrow (1+m^2)x^2 + (20+32m^2)x + (96+256m^2) = 0$

$b^2 - 4ac = 0 \Rightarrow \left[4(5+8m^2)\right]^2 - 4(1+m^2).4(24+64m^2) = 0$

$\Rightarrow 64m^4 + 80m^2 + 25 - 24 - 64m^2 - 24m^2 - 64m^4 = 0$

$\Rightarrow -8m^2 + 1 = 0$

$\Rightarrow m^2 = \frac{1}{8}$

$\Rightarrow m = \frac{1}{2\sqrt{2}}$

$4\sqrt{2}(m+c) = 4\sqrt{2} \times 17m = 4\sqrt{2} \times 17 \times \frac{1}{2\sqrt{2}} = 34$

Q.4 If the shortest distance between the lines $\vec{r}_1 = \alpha\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$, $\lambda \in \mathbf{R}$, $\alpha > 0$ and $\vec{r}_2 = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$, $\mu \in \mathbf{R}$ is 9, then α is equal to ________.

Given --
Answer :

Ans: 21.00

Sol: $\vec{a}_1 = \alpha\hat{i} + 2\hat{j} + 2\hat{k}$, $\vec{b}_1 = \hat{i} - 2\hat{j} + 2\hat{k}$

$\vec{a}_2 = -4\hat{i} - \hat{k}$, $\vec{b}_2 = 3\hat{i} - 2\hat{j} - 2\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix} = 8\hat{i} + 8\hat{j} + 4\hat{k}$$

$\vec{a}_2 - \vec{a}_1 = -(4+\alpha)\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$

$$S.D = \left|\frac{(\vec{b}_1 \times \vec{b}_2)(\vec{a}_2 - \vec{a1})}{|\vec{b}_1 \times \vec{b}_2|}\right| = \frac{|8(4+\alpha) + 8(2) + 4(3)|}{\sqrt{8^2 + 8^2 + 4^2}} = \frac{|60 + 8\alpha|}{12} = 9$$

$\Rightarrow |60 + 8\alpha| = 108$

$60 + 8\alpha = \pm 108$

$8\alpha = 60 \pm 108$

$8\alpha = 168$ or -48

$\alpha = 21$ $(\because \alpha > 0)$

Q.5

Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ and $B = 7A^{20} - 20A^7 + 2I$, where I is an identity matrix of order 3×3.

If $B = [b_{ij}]$, then b_{13} is equal to ________.

Given --
Answer :

Ans: 910.00

Sol: $A^2 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}^2 = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

$A^3 = A^2.A = \begin{bmatrix} 1 & -3 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$

$A^4 = A^3.A = \begin{bmatrix} 1 & -4 & 6 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$

$A^5 = A^4.A = \begin{bmatrix} 1 & -5 & 10 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix}$

$A^6 = A^5.A = \begin{bmatrix} 1 & -6 & 15 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$

Clearly

$A^n = \begin{bmatrix} 1 & -n & ? \\ 0 & 1 & -n \\ 0 & 0 & 1 \end{bmatrix}$

a_{13} is given by ,
Consider the pattern 0,1,3,6,10,15,….

$a_n = 0 + (1 + 2 + 3 + \text{........ to } (n-1)\text{term}) = \frac{n(n-1)}{2}$

$\therefore A^n = \begin{bmatrix} 1 & -n & \frac{n(n-1)}{2} \\ 0 & 1 & -n \\ 0 & 0 & 1 \end{bmatrix}$

a_{13} in $7A^{20}$ is $7 \times \frac{20(20-1)}{2} = 1330$

a_{13} in $20A^7$ is $20 \times \frac{7(7-1)}{2} = 420$

a_{13} in $2I$ is 0

$\therefore b_{13} = 1330 - 420 + 0 = 910$

Q.6

The number of rational terms in the binomial expansion of $\left(4^{\frac{1}{4}} + 5^{\frac{1}{6}}\right)^{120}$ is ________.

