SOLUTIONS & ANSWERS FOR JEE MAINS-2021
20th July Shift 2

[PHYSICS, CHEMISTRY & MATHEMATICS]

PART – A – PHYSICS

Section A

Q.1 Consider a binary star system of star A and star B with masses m_A and m_B revolving in a circular orbit of radii r_A and r_B, respectively. If T_A and T_B are the time period of star A and star B, respectively, then :

Options

1. $\frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^{\frac{3}{2}}$
2. $T_A > T_B$ (if $r_A > r_B$)
3. $T_A > T_B$ (if $m_A > m_B$)
4. $T_A = T_B$

Ans: $T_A = T_B$

Sol: $T_A = T_B$ (angular speed same)

Q.2 At an angle of 30° to the magnetic meridian, the apparent dip is 45°. Find the true dip :

Options

1. $\tan^{-1}\frac{\sqrt{3}}{2}$
2. $\tan^{-1}\frac{1}{\sqrt{3}}$
3. $\tan^{-1}\sqrt{3}$
4. $\tan^{-1}\frac{2}{\sqrt{3}}$

Ans: $\tan^{-1}\frac{\sqrt{3}}{2}$

Sol: $\delta' = 45°$ (apparent dip)

$\theta = 30°$

True dip, $\delta = ?$

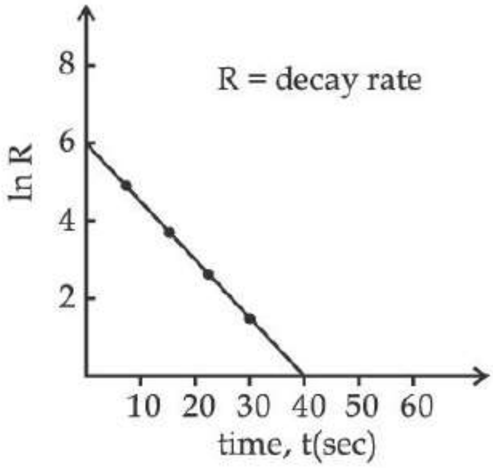
$$\tan\delta' = \frac{V}{H\cos\theta} = \frac{V}{H\cos 30}$$

$$\tan\delta' = \frac{V}{H} \times \frac{2}{\sqrt{3}}$$

$$\tan\delta = \frac{V}{H} \Rightarrow \tan\delta' = \tan\delta \times \frac{2}{\sqrt{3}} \Rightarrow \tan\delta = \tan 45\frac{\sqrt{3}}{2}$$

$\delta = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Q.3 For a certain radioactive process the graph between ln R and t(sec) is obtained as shown in the figure. Then the value of half life for the unknown radioactive material is approximately :



Options
1. 9.15 sec
2. 2.62 sec
3. 6.93 sec
4. 4.62 sec

Ans: 4.62 sec

Sol: $R = R_0\ e^{-\lambda t}$

$\ell nR = \ell n\ (R_0 e^{-\lambda t})$

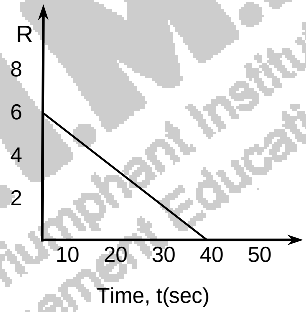
$\ell nR = \ell nR_0 + \ell n\ e^{-\lambda t}$

$\ell nR = \ell nR_0 - \lambda t$

slope of graph = $-\lambda$

$= \frac{-6}{40}$

$\lambda = \frac{3}{20}$



Half life $t_{1/2} = \frac{\ell n2}{\lambda} = \ell = \frac{\ell n2 \times 20}{3}$

= 4.62 sec

Q.4 Two vectors $\vec{P}$ and $\vec{Q}$ have equal magnitudes. If the magnitude of $\vec{P} + \vec{Q}$ is n times the magnitude of $\vec{P} - \vec{Q}$, then angle between $\vec{P}$ and $\vec{Q}$ is :

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Options

1. $\cos^{-1}\left(\frac{n-1}{n+1}\right)$

2. $\sin^{-1}\left(\frac{n-1}{n+1}\right)$

3. $\cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

4. $\sin^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

Ans: $\cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

Sol: Let $|\vec{P}| = |\vec{Q}| = a$

$$|\vec{P}+\vec{Q}| = n|\vec{P}-\vec{Q}| \Rightarrow |\vec{P}+\vec{Q}| = n^2|\vec{P}-\vec{Q}|^2$$

$$|\vec{P}|^2 + |\vec{Q}|^2 + 2|\vec{P}||\vec{Q}|\cos\theta = n^2\left(|\vec{P}|^2 + |\vec{Q}|^2 - 2|\vec{P}||\vec{Q}|\cos\theta\right)$$

$2a^2 + 2a^2 \cos\theta = n^2 (2a^2 - 2a^2 \cos\theta)$

$1 + \cos\theta = n^2 (1 - \cos\theta)$

$$\frac{1+\cos\theta}{1-\cos\theta} = n^2$$

By componendo and dividendo rule,

$$\frac{(1+\cos\theta)+(1-\cos\theta)}{(1+\cos\theta)-(1-\cos\theta)} = \frac{n^2+1}{n^2-1}$$

$$\frac{2}{2\cos\theta} = \frac{n^2+1}{n^2-1}$$

$$\cos\theta = \frac{n^2-1}{n^2+1}$$

$$\theta = \cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$$

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Q.5 Two small drops of mercury each of radius R coalesce to form a single large drop. The ratio of total surface energy before and after the change is :

Options

1. $2 : 1$

2. $1 : 2^{\frac{1}{3}}$

3. $1 : 2$

4. $2^{\frac{1}{3}} : 1$

Ans: $2^{\frac{1}{3}} : 1$

Sol: Let R' be the radius of single large drop

$$\frac{4}{3}\pi R'^3 = 2\times\frac{4}{3}\pi R^3 \qquad R \rightarrow \text{radius of small drop}$$

$R'^3 = 2R^3$

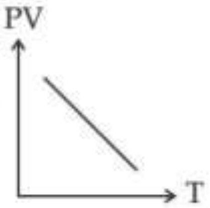
$R' = 2^{1/3}\,R$

$$\frac{\text{Total surface energy before}}{\text{Total surface energy after}} = \frac{2\times S\times 4\pi R^2}{S\times 4\pi R^{12}}$$

$$= \frac{2R^2}{\left(2^{1/3}R\right)^2} = \frac{2}{2^{2/3}} = 2^{1/3}$$

Q.6 Which of the following graphs represent the behavior of an ideal gas ? Symbols have their usual meaning.

Options

1.


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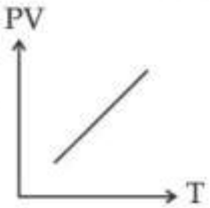

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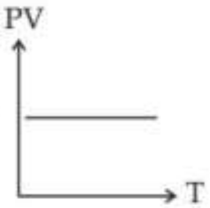

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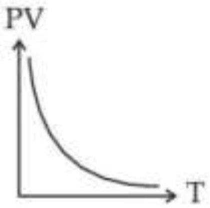
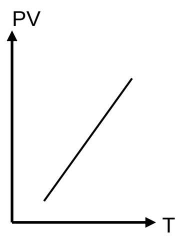


Ans:


Sol: PV = nRT

$PV \propto T$

Graph is a straight line

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Q.7 An electron having de-Broglie wavelength λ is incident on a target in a X-ray tube. Cut-off wavelength of emitted X-ray is :

Options

1. $\frac{2mc\lambda^2}{h}$
2. $\frac{hc}{mc}$
3. 0
4. $\frac{2m^2c^2\lambda^2}{h^2}$

Ans: $\frac{2mc\lambda^2}{h}$

Sol: $\lambda = \frac{h}{P} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2meV}} \Rightarrow \lambda^2 = \frac{h^2}{2meV}$

$eV = \frac{h^2}{2m\lambda^2} - - - - - (1)$

$E = \frac{hc}{\lambda}$

$\lambda_{cut\ off} = \frac{hc}{E} = \frac{hc}{eV}$

$\lambda_{cut\ off} = \frac{hc}{\frac{h^2}{2m\lambda^2}} = \frac{2mhc\lambda^2}{h^2} = \frac{2mc\lambda^2}{h}$

Q.8 A satellite is launched into a circular orbit of radius R around earth, while a second satellite is launched into a circular orbit of radius 1.02R. The percentage difference in the time periods of the two satellites is :

Options

1. 2.0
2. 0.7
3. 1.5
4. 3.0

Ans: 3.0

Sol: $T^2 \propto R^3$

$T^2 = KR^3$ ----------- (1)

Change in radius dR = 1.02 R – R = 0.02 R (2% of R)

Equation (1) $\Rightarrow$ 2T = K3R^2 $\frac{dR}{dT}$ – – – –(2)

$(1) \div (2) \frac{T^2}{2T} = \frac{KR^3}{K3R^2 \frac{dR}{dT}}$

$\Rightarrow \frac{dT}{T} = \frac{3}{2}\frac{dR}{R}$

dR = 0.02

$\frac{dT}{T} = \frac{3}{2} \times \frac{0.02R}{R}$

$\frac{dT}{T} = \frac{3}{2} \times 0.02$

In percentage, $\frac{dT}{T} \times 100 = \frac{3}{2} \times 0.02 \times 100 = 3\%$

Q.9 A particle is making simple harmonic motion along the X-axis. If at a distances x_1 and x_2 from the mean position the velocities of the particle are v_1 and v_2 respectively. The time period of its oscillation is given as :

Options

1. $T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 + v_2^2}}$

2. $T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$

3. $T = 2\pi\sqrt{\frac{x_2^2 + x_1^2}{v_1^2 - v_2^2}}$

4. $T = 2\pi\sqrt{\frac{x_2^2 + x_1^2}{v_1^2 + v_2^2}}$

Ans: $T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{V_1^2 - V_2^2}}$

Sol: $V_1 = \omega\sqrt{A^2 - x_1^2} \Rightarrow \left(\frac{V_1}{\omega}\right)^2 = A^2 - x_1^2 ------(1)$

$V_2 = \omega\sqrt{A^2 - x_2^2} \Rightarrow \left(\frac{V_2}{\omega}\right)^2 = A^2 - x_2^2 ------(2)$

$(1) - (2) \Rightarrow \left(\frac{V_1}{\omega}\right)^2 - \left(\frac{V_2}{\omega}\right)^2 = x_2^2 - x_1^2$

$\frac{1}{\omega}\left[V_1^2 - V_2^2\right] = x_2^2 - x_1^2$

$\omega^2 = \frac{V_1^2 - V_2^2}{x_2^2 - x_1^2}$

$\omega = \sqrt{\frac{V_1^2 - V_2^2}{x_2^2 - x_1^2}}$

$\frac{2\pi}{T} = \sqrt{\frac{V_1^2 - V_2^2}{x_2^2 - x_1^2}}$

$T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{V_1^2 - V_2^2}}$

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Q.10 The magnetic susceptibility of a material of a rod is 499. Permeability in vacuum is $4\pi \times 10^{-7}$ H/m. Absolute permeability of the material of the rod is :

Options

1. $2\pi \times 10^{-4}$ H/m

2. $\pi \times 10^{-4}$ H/m

3. $3\pi \times 10^{-4}$ H/m

4. $4\pi \times 10^{-4}$ H/m

Ans: $2\pi \times 10^{-4}$ H/m

Sol: $x = 499$
$\mu_0 = 4\pi \times 10^{-7}$ H/m
$\mu_r = 1 + x = 1 + 499 = 500$
$\mu = \mu_r \mu_0$
$\mu = 500 \times 4\pi \times 10^{-7}$ H/m
$\mu = 2\pi \times 10^{-4}$ H/m

Q.11 With what speed should a galaxy move outward with respect to earth so that the sodium-D line at wavelength 5890 Å is observed at 5896 Å ?

Options
1. 296 km/sec
2. 336 km/sec
3. 306 km/sec
4. 322 km/sec

Ans: 306 km/sec

Sol: $\lambda = 5890$ Å
$\Delta\lambda = 6$ Å

$$V = C \times \frac{\Delta\lambda}{\lambda}$$

$$= 3 \times 10^{8} \times \frac{6A^\circ}{5890A^\circ}$$

$= 305.6$ km/s $\cong 306$ km/s

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Q.12 A body at rest is moved along a horizontal straight line by a machine delivering a constant power. The distance moved by the body in time 't' is proportional to :

Options
1. $t^{\frac{3}{2}}$
2. $t^{\frac{3}{4}}$
3. $t^{\frac{1}{2}}$
4. $t^{\frac{1}{4}}$

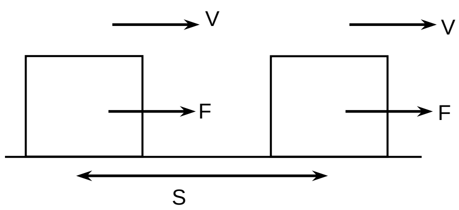
Ans: $t^{\frac{3}{2}}$

Sol: Power, $\vec{P} = \vec{F} \cdot \vec{V}$
$P = FV \cos 0^\circ$
$P = FV$
$P = FV$ (constant)

FV = constant, K $\Rightarrow F = \frac{K}{V}$ – – – –(1)

$$F = m\frac{dV}{dt} \text{– – – –(2)}$$

From (2) and (1), $m\frac{dV}{dt} = \frac{K}{V}$



$$\int_0^V VdV = \int_0^t \frac{K}{m} dt$$

$$\frac{V^2}{2} = \frac{K}{m} t$$

$$V^2 = \frac{2K}{m} t \Rightarrow V = \sqrt{\frac{2K}{m} t}$$

$$\frac{dx}{dt} = \sqrt{\frac{2K}{m} t}$$

$$\int_0^S dx = \sqrt{\frac{2K}{m}} \int_0^t t^{1/2} dt$$

$$[x]_0^S = \sqrt{\frac{2K}{m}} \left[\frac{t^{1/2}}{3/2}\right]_0^t$$

$$S = \frac{2}{3}\sqrt{\frac{2K}{m}}\ t^{3/2}$$

$$S \propto t^{3/2}$$

Q.13 The correct relation between the degrees of freedom f and the ratio of specific heat γ is :

Options

1. $f = \frac{1}{\gamma + 1}$
2. $f = \frac{2}{\gamma + 1}$
3. $f = \frac{2}{\gamma - 1}$
4. $f = \frac{\gamma + 1}{2}$

Ans: $f = \frac{2}{\gamma - 1}$

Sol: $f = \frac{2}{\gamma - 1}$

$$C_V = \frac{f}{2} R$$

$$C_P = \left(\frac{f}{2} + 1\right) R$$

$$\gamma = \frac{C_p}{C_V} = 1 + \frac{2}{f}$$

$$\Rightarrow f = \frac{2}{\gamma - 1}$$

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Q.14 For a series LCR circuit with $R = 100\ \Omega$, $L = 0.5$ mH and $C = 0.1$ pF connected across 220 V – 50 Hz AC supply, the phase angle between current and supplied voltage and the nature of the circuit is :

Options 1. ≈ 90°, predominantly capacitive circuit

2. ≈ 90°, predominantly inductive circuit

3. 0°, resistive circuit

4. 0°, resonance circuit

Ans: ≈ 90°, predominantly capacitive circuit

Sol: R = 100 Ω
L = 0.5 mH
C = 0.1 PF
$X_L = \omega L = 50\pi \times 10^{-3} = 0.157$ [$\omega = 2\pi f = 2\pi \times 50$]

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 50 \times 0.1 \times 10^{-12}} = 3.18 \times 10^{10}$$

$X_C >> X_L$ Circuit is capacitive in nature
$|X_C - X_L| >> R$
Phase angle between current and supplied voltage ≅ 90°

Q.15 If the Kinetic energy of a moving body becomes four times its initial Kinetic energy, then the percentage change in its momentum will be :

Options 1. 300%

2. 400%

3. 100%

4. 200%

Ans: 100%

Sol: KE = $\frac{1}{2}mV^2 = \frac{P^2}{2M}$

KE' = 4KE

$$\frac{1}{2}mV'^2 = 4\frac{1}{2}mV^2$$

V' = 2V
P' = mV' = m (2V) = 2(mV) = 2P
OR
$P^2 \propto E$

$$\left(\frac{P_1}{P_2}\right) = \left(\frac{E_1}{E_2}\right) = \left(\frac{E_1}{2E_1}\right)$$

$$\frac{P_1}{P_2} = \frac{1}{2}$$

$P_2 = 2P_1$

% change in momentum = $\frac{P_2 - P_1}{P_1} \times 100 = (2 - 1) \times 100 = 100$

Q.16 The length of a metal wire is l_1, when the tension in it is T_1 and is l_2 when the tension is T_2. The natural length of the wire is :

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Options

1. $\dfrac{l_1 T_2 + l_2 T_1}{T_2 + T_1}$

2. $\dfrac{l_1 + l_2}{2}$

3. $\sqrt{l_1 \, l_2}$

4. $\dfrac{l_1 T_2 - l_2 T_1}{T_2 - T_1}$

Ans: $\dfrac{\ell_1 T_2 - \ell_2 T_1}{T_2 - T_1}$

Sol: $Y = \dfrac{F/A}{\Delta\ell/\ell}$ Or $\dfrac{T/A}{\Delta\ell/\ell}$

Where $\ell \rightarrow$ initial length

$A \rightarrow$ Area of cross section

$\dfrac{T}{A} = Y\dfrac{\Delta\ell}{\ell}$

$\dfrac{T_1}{A} = Y\dfrac{(\ell_1 - \ell)}{\ell}$ ----(1)

$\dfrac{T_2}{A} = Y\dfrac{(\ell_2 - \ell)}{\ell}$ ----(2)

(1)/ (2) $\Rightarrow \dfrac{T_1}{T_2} = \dfrac{\ell_1 - \ell}{\ell_2 - \ell}$

$T_1\ell_2 - T_1\ell = T_2\ell_1 - T_2\ell_2$

$(T_2 - T_1)\ell = T_2\ell_1 - T_1\ell_2$

$\therefore \ell = \dfrac{T_2\ell_1 - T_1\ell_2}{T_2 - T_1}$

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Q.17 In an electromagnetic wave the electric field vector and magnetic field vector are given as $\vec{E} = E_0\hat{i}$ and $\vec{B} = B_0\hat{k}$ respectively. The direction of propagation of electromagnetic wave is along :

Options

1. $\hat{j}$

2. $(\hat{k})$

3. $(-\hat{k})$

4. $(-\hat{j})$

Ans: $(-\hat{j})$

Sol: Direction of propagation is in the direction of $\vec{E}\times\vec{B}$

$\vec{E}\times\vec{B} = E_0B_0\,(\hat{i}\times\hat{k}) = E_0B_0\,(-\hat{j})$

Q.18 A boy reaches the airport and finds that the escalator is not working. He walks up the stationary escalator in time t_1. If he remains stationary on a moving escalator then the escalator takes him up in time t_2. The time taken by him to walk up on the moving escalator will be :

Options

1. $\frac{t_1 t_2}{t_2 - t_1}$
2. $\frac{t_1 + t_2}{2}$
3. $t_2 - t_1$
4. $\frac{t_1 t_2}{t_2 + t_1}$

Ans: $\frac{t_1 t_2}{t_2 + t_1}$

Sol: Let 'ℓ' be the length of escalator

Speed of boy w.r.t escalator = $\frac{\ell}{t_1}$

Speed of escalator = $\frac{\ell}{t_2}$

Speed of boy w.r.t ground when escalator is moving = $\frac{\ell}{t_1} + \frac{\ell}{t_2}$

Time taken by the boy to walk on the moving escalator = $\frac{\ell}{\frac{\ell}{t_1} + \frac{\ell}{t_2}} = \frac{t_1 t_2}{t_1 + t_2}$

Q.19 A body rolls down an inclined plane without slipping. The kinetic energy of rotation is 50% of its translational kinetic energy. The body is :

Options

1. Solid cylinder
2. Ring
3. Solid sphere
4. Hollow cylinder

Ans: solid cylinder

Sol: $KE_{rotation}$ = 50% of $KE_{translation}$

$\frac{1}{2} mV^2 \left(\frac{K^2}{R^2}\right) = 50\% \text{ of } \frac{1}{2} mV^2$

$\frac{K^2}{R^2} = \frac{1}{2}$ $\quad\quad K^2 = \frac{I}{M}$

$K^2 = R^2$

$\frac{I}{M} = \frac{1}{2} R^2$

$I = \frac{1}{2} MR^2$

∴ the body is solid cylinder

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Q.20 If time (t), velocity (v), and angular momentum (l) are taken as the fundamental units. Then the dimension of mass (m) in terms of t, v, and l is :

Options
1. $[\, t^{1} v^{2} l^{-1} \,]$
2. $[\, t^{-2} v^{-1} l^{1} \,]$
3. $[\, t^{-1} v^{1} l^{-2} \,]$
4. $[\, t^{-1} v^{-2} l^{1} \,]$

Ans: $[t^{-1} V^{-2} \ell^{1}]$

Sol: $M \propto t^a V^b \ell^c$

$M = K t^a V^b \ell^c$

$[M] = [t^a V^b \ell^c]$

$M'L^0T^0 = T^a (LT^{-1})^b (ML^2 T^{-1})^c$

$M'L^0T^0 = M^c L^{b+2c} T^{a-b-c}$

Comparing, $c = 1$

$b + 2c = 0$

$b = -2$

$a - b - c = 0$

$a + 2 - 1 = 0$

$a = -1$

$m = t^{-1}V^{-2}\ell^{1}$

Section B

Q.1 Two bodies, a ring and a solid cylinder of same material are rolling down without slipping an inclined plane. The radii of the bodies are same. The ratio of velocity of the centre of mass at the bottom of the inclined plane of the ring to that of the cylinder is $\frac{\sqrt{x}}{2}$. Then, the value of x is __________.