Given --
Answer :

Ans: 11.00

Sol: $\left(4^{1/4} + 5^{1/6}\right)^{120}$

$$T_{r+1} = {}^{120}C_r \left(4^{1/4}\right)^{120-r} . \left(5^{1/6}\right)^{r} = {}^{120}C_r\, 4^{\left(30-\frac{r}{4}\right)} . \left(5^{r/6}\right)$$

$\therefore$ r is a multiple of 4 and 6

$\therefore$ r is a multiple of 12

$\therefore r = 0, 12(1), 12(2), 12(3), \ldots\ldots. 12(10)$

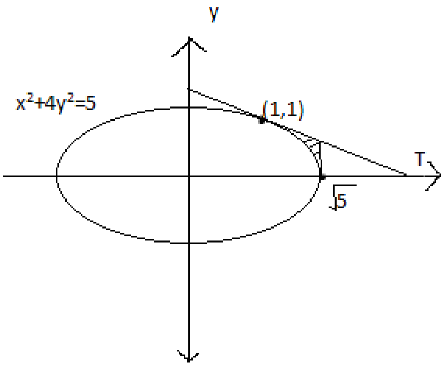
$\therefore$ Ans : 11.00

Q.7 Let T be the tangent to the ellipse $E : x^2 + 4y^2 = 5$ at the point P(1, 1). If the area of the region bounded by the tangent T, ellipse E, lines $x = 1$ and $x = \sqrt{5}$ is $\alpha\sqrt{5} + \beta + \gamma \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$, then $|\alpha + \beta + \gamma|$ is equal to __________.

Given --
Answer :

Ans: 1.25

Sol:



Tangent at (1,1) is $x(1) + 4.y(1) = 5$

$$\text{Area} = \int_1^{\sqrt{5}} \left[\frac{(5-x)}{4} - \frac{\sqrt{5-x^2}}{2}\right] dx$$

$$= \frac{5}{4}[x]_1^{\sqrt{5}} - \frac{1}{4}\left[\frac{x^2}{2}\right]_1^{\sqrt{5}} - \frac{1}{2}\int_1^{\sqrt{5}} \sqrt{5-x^2}\, dx$$

$$= \frac{5}{4}(\sqrt{5}-1) - \frac{1}{8}(5-1) - \frac{1}{2}\int_1^{\sqrt{5}} \sqrt{5-x^2}\, dx$$

$$== \frac{5\sqrt{5}}{4} - \frac{5}{4} - \frac{1}{2} - \frac{1}{2}\left[\frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)\right]_1^{\sqrt{5}}$$

$$= \frac{5\sqrt{5}}{4} - \frac{7}{4} - \frac{1}{2}\left[\left(0 + \frac{5}{2}.\frac{\pi}{2}\right) - \left(\frac{1}{2}\sqrt{4} + \frac{5}{2}\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right]$$

$$= \frac{5\sqrt{5}}{4} - \frac{7}{4} - \frac{1}{2}\left[\frac{5\pi}{4} - 1 - \frac{5}{2}\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)\right]$$

$$= \frac{5\sqrt{5}}{4} - \frac{7}{4} - \frac{5\pi}{8} + \frac{1}{2} + \frac{5}{4}\sin^{-1}\left(\frac{1}{\sqrt{5}}\right) = \frac{5\sqrt{5}}{4} - \frac{5}{4} - \frac{5\pi}{8} + \frac{5}{4}\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right]$$

$$= \frac{5\sqrt{5}}{4} - \frac{5}{4} - \frac{5\pi}{8} + \frac{5\pi}{8} - \frac{5}{4}\cos^{-1}\left(\frac{1}{\sqrt{5}}\right) = \frac{5\sqrt{5}}{4} - \frac{5}{4} - \frac{5}{4}\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

$$\therefore |\alpha + \beta + \gamma| = \frac{5}{4}$$

Q.8 Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three mutually perpendicular vectors of the same magnitude and equally inclined at an angle θ, with the vector $\vec{a}+\vec{b}+\vec{c}$. Then $36\cos^2 2\theta$ is equal to ________.