Ans: 3.00

Sol: $mgh = \frac{1}{2} I\omega^2$

$$mgh = \frac{1}{2}\left(2mR^2\right)\frac{V_{ring}^2}{R^2}$$

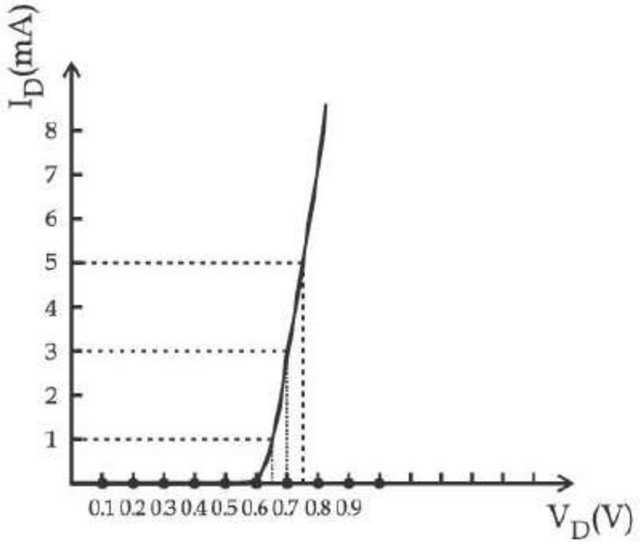
$$V_{ring} = \sqrt{2gh}$$

$$mgh = \frac{1}{2}\left(\frac{3}{2}mR^2\right)\frac{V_{Cylinder}^2}{R^2}$$

$$V_{cylinder} = \sqrt{\frac{4gh}{3}}$$

$$\frac{V_{ring}}{V_{cylinder}} = \frac{\sqrt{3}}{2} \qquad \therefore x = 3$$

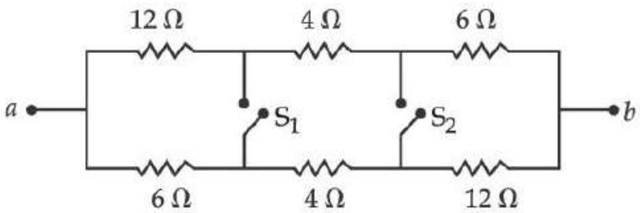
Q.2 For the forward biased diode characteristics shown in the figure, the dynamic resistance at $I_D = 3$ mA will be __________ Ω.



Ans: 25.00

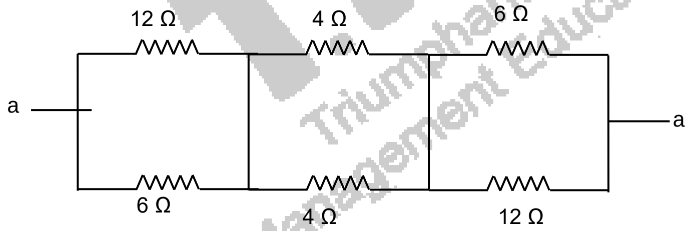
Sol: Dynamic resistance $R_d = \frac{dV}{di} = \frac{0.75 - 0.65}{(5-1) \times 10^{-3}} = 25\ \Omega$

Q.3 In the given figure switches S_1 and S_2 are in open condition. The resistance across *ab* when the switches S_1 and S_2 are closed is __________ Ω.



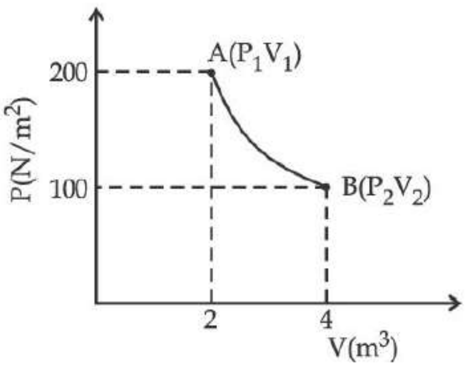
Ans: 10

Sol:



$$R_{ab} = \frac{12 \times 6}{12 + 6} + \frac{4 \times 4}{4 + 4} + \frac{6 \times 12}{6 + 12} = 4 + 2 + 4 = 10\ \Omega$$

Q.4 One mole of an ideal gas at 27°C is taken from A to B as shown in the given PV indicator diagram. The work done by the system will be ________ $\times 10^{-1}$ J.
[Given : R = 8.3 J/mole K, ln2 = 0.6931] (Round off to the nearest integer)



Ans: 17528.00

Sol: Process is isothermal

$W = nRT\, \ell n\left(\frac{V_2}{V_1}\right) = 1 \times 8.3 \times 300 \times \ell n\left(\frac{4}{2}\right) = 17528 \times 10^{-1}$ J

Q.5 A radioactive substance decays to $\left(\frac{1}{16}\right)^{th}$ of its initial activity in 80 days. The half life of the radioactive substance expressed in days is ________.

Ans: 20.00

Sol: $N_0 \xrightarrow{t_{1/2}} \frac{N_0}{2}$

$\frac{N_0}{2} \xrightarrow{t_{1/2}} \frac{N_0}{4}$

$\frac{N_0}{4} \xrightarrow{t_{1/2}} \frac{N_0}{8}$

$\frac{N_0}{8} \xrightarrow{t_{1/2}} \frac{N_0}{16}$

$4t_{1/2} = 80$

$t_{1/2} = \frac{80}{4} = 20 \text{ days}$

Q.6 A body rotating with an angular speed of 600 rpm is uniformly accelerated to 1800 rpm in 10 sec. The number of rotations made in the process is ________.

Ans: 200.00

Sol: $\omega_f = \omega_0 + \alpha t$

$\omega_f - \omega_0 = \alpha t$

$\alpha = 1200 \times 6$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\theta = 600 \times \frac{10}{60} + \frac{1}{2} \times 1200 \times 6 \times \frac{1}{36}$

$\theta = 200$

Q.7 A certain metallic surface is illuminated by monochromatic radiation of wavelength λ. The stopping potential for photoelectric current for this radiation is $3V_0$. If the same surface is illuminated with a radiation of wavelength 2λ, the stopping potential is V_0. The threshold wavelength of this surface for photoelectric effect is _________λ.

Ans: 4.00

Sol: $\frac{hc}{\lambda} = \phi + KE$

KE = eV_0

$e(3V_0) = \frac{hc}{\lambda} - \phi --- (1)$

$e(V_0) = \frac{hc}{2\lambda} - \phi --- (2)$

$\frac{3}{2}\frac{hc}{\lambda} - 3\phi = \frac{hc}{\lambda} - \phi$

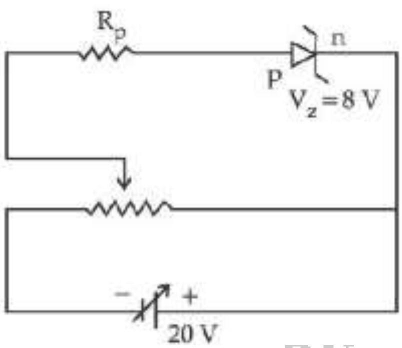
$2\phi = \frac{1}{2}\frac{hc}{\lambda}$

$\phi = \frac{1}{4}\frac{hc}{\lambda}$

$\frac{hc}{\lambda_0} = \frac{1}{4}\frac{hc}{\lambda}$

Threshold wavelength, $\lambda_0 = 4\lambda$

Q.8 A zener diode having zener voltage 8 V and power dissipation rating of 0.5 W is connected across a potential divider arranged with maximum potential drop across zener diode is as shown in the diagram. The value of protective resistance R_p is __________ Ω.



Ans: 192.00

Sol: P = V I

0.5 = 8I

$I = \frac{1}{16}$ A

20 = 8 + IR_P

IR_P = 12

$R_P = \frac{12}{\frac{1}{16}} = 192\Omega$

Q.9 A series LCR circuit of R = 5 Ω, L = 20 mH and C = 0.5 μF is connected across an AC supply of 250 V, having variable frequency. The power dissipated at resonance condition is _________ $\times 10^2$ W.

Ans: 125.00

Sol: $X_L = X_C$ (Resonance)
R = 5 Ω
L = 20 mH
C = 0.5 μF
Z = R (under resonance)

$$I_{rms} = \frac{V}{Z}$$

$$I_{rms} = \frac{V}{R}$$

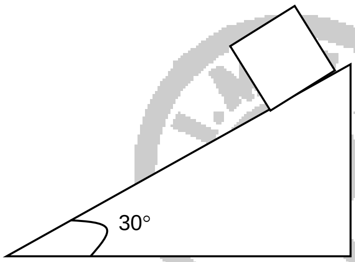
$$\text{Power} = \frac{V^2}{R} = \frac{250 \times 250}{5}$$

$= 125 \times 10^2$ W

Q.10 A body of mass '*m*' is launched up on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of friction between the body and plane is $\frac{\sqrt{x}}{5}$ if the time of ascent is half of the time of descent. The value of x is __________.

Ans: 3.00

Sol:



Let t_a be the time of ascent and t_b be the time of descent;

$$t_a = \frac{1}{2} t_d$$

$$\sqrt{\frac{25}{a_a}} = \frac{1}{2}\sqrt{\frac{25}{a_d}} \text{ ----(1)}$$

$$a_a = g \sin\theta + \mu g \cos\theta = g \sin 30 + \frac{\sqrt{3}}{2}\mu g = \frac{g}{2} + \frac{\sqrt{3}}{2}\mu g$$

$$a_d = g \sin\theta - \mu g \cos\theta = g \sin 30 - \mu g \cos 30 = \frac{g}{2} - \frac{\sqrt{3}}{2}\mu g$$

(1) $\Rightarrow a_a = 4\, a_d$

$$\frac{g}{2} + \frac{\sqrt{3}}{2}\mu g = 4\left[\frac{g}{2} - \frac{\sqrt{3}}{2}\mu g\right]$$

$$\frac{5\sqrt{3}}{2}\mu g = \frac{3g}{2}$$

$$\mu = \frac{\sqrt{3}}{5}$$

x = 3

PART – B – CHEMISTRY

Section A

Q.1 Which one of the following gases is reported to retard photosynthesis ?

Options
1. CFCs
2. CO
3. CO_2
4. NO_2

Ans: NO_2

Sol: Higher concentrations of NO_2 damage the leaves of plants and retard the rate of photosynthesis

Q.2 Consider two chemical reactions (A) and (B) that take place during metallurgical process :

(A) $ZnCO_{3(s)} \xrightarrow{\Delta} ZnO_{(s)} + CO_{2(g)}$

(B) $2ZnS_{(s)} + 3O_{2(g)} \xrightarrow{\Delta} 2ZnO_{(s)} + 2SO_{2(g)}$

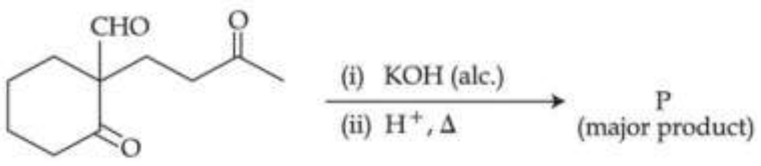
The **correct** option of names given to them respectively is :

Options
1. (A) is calcination and (B) is roasting
2. (A) is roasting and (B) is calcination
3. Both (A) and (B) are producing same product so both are calcination
4. Both (A) and (B) are producing same product so both are roasting

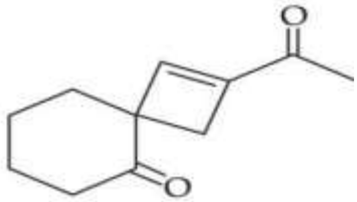
Ans: (A) is calcination and (B) is roasting

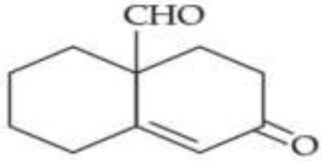
Sol: Calcination is usually employed for hydroxides and carbonate ores whereas roasting is used for converting sulphide ores into their respective oxides form

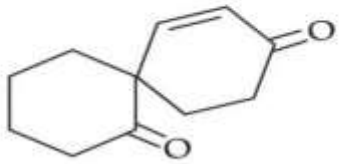
Q.3 The major product (P) in the following reaction is :

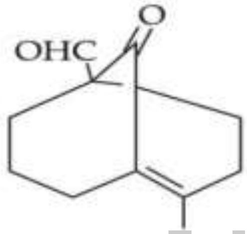


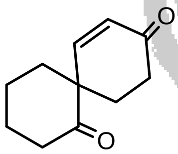
Options

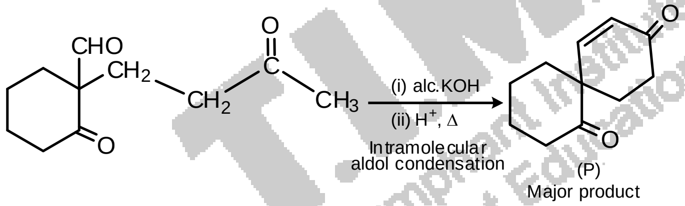
1. 

2. 

3. 

4. 

Ans: 

Sol: 

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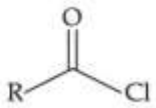
Q.4 In Carius method, halogen containing organic compound is heated with fuming nitric acid in the presence of :

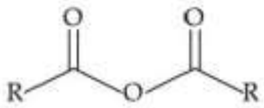
Options
1. $AgNO_3$
2. HNO_3
3. $CuSO_4$
4. $BaSO_4$

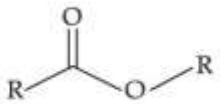
Ans: $AgNO_3$

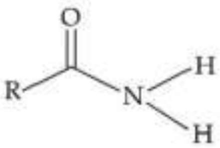
Sol: In Carius method, organic compound containing halogen is heated with fuming HNO_3 in the presence of $AgNO_3$. The halogen present forms the corresponding silver halides

Q.5

(A) 

(B) 

(C) 

(D) 

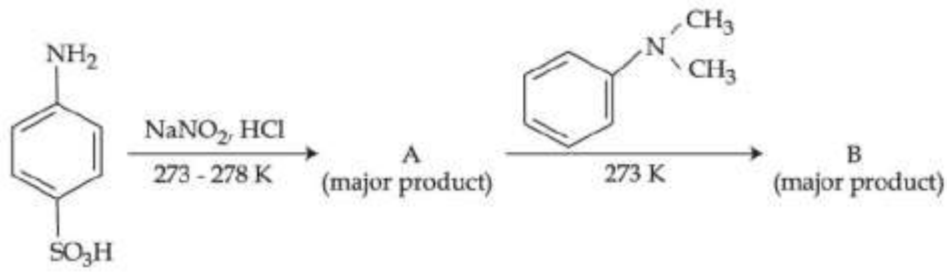
The **correct** order of their reactivity towards hydrolysis at room temperature is :

Options
1. (D) > (A) > (B) > (C)
2. (A) > (B) > (C) > (D)
3. (D) > (B) > (A) > (C)
4. (A) > (C) > (B) > (D)

Ans: (A) > (B) > (C) > (D)

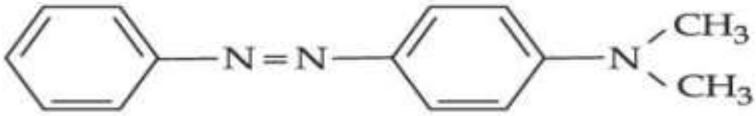
Sol: The reactivity of acid derivatives towards hydrolysis at room temperature is of the order
Acid chloride > acid anhydride > esters > amide

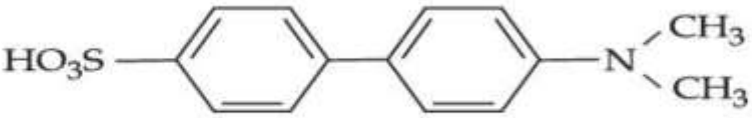
Q.6

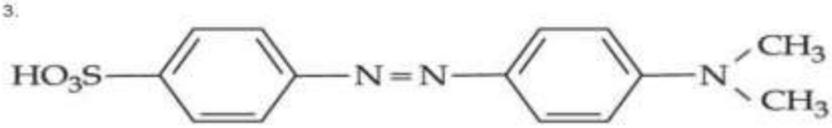


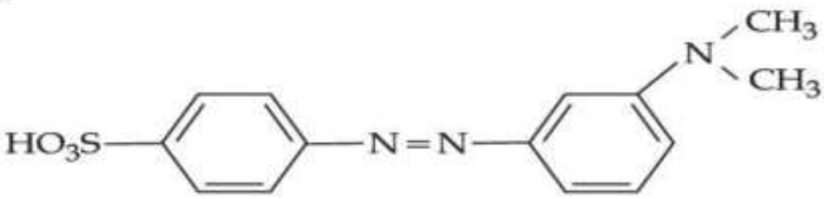
Consider the above reaction, compound B is :

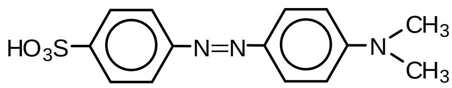
Options

1. 

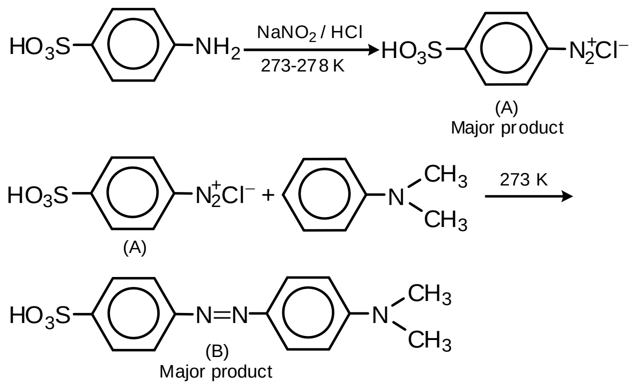
2. 

3. 

4. 

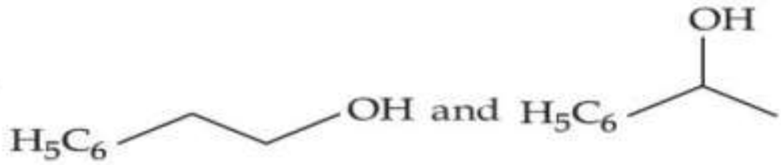
Ans: 

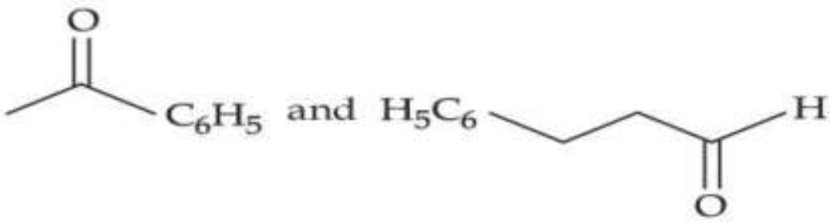
Sol:

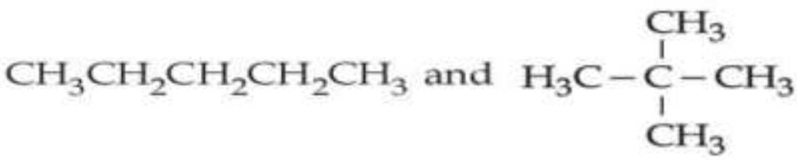


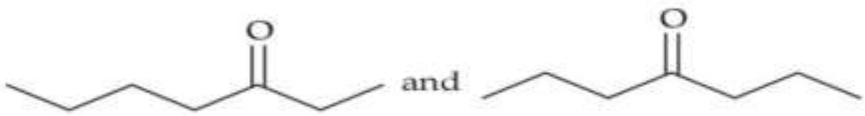
Q.7 Which one of the following pairs of isomers is an example of metamerism ?