Given **4**
Answer :

Ans: 4.00

Sol:

$$Let\,|\vec{a}| = |\vec{b}| = |\vec{c}| = k$$

$$|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}| + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 3k^2$$

$$\Rightarrow |\vec{a}+\vec{b}+\vec{c}| = \sqrt{3}k.$$

$$\vec{a}.(\vec{a}+\vec{b}+\vec{c}) = |\vec{a}|.|\vec{a}+\vec{b}+\vec{c}|.\cos\theta$$

$$\Rightarrow |\vec{a}|^2 + 0 + 0 = |\vec{a}|.|\vec{a}+\vec{b}+\vec{c}|.\cos\theta$$

$$\Rightarrow k^2 = k.\sqrt{3}k.\cos\theta$$

$$\therefore \cos\theta = \frac{1}{\sqrt{3}}$$

$$\cos 2\theta = 2\cos^2\theta - 1 = \frac{2}{3} - 1 = -\frac{1}{3}$$

$$36\cos^2 2\theta = (6\cos 2\theta)^2$$

$$= \left(6 \times -\frac{1}{3}\right)^2 = (-2)^2 = 4$$

Q.9 Let P be a plane passing through the points (1, 0, 1), (1, −2, 1) and (0, 1, −2). Let a vector $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ be such that $\vec{a}$ is parallel to the plane P, perpendicular to $(\hat{i}+2\hat{j}+3\hat{k})$ and $\vec{a}\cdot(\hat{i}+\hat{j}+2\hat{k}) = 2$, then $(\alpha-\beta+\gamma)^2$ equals ________.

Given --
Answer :

Ans: 81.00

Sol: The required plane is $\begin{vmatrix} x-1 & y-0 & z-1 \\ 0 & -2 & 0 \\ -1 & 1 & -3 \end{vmatrix} = 0$

$$(x-1)(6) - y(0) + (z-1)(-2) = 0$$

$$6x - 6 - 2z + 2 = 0$$

$$6x - 2z - 4 = 0$$

$$\Rightarrow 3x - z - 2 = 0$$

$$\vec{n} = 3\hat{i} + 0\hat{j} - \hat{k} \quad \text{(Normal vector)}$$

$$\vec{a}.\vec{n} = 0$$

$$\Rightarrow (\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}).(3\hat{i} + 0\hat{j} - \hat{k}) = 0$$

$$\Rightarrow 3\alpha - \gamma = 0 \qquad -(1)$$

$$\vec{a}.(\hat{i} + 2\hat{j} + 3\hat{k}) = 0$$

$$\Rightarrow \alpha + 2\beta + 3\gamma = 0 \qquad -(2)$$

$$\vec{a}.(\hat{i} + \hat{j} + 2\hat{k}) = 2$$

$$\Rightarrow \alpha + \beta + 2\gamma = 2 \qquad -(3)$$

$$(2) - 2\times(3) \Rightarrow -\alpha - \gamma = -4$$

$$\Rightarrow \alpha + \gamma = 4 \quad \text{and}$$

$$3\alpha - \gamma = 0$$

$$\Rightarrow \alpha = 1, \gamma = 3, \beta = -5$$

$$\therefore (\alpha - \beta + \gamma)^2 = [1 - (-5) + 3]^2 = 81$$

Q.10

If the value of $\lim_{x\to 0}(2-\cos x\sqrt{\cos 2x})^{\left(\frac{x+2}{x^2}\right)}$ is equal to e^a, then a is equal to ________.

Given Answer : **2**

Ans: 3.00

Sol:

$$\lim_{x\to 0}\left[2-\cos x\sqrt{\cos 2x}\right]^{\left(\frac{x+2}{x^2}\right)}$$

$$=\lim_{x\to 0}\left[(2-\cos x\sqrt{\cos 2x})^{\frac{1}{x^2}}\right]^{x+2}$$

$$=\lim_{x\to 0}\left[(2-\cos x\sqrt{\cos 2x})^{\frac{1}{x^2}}\right]^{2}=A$$

$$\log A=\lim_{x\to 0}\frac{2}{x^2}.\log\left[2-\cos x\sqrt{\cos 2x}\right]$$

$$=2.\lim_{x\to 0}\left[\frac{\frac{-1}{(2-\cos x\sqrt{\cos 2x})}\left\{\cos x\frac{1}{2\sqrt{\cos 2x}}(-\sin 2x).2+\sqrt{\cos 2x}(-\sin x)\right\}}{2x}\right]$$

$$=\frac{-1}{(2-1)}\left\{\left(\frac{1}{\sqrt{1}}\right).(-2)-\sqrt{1}.(1)\right\}$$

$$=-(-2-1)=3$$

$$\therefore A=e^3$$

$$\therefore a=3$$