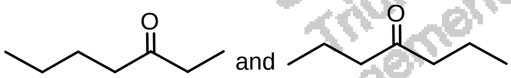
Options

1. 

2. 

3. 

4. 

Ans: 

Sol: $CH_3-CH_2-CH_2-\overset{O}{\overset{\|}{C}}-CH_2-CH_3$ (Heptan-3-one) and $CH_3-CH_2-CH_2-\overset{O}{\overset{\|}{C}}-CH_2-CH_2-CH_3$ (Heptan-4-one) exhibits metamerism

Q.8 Metallic sodium does not react normally with :

Options

1. tert-butyl alcohol
2. But-2-yne
3. Ethyne
4. gaseous ammonia

Ans: But-2-yne

Sol: But-2-yne does not contain any acidic hydrogen. So it does not react with metallic sodium

Q.9 Outermost electronic configuration of a group 13 element, E, is $4s^2$, $4p^1$. The electronic configuration of an element of p-block period-five placed diagonally to element, E is :

Options
1. $[Kr]\ 4d^{10}\ 5s^2\ 5p^2$
2. $[Ar]\ 3d^{10}\ 4s^2\ 4p^2$
3. $[Xe]\ 5d^{10}\ 6s^2\ 6p^2$
4. $[Kr]\ 3d^{10}\ 4s^2\ 4p^2$

Ans: $[Kr]\ 4d^{10}\ 5s^2\ 5p^2$

Sol: Group 13 element with electronic configuration $4s^2\ 4p^1$ is Ga. The element placed diagonally to Ga in the fifth period is Sn (Z = 50) – $[Kr]\ 5s^2\ 4d^{10}\ 5p^2$

Q.10 Which one of the following statements is **not** true about enzymes ?

Options
1. Enzymes are non-specific for a reaction and substrate.
2. Almost all enzymes are proteins.
3. The action of enzymes is temperature and pH specific.
4. Enzymes work as catalysts by lowering the activation energy of a biochemical reaction.

Ans: Enzymes are non-specific for a reaction and substrate

Sol: Each enzyme is specific for a given reaction

Q.11 The single largest industrial application of dihydrogen is :

Options
1. Manufacture of metal hydrides
2. Rocket fuel in space research
3. In the synthesis of ammonia
4. In the synthesis of nitric acid

Ans: In the synthesis of ammonia

Sol: The largest single use of dihydrogen is in the synthesis of ammonia which is used in the manufacture of nitric acid and nitrogenous fertilizer

Q.12 Benzene on nitration gives nitrobenzene in presence of HNO_3 and H_2SO_4 mixture, where :

Options
1. HNO_3 acts as an acid and H_2SO_4 acts as a base
2. HNO_3 acts as a base and H_2SO_4 acts as an acid
3. both H_2SO_4 and HNO_3 act as an acids
4. both H_2SO_4 and HNO_3 act as a bases

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Ans: HNO_3 acts as a base and H_2SO_4 acts as acid

Sol: In the nitrating mixture, HNO_3 acts as a base and H_2SO_4 acts as an acid

$$H{-}HSO_4 + O_2N{-}OH \longrightarrow HSO_4^{\ominus} + \overset{\oplus}{N}O_2 + H_2O$$

Q.13 Bakelite is a cross-linked polymer of formaldehyde and :

Options
1. Dacron
2. Buna-S
3. PHBV
4. Novolac

Ans: Novolac

Sol: Novolac on heating with HCHO undergoes cross linking to form an infusible solid mass called bakelite

Q.14 The hybridisations of the atomic orbitals of nitrogen in NO_2^-, NO_2^+ and NH_4^+ respectively are :

Options
1. sp, sp^2 and sp^3
2. sp^3, sp^2 and sp
3. sp^3, sp and sp^2
4. sp^2, sp and sp^3

Ans: sp^2, sp and sp^3

Sol: In NO_2^-, No. of hybrid orbitals $= \frac{1}{2}[5+1] = 3$

Hence, sp^2 hybridisation

In NO_2^+, No. of hybrid orbitals $= \frac{1}{2}[5-1] = 2$

Hence, sp hybridisation

In NH_4^+, No. of hybrid orbitals $= \frac{1}{2}[5+4-1] = 4$

Hence, sp^3 hybridisation

Q.15 Spin only magnetic moment of an octahedral complex of Fe^{2+} in the presence of a strong field ligand in BM is :

Options
1. 2.82
2. 4.89
3. 0
4. 3.46

Ans: 0

Sol: $Fe^{2+} \rightarrow [Ar]\ 3d^6\ 4s^0$

In presence of strong ligands, pairing takes place

Hence, $Fe^{2+} \rightarrow$ 3d: | ↑↓ | ↑↓ | ↑↓ | | |

$\therefore$ No. of unpaired electrons = 0

$\therefore$ Magnetic moment, $\mu = 0$

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Q.16 A solution is 0.1 M in Cl^- and 0.001 M in CrO_4^{2-}. Solid $AgNO_3$ is gradually added to it. Assuming that the addition does not change in volume and $K_{sp}(AgCl) = 1.7 \times 10^{-10}$ M^2 and $K_{sp}(Ag_2CrO_4) = 1.9 \times 10^{-12}$ M^3.

Select **correct** statement from the following :

Options

1. Ag_2CrO_4 precipitates first as its K_{sp} is low.
2. Ag_2CrO_4 precipitates first because the amount of Ag^+ needed is low.
3. AgCl will precipitate first as the amount of Ag^+ needed to precipitate is low.
4. AgCl precipitates first because its K_{sp} is high.

Ans: AgCl will precipitate first as the amount of Ag^+ needed to precipitate is low

Sol: $AgCl \rightarrow Ag^+ + Cl^-$

$Ag_2CrO_4 \rightarrow 2Ag^+ + CrO_4^{2-}$

$\underset{1.7\times10^{-10}}{K_{sp}\,(AgCl)} = [Ag+]\underset{10^{-1}}{[Cl^-]}$

$\therefore$ Solubility of $[Ag^+] = \dfrac{1.7\times10^{-10}}{10^{-1}} = 1.7\times10^{-9}$

$K_{sp}\,(Ag_2CrO_4) = [Ag^+]^2\ [CrO_4^{2-}]$

$1.9 \times 10^{-12} \quad = [Ag^+]^2 \times 10^{-3}$

$\therefore$ Solubility of $[Ag^+] = \sqrt{\dfrac{1.9\times10^{-12}}{10^{-3}}}$

$= \sqrt{1.9\times10^{-9}}$

$= 4.4 \times 10^{-5}$

Since the solubility of Ag^+ in AgCl is less, that will precipitate faster

Q.17 Cu^{2+} salt reacts with potassium iodide to give :

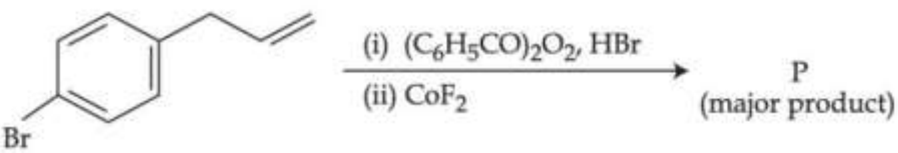
Options

1. Cu_2I_3
2. CuI
3. Cu_2I_2
4. $Cu(I_3)_2$

Ans: Cu_2I_2

Sol: $2Cu^{2+} + 4I^- \rightarrow Cu_2I_{2(s)} + I_2$

Q.18



Major product P of above reaction, is :

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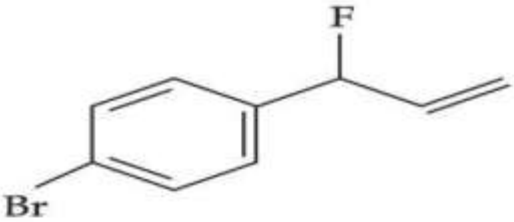
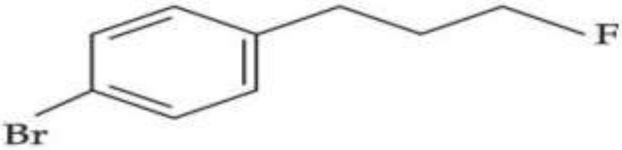
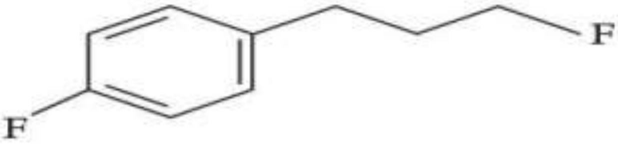
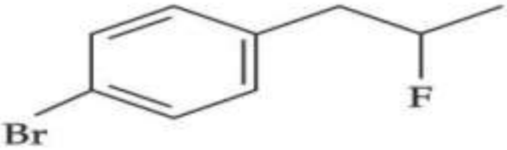
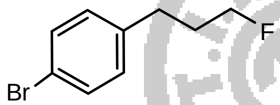
Options

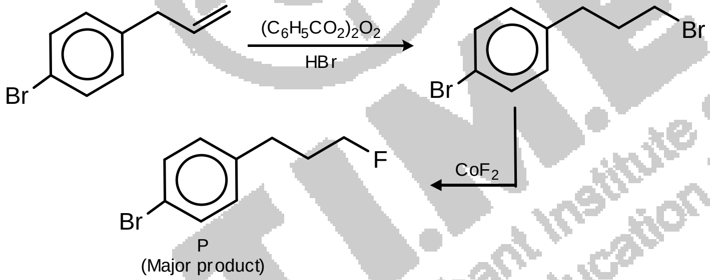
1. 

2. 

3. 

4. 

Ans: 

Sol: 

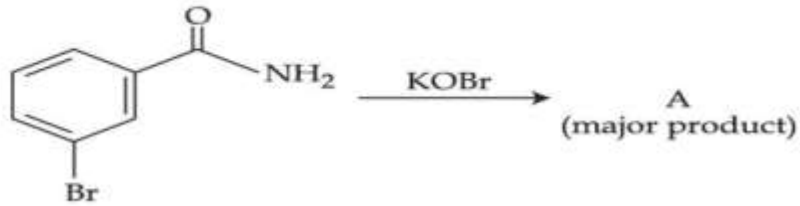
Q.19 Which one of the following species **doesn't** have a magnetic moment of 1.73 BM, (spin only value) ?

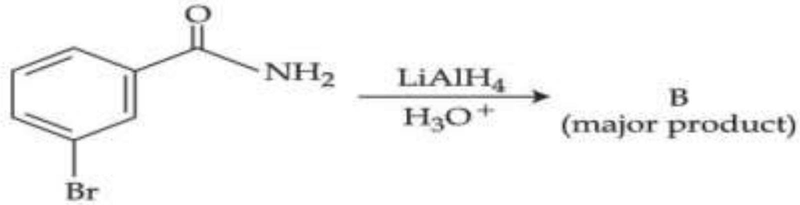
Options
1. $[Cu(NH_3)_4]Cl_2$
2. O_2^+
3. CuI
4. O_2^-

Ans: CuI

Sol: Magnetic moment of 1.7 BM suggest the presence of one unpaired electron, In CuI, Cu is in +1 oxidation state and hence 0 unpaired electrons

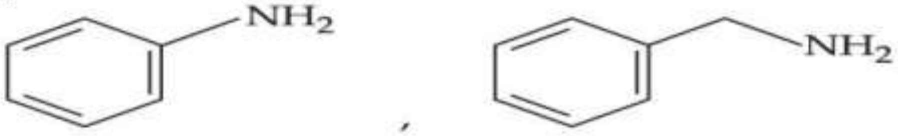
Q.20



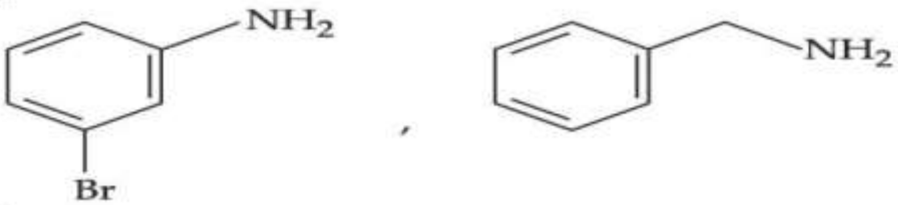


In the above reactions, product A and product B respectively are :

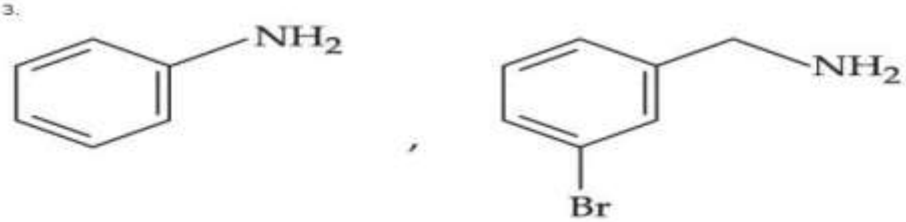
Options 1.



2.



3.



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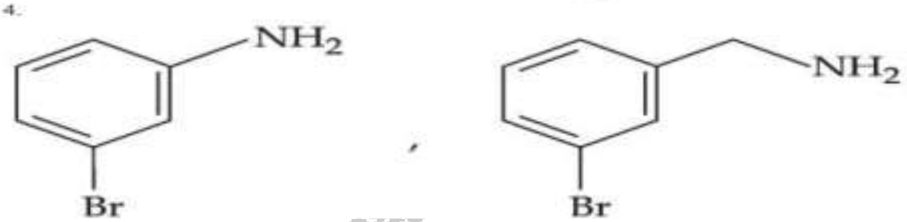
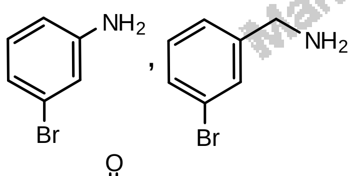
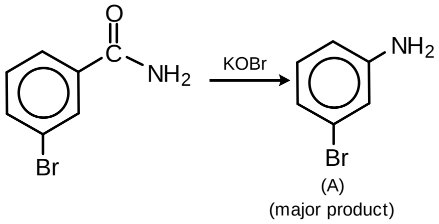
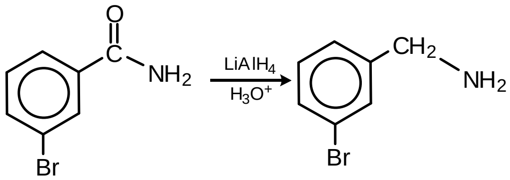


Ans:



Sol:





Section B

Q.1 The vapour pressures of A and B at 25°C are 90 mm Hg and 15 mm Hg respectively. If A and B are mixed such that the mole fraction of A in the mixture is 0.6, then the mole fraction of B in the vapour phase is $x \times 10^{-1}$. The value of x is __________. (Nearest integer)

Ans: 1

Sol: Mole fraction of B in the vapour phase, $x_B = \frac{P_B}{P_{total}}$

i.e., $x \times 10^{-1} = \frac{P_B}{P_{Total}} = \frac{P_B^\circ . x_B}{P_B^\circ x_B + P_A^\circ x_A}$

$= \frac{15 \times 0.4}{15 \times 0.4 + 90 \times 0.6} = \frac{6}{6+54}$

$= \frac{6}{60} = 10^{-1} = 1 \times 10^{-1}$

$\therefore x = 1$

Q.2 For a given chemical reaction A → B at 300 K the free energy change is -49.4 kJ mol^{-1} and the enthalpy of reaction is 51.4 kJ mol^{-1}. The entropy change of the reaction is __________ J K^{-1} mol^{-1}.

Ans: 336

Sol: $\Delta G = -49.4$ kJ mol^{-1}

$\Delta H = 51.4$ kJ mol^{-1}

$\Delta G = \Delta H - T\Delta S$

$-49.4 \times 10^3 = 51.4 \times 10^{-3} - 300 \times x$

$-49400 = 51400 - 300$ x

300 x = 51400 + 49400

300 x = 100800

$x = \frac{100800}{300} = 336 \text{ J } K^{-1} mol^{-1}$

Q.3 100 ml of 0.0018% (w/v) solution of Cl^- ion was the minimum concentration of Cl^- required to precipitate a negative sol in one h. The coagulating value of Cl^- ion is __________. (Nearest integer)

Ans: **1**

Sol: Weight of Cl^- ion present in 100 mL of soln = 1.08×10^{-3} g

$\therefore$ No. of moles $= \frac{1.8 \times 10^{-3}}{35.5} = 0.05 \times 10^{-3}$

= 0.05 milli moles

i.e., 0.05 milli moles of $Cl^- \rightarrow$ 100 mL

$\therefore$ 0.5 milli moles of $Cl^- \rightarrow$ 1 L

Coagulating value = 0.5

Answer = 1 (nearest integer)

Q.4 When 0.15 g of an organic compound was analyzed using Carius method for estimation of bromine, 0.2397 g of AgBr was obtained. The percentage of bromine in the organic compound is __________. (Nearest integer)

$$\begin{bmatrix} \text{Atomic mass :} \\ \text{Silver} = 108 \\ \text{Bromine} = 80 \end{bmatrix}$$

Ans: 68

Sol: % of bromine in the organic compound $= \frac{80}{188} \times \frac{\text{Wt. of AgBr}}{\text{Wt. of OC}} \times 100$

$$= \frac{80}{188} \times \frac{0.2397}{0.15} \times 100 = 68\%$$

Q.5 Diamond has a three dimensional structure of C atoms formed by covalent bonds. The structure of diamond has face centred cubic lattice where 50% of the tetrahedral voids are also occupied by carbon atoms. The number of carbon atoms present per unit cell of diamond is __________.

Ans: 8

Sol: Diamond has ZnS structure
No. of C atoms occupying fcc structure = 4
No. of C atoms occupying half of the tetrahedral voids = 4
$\therefore$ Total number of carbons atoms present per unit cell of diamond = 4 +4 = 8

Q.6 4 g equimolar mixture of NaOH and Na_2CO_3 contains x g of NaOH and y g of Na_2CO_3. The value of x is __________ g.

(Nearest integer)

Ans: 1

Sol: Mass of NaOH = x gm
$\therefore$ Mass of Na_2CO_3 = (4 – x) gm
No. of moles of NaOH $= \frac{x}{40}$
No. of moles of Na_2CO_3 $= \frac{4-x}{106}$
Since equimolar amount of NaOH and Na_2CO_3 is taken,
$\frac{x}{40} = \frac{4-x}{106}$
106 x = 160 – 40 x
106 + 40 x = 160
146 x = 160
$x = \frac{160}{146} = 1.1\text{g}$
$\therefore$ Mass of NaOH = 1.1 g

Q.7 An aqueous solution of $NiCl_2$ was heated with excess sodium cyanide in presence of strong oxidizing agent to form $[Ni(CN)_6]^{2-}$. The total change in number of unpaired electrons on metal centre is __________.

Ans: 2

Sol: In $NiCl_2$, $Ni^{2+} \rightarrow [Ar]\ 3d^8\ 4s^0$
Number of unpaired electrons = 2
In $[Ni(CN)_6]^{2-}$, $Ni^{4+} \rightarrow [Ar]\ 3d^6\ 4s^0$
Since it is a strong ligand, pairing of electrons takes place. Hence all the electrons undergo pairing
Number of unpaired electrons = 0
$\therefore$ Change in the number of unpaired electrons 2 − 0 = 2

Q.8 $PCl_5(g) \rightarrow PCl_3(g) + Cl_2(g)$

In the above first order reaction the concentration of PCl_5 reduces from initial concentration 50 mol L^{-1} to 10 mol L^{-1} in 120 minutes at 300 K. The rate constant for the reaction at 300 K is $x \times 10^{-2}\ min^{-1}$. The value of x is ________.

[Given $\log 5 = 0.6989$]

Ans: 1

Sol: For a first order reaction,

$$k = \frac{2.303}{t}\log\frac{[R]_0}{[R]} = \frac{2.303}{120}\times\log\frac{50}{10} = \frac{2.303}{120}\times\log 5$$

$$= \frac{2.303}{120}\times 0.6989 = 0.0134\ min^{-1} = 1.34 \times 10^{-2}\ min^{-1}$$

Q.9 The wavelength of electrons accelerated from rest through a potential difference of 40 kV is $x \times 10^{-12}$ m. The value of x is ________. (Nearest integer)

Given : Mass of electron $= 9.1 \times 10^{-31}$ kg
Charge on an electron $= 1.6 \times 10^{-19}$ C
Planck's constant $= 6.63 \times 10^{-34}$ Js

Ans: 6

Sol: $$\lambda = \frac{h}{\sqrt{2meV}} = \frac{6.63\times10^{-34}}{\sqrt{2\times1.6\times10^{-19}\times9.1\times10^{-31}\times40\times10^{3}}}$$

$$= 0.614 \times 10^{-11}\ m = 6.14 \times 10^{-12}\ m$$

Q.10 Potassium chlorate is prepared by electrolysis of KCl in basic solution as shown by following equation.

$$6\ OH^- + Cl^- \rightarrow ClO_3^- + 3\ H_2O + 6\ e^-$$

A current of xA has to be passed for 10 h to produce 10.0 g of potassium chlorate. The value of x is ________. (Nearest integer)

(Molar mass of $KClO_3 = 122.6\ g\ mol^{-1}$, F = 96500 C)

Ans: 1

Sol: For 1 mole $KClO_3 \Rightarrow$ 6F required
i.e., 122.5 g $KClO_3 \rightarrow 6 \times 96500$ C
Total charge passing through the solution = I × t = x × 10 × 60 × 60 C
122.5 g $KClO_3 \rightarrow 6 \times 96500$ C
10 g $KClO_3 \rightarrow x \times 10 \times 60 \times 60$

$$x = \frac{10\times6\times96500}{122.5\times10\times60\times60} = 1.312\ A$$

PART – C – MATHEMATICS

Section A

Q.1 For the natural numbers m, n, if $(1-y)^m (1+y)^n = 1 + a_1 y + a_2 y^2 + ... + a_{m+n} y^{m+n}$ and $a_1 = a_2 = 10$, then the value of $(m+n)$ is equal to :

Options
1. 100
2. 64
3. 80
4. 88

Ans: 80

Sol: $(1-y)^m(1+y)^n = 1 + a_1 y + a_2 y^2 + ...$

a_1 is coefficient of $y^2 = 1.{}^nC_1 - {}^mC_1.1 = 10$

$\Rightarrow n - m = 10..........\ (1)$

a_2 is coefficient of $y^2 = {}^mC_2 + {}^nC_2 - {}^mC_1.{}^nC_1 = 10$

$\Rightarrow m(m-1) + n(n-1) - 2mn = 10.....(2)$

From (1) and (2)

$m^2 + n^2 - 2mn - (m+n) = 20$

$(m-n)^2 - (m+n) = 20$

$(10)^2 - (m+n) = 20$

$(m+n) = 80$

Q.2 Let $f : \mathbf{R} - \left\{\frac{\alpha}{6}\right\} \to \mathbf{R}$ be defined by $f(x) = \frac{5x+3}{6x-\alpha}$. Then the value of α for which $(fof)(x) = x$, for all $x \in \mathbf{R} - \left\{\frac{\alpha}{6}\right\}$, is :

Options
1. 8
2. 5
3. No such α exists
4. 6

Ans: 5

Sol: $f(x) = \frac{5x+3}{6x-\alpha}$

$$f(f(x)) = \frac{5\left(\frac{5x+3}{6x-\alpha}\right)+3}{6\left(\frac{5x+3}{6x-\alpha}\right)-\alpha} = x$$

$$= \frac{25x+15+18x-3\alpha}{30x+18-6x\alpha+\alpha^2} = x$$

Simplifying, we get

$\Rightarrow 6(5-\alpha)x^2 + (\alpha-5)(\alpha+5)x + 3(\alpha-5) = 0$

$\Rightarrow \alpha = 5$

Q.3 Let in a right angled triangle, the smallest angle be θ. If a triangle formed by taking the reciprocal of its sides is also a right angled triangle, then $\sin\theta$ is equal to :

Options

1. $\frac{\sqrt{5}-1}{4}$
2. $\frac{\sqrt{2}-1}{2}$
3. $\frac{\sqrt{5}-1}{2}$
4. $\frac{\sqrt{5}+1}{4}$

Ans: $\frac{\sqrt{5}-1}{2}$

Sol: Let a>b>c

$\sin\theta = \frac{c}{a}$

$\frac{1}{a} < \frac{1}{b} < \frac{1}{c}$

$\frac{1}{c^2} = \frac{1}{a^2} + \frac{1}{b^2}$

$1 = \frac{c^2}{a^2} + \frac{c^2}{b^2}$

$1 = \frac{c^2}{a^2} + \frac{c^2}{a^2 - c^2}$ [As $a^2 = b^2 + c^2$

$1 = \frac{1-\sin^2\theta + 1}{\operatorname{cosec}^2\theta - 1} \Rightarrow \sin^2\theta + \operatorname{cosec}^2\theta = 3$

Let $sin^2\theta = u$

$u + \frac{1}{u} = 3$

Solving above equation we get

$u = \frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2} (not\ acceptable)$

so, $\sin\theta = \sqrt{\frac{3-\sqrt{5}}{2}} \Rightarrow \sin\theta = \frac{\sqrt{5}-1}{2}$

Q.4 If $f : \mathbf{R} \to \mathbf{R}$ is given by $f(x) = x + 1$, then the value of

$$\lim_{n\to\infty} \frac{1}{n}\left[f(0) + f\left(\frac{5}{n}\right) + f\left(\frac{10}{n}\right) + ... + f\left(\frac{5(n-1)}{n}\right)\right], \text{ is :}$$

Options

1. $\frac{1}{2}$
2. $\frac{5}{2}$
3. $\frac{3}{2}$
4. $\frac{7}{2}$

Ans: $\frac{7}{2}$

Sol: $= \lim_{n\to\infty} \frac{1}{n}\sum_{r=0}^{n=1} f\left(\frac{5r}{n}\right) = \int_0^1 f(5x)dx = \int_0^1 (5x+1)dx$

$= \left(\frac{5x^2}{2}+x\right)\Big|_0^1 = \frac{5}{2}+1=\frac{7}{2}$

Q.5 Let r_1 and r_2 be the radii of the largest and smallest circles, respectively, which pass through the point $(-4, 1)$ and having their centres on the circumference of the circle $x^2+y^2+2x+4y-4=0$. If $\frac{r_1}{r_2} = a + b\sqrt{2}$, then $a+b$ is equal to :

Options
1. 5
2. 11
3. 7
4. 3

Ans: 5

Sol: $x^2 + y^2 + 2x + 4y - 4 = 0$
$(x + 1)^2 + (y + 2)^2 = 3^3$
General point on circumference $(3 \cos \theta - 1, 3 \sin \theta - 2)$
As centre of circle is $(-1, -2)$

So, $r = \sqrt{(3\cos\theta+3)^2+(3\sin\theta-3)^2}$

$= 3\sqrt{\cos^2\theta+1+2\cos\theta+\sin^2\theta+1-2\sin\theta}$

$= 3\sqrt{3+2(\cos\theta-\sin\theta)}$

$\frac{r_1}{r_2} = 3 + 2\sqrt{2}$

On comparing with $\frac{r_1}{r_2} = a + b\sqrt{2}$

$a + b = 5$

Q.6 Let $g(t) = \int_{-\pi/2}^{\pi/2} \cos\left(\frac{\pi}{4}t + f(x)\right)dx$, where $f(x) = \log_e\left(x+\sqrt{x^2+1}\right)$, $x \in \mathbf{R}$. Then which one of the following is correct ?

Options
1. $g(1) + g(0) = 0$
2. $\sqrt{2}\, g(1) = g(0)$
3. $g(1) = g(0)$
4. $g(1) = \sqrt{2}\, g(0)$

Ans: $\sqrt{2}\, g(1) = g(0)$

Sol: $g(t) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\left(\frac{\pi}{4}t + f(x)\right)dx, f(x) = \log_e\left(x+\sqrt{x^2+1}\right)$

Putting t = 1 in g(t) , we get

$$g(1) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\left(\frac{\pi}{4} + \log_e\left(x + \sqrt{x^2+1}\right)\right)dx,$$

Putting t=0 in g(t), we get

$$g(0) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\left[\log_e\left(x + \sqrt{x^2+1}\right)\right]$$

$$g(1) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[\frac{1}{\sqrt{2}}\cos\left[\log_e\left(x + \sqrt{x^2+1}\right)\right] - \frac{1}{\sqrt{2}}\sin\left[\log_e\left(x + \sqrt{x^2+1}\right)\right]\right]dx$$

Since, $\left[\sin\left(\log_e\left(x + \sqrt{x^2+1}\right)\right)\right]$ is odd function

So $g(1) = \frac{1}{\sqrt{2}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\left(\log_e\left(x + \sqrt{x^2+1}\right)\right)dx$

$\sqrt{2}g(1) = g(0)$

Q.7 Consider the following three statements :

(A) If $3+3=7$ then $4+3=8$.

(B) If $5+3=8$ then earth is flat.

(C) If both (A) and (B) are true then $5+6=17$.

Then, which of the following statements is correct ?

Options
1. (A) and (B) are false while (C) is true
2. (A) is true while (B) and (C) are false
3. (A) is false, but (B) and (C) are true
4. (A) and (C) are true while (B) is false

Ans: (A) and (C) are true while (B) is false

Sol: By the truth value of $p \to q$
A is true, B is false, C is true.

Q.8
Consider the line L given by the equation $\frac{x-3}{2} = \frac{y-1}{1} = \frac{z-2}{1}$. Let Q be the mirror image of the point $(2, 3, -1)$ with respect to L. Let a plane P be such that it passes through Q, and the line L is perpendicular to P. Then which of the following points is on the plane P ?

Options
1. (1, 1, 2)
2. (−1, 1, 2)
3. (1, 1, 1)
4. (1, 2, 2)

Ans: (1, 2, 2)

Sol: Let A (2, 3, –1)

Let image of A (2, 3, –1) in the line mirror

$\frac{x-3}{2}=\frac{y-1}{1}=\frac{z-2}{1}$ is $Q(\alpha,\beta,\gamma)$

$\Rightarrow\left(\frac{\alpha+2}{2},\frac{\beta+3}{2},\frac{\gamma-1}{2}\right)$ lies on $\frac{x-3}{2}=\frac{y-1}{1}=\frac{z-2}{1}$

$\Rightarrow\frac{\alpha-4}{4}=\frac{\beta+1}{2}=\frac{\gamma-5}{2}$.........(1)

Also AQ $\perp$ to given line (L)

$\Rightarrow 2\alpha+\beta+\gamma-6=0$.......... ..(2)

by solving (1) and (2)

we get $\alpha=2,\beta=-2,\gamma=4$

$\Rightarrow Q(2,-2,4)$

Now equation of plane P which passes through Q(1,-1,5)

and perpendicular to the line L is

2x+y+z = 6

Hence point (1, 2, 2) lies in it

Q.9 In a triangle ABC, if $|\overrightarrow{BC}|=3$, $|\overrightarrow{CA}|=5$ and $|\overrightarrow{BA}|=7$, then the projection of the vector $\overrightarrow{BA}$ on $\overrightarrow{BC}$ is equal to :

Options

1. $\frac{15}{2}$
2. $\frac{11}{2}$
3. $\frac{19}{2}$
4. $\frac{13}{2}$

Ans: $\frac{11}{2}$

Sol: Projection of $\overrightarrow{BA}$ on $\overrightarrow{BC}$ $=7\cos\theta$ $=7\left[\frac{7^2+3^2-5^2}{2\times7\times3}\right]=\frac{11}{2}$

Q.10 The lines $x=ay-1=z-2$ and $x=3y-2=bz-2$, $(ab\neq0)$ are coplanar, if :

Options

1. $a=1,\ b\in\mathbf{R}-\{0\}$
2. $b=1,\ a\in\mathbf{R}-\{0\}$
3. $a=2,\ b=3$
4. $a=2,\ b=2$

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Ans: $b=1, a\in R-\{0\}$

Sol: $\dfrac{x-0}{1}=\dfrac{y-\frac{1}{a}}{\frac{1}{a}}=\dfrac{z-2}{1}$ ......(1)

$\dfrac{x-0}{1}=\dfrac{y-\frac{2}{3}}{\frac{1}{3}}=\dfrac{z-\frac{2}{b}}{\frac{1}{b}}$ ......(2)

For coplanar:

$$\begin{vmatrix} 0 & \frac{2}{3}-\frac{1}{a} & \frac{2}{b}-2 \\ 1 & \frac{1}{a} & 1 \\ 1 & \frac{1}{3} & \frac{1}{b} \end{vmatrix}=0$$

$\Rightarrow 3-3b-2a+2ab+2a-6-2ab+6b=0$

$\Rightarrow 3b-3=0$

$\Rightarrow b=1, a\in R-\{0\}$

Q.11 If $[x]$ denotes the greatest integer less than or equal to x, then the value of the integral $\int_{-\pi/2}^{\pi/2}[[x]-\sin x]\,dx$ is equal to :

Options
1. 0
2. 1
3. $-\pi$
4. π

Ans: $-\pi$

Sol: $\sum_{-\pi/2}^{\pi/2}[[x]-\sin x]dx$

$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}[[x]-\sin x]dx$

$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}([-\sin x]+[x])dx \quad (\because [x+I]=[x]+I)$

$=\int_{0}^{\frac{\pi}{2}}([-\sin x]+[x]+[\sin x]+[-x])dx$

$\int_{0}^{\frac{\pi}{2}}(-1-1)dx \quad \{\because [x]+[-x]=-1, x\notin I\}$

$=-2(x)\Big|_{0}^{\frac{\pi}{2}}$

$=-\pi$

Q.12

Let $y=y(x)$ satisfies the equation $\frac{dy}{dx}-|A|=0,$ for all $x>0$, where $A=\begin{bmatrix} y & \sin x & 1 \\ 0 & -1 & 1 \\ 2 & 0 & \frac{1}{x} \end{bmatrix}$.

If $y(\pi)=\pi+2$, then the value of $y\left(\frac{\pi}{2}\right)$ is :

Options

1. $\frac{\pi}{2}-\frac{1}{\pi}$
2. $\frac{\pi}{2}+\frac{4}{\pi}$
3. $\frac{\pi}{2}-\frac{4}{\pi}$
4. $\frac{3\pi}{2}-\frac{1}{\pi}$

Ans: $\frac{\pi}{2}+\frac{4}{\pi}$

Sol: $|A|=y\left(-\frac{1}{x}\right)-\sin x(-2)+1(+2)$

$|A|=-\frac{y}{x}+2\sin x+2$

$\Rightarrow \frac{dy}{dx}-\left(-\frac{y}{x}+2\sin x+2\right)=0$

$\Rightarrow \frac{dy}{dx}+\frac{y}{x}-2(\sin x+1)=0$

$I.F=e^{\int\frac{1}{x}dx}=x$

Solution is

$yx=\int 2(\sin x+1)x dx$

$\Rightarrow yx=x^2-2x\cos x+2\sin x+c$

At $x=\pi, y=\pi+2$

$\Rightarrow c=0$

At $x=\frac{\pi}{2}$

$\Rightarrow y.\frac{\pi}{2}=\left(\frac{\pi}{2}\right)^2-0+2+0 \Rightarrow y.\frac{\pi}{2}=\left(\frac{\pi}{2}\right)^2+2$

$\Rightarrow y.=\left(\frac{\pi}{2}\right)^2+\frac{4}{\pi}$

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Q.13 If sum of the first 21 terms of the series $\log_{9^{1/2}} x + \log_{9^{1/3}} x + \log_{9^{1/4}} x + \ldots$, where $x > 0$ is 504, then x is equal to :

Options
1. 81
2. 7
3. 243
4. 9

Ans: 81

Sol: $2\log_9 x + 3\log_9 x + 4\log_9 x \ldots\ldots 21 \text{terms}$

$= (2+3+4+5+\ldots\ldots+22)\log_9 x = \frac{21}{2}(2+22)\log_9 x$

$= 21 \times 12 \log_9 x$

$= 252\log_9 x = 504$

$\log_9 x = 2 \Rightarrow x = 81$

Q.14 If the mean and variance of six observations 7, 10, 11, 15, a, b are 10 and $\frac{20}{3}$, respectively, then the value of $|a-b|$ is equal to :

Options
1. 1
2. 9
3. 7
4. 11

Ans: 1

Sol: Mean=10

$\frac{7+10+11+15+a+b}{6} = 10$

a + b = 17 …………..(1)

Variance $= \frac{20}{3}$

$\frac{49+100+121+225+a^2+b^2}{6} - 100 = \frac{20}{3}$

$a^2 + b^2 = 145$ ………….. (2)

From (1) and (2)

$(a-b)^2 = 289 - 288 = 1$

$|a - b| = 1$

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Q.15

The value of $\tan\left(2\tan^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)\right)$ is equal to :

Options

1. $\frac{220}{21}$
2. $\frac{151}{63}$
3. $\frac{-181}{69}$
4. $\frac{-291}{76}$

Ans: $\frac{220}{21}$

Sol: $\tan\left(\tan^{-1}\frac{\frac{6}{5}}{1-\frac{9}{25}}+\tan^{-1}\frac{5}{12}\right)$

$\tan\left(\tan^{-1}\left(\frac{15}{8}\right)+\tan^{-1}\left(\frac{5}{12}\right)\right)=\frac{\frac{15}{8}+\frac{5}{12}}{1-\frac{15}{8}\cdot\frac{5}{12}}=\frac{220}{21}$

Q.16

If the real part of the complex number $(1-\cos\theta+2i\sin\theta)^{-1}$ is $\frac{1}{5}$ for $\theta\in(0,\pi)$, then the value of the integral $\int_0^\theta \sin x\,dx$ is equal to :

Options

1. -1
2. 1
3. 2
4. 0

Ans: 1

Sol: $z=(1-\cos\theta+2i\sin\theta)^{-1}$

$\mathrm{Re}(z)=\frac{1-\cos\theta}{(1-\cos\theta)^2+4\sin^2\theta}=\frac{1}{5}$

$\Rightarrow \theta=0,\frac{\pi}{2}$

But $\theta\in(0,\pi)$

So, $\theta=\frac{\pi}{2}$

So, $\int_0^{\frac{\pi}{2}}\sin x\,dx=1$

Q.17 Let P be a variable point on the parabola $y=4x^2+1$. Then, the locus of the mid-point of the point P and the foot of the perpendicular drawn from the point P to the line $y=x$ is :

Options
1. $2(3x-y)^2+(x-3y)+2=0$
2. $(3x-y)^2+(x-3y)+2=0$
3. $(3x-y)^2+2(x-3y)+2=0$
4. $2(x-3y)^2+(3x-y)+2=0$

Ans: $2(3x-y)^2 + (x-3y) + 2 = 0$

Sol: Let $A(t, 4t^2+1)$

Foot of perpendiculars from A to y=x is

$B\left(\frac{4t^2+t+1}{2}, \frac{4t^2+t+1}{2}\right)$

Midpoint of A and B is

$\left(\frac{4t^2+3t+1}{4}, \frac{12t^2+t+3}{4}\right)$

Locus of midpoint is $2(3x-y)^2+(x-3y)+2=0$

Q.18 The value of $k \in \mathbf{R}$, for which the following system of linear equations

$$3x-y+4z=3,$$
$$x+2y-3z=-2,$$
$$6x+5y+kz=-3,$$

has infinitely many solutions, is :

Options
1. 5
2. -3
3. -5
4. 3

Ans: -5

Sol: $\Delta = \begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & k \end{vmatrix}$

$\Delta = 3(2k+15)+1(k+18)+4(5-12)=0 \Rightarrow k=-5$

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Q.19 The sum of all the local minimum values of the twice differentiable function $f: \mathrm{R} \to \mathrm{R}$ defined by $f(x)=x^3-3x^2-\frac{3f''(2)}{2}x+f''(1)$ is :

Options
1. 0
2. -27
3. 5
4. -22

Ans: -27

Sol: $f(x) = 3x^2 - 6x - \frac{3}{2}f''(2)$

f''(x) = 6x – 6

f''(1) = 0 & f''(2) = 6

Then the local minimum value f'(x) = 0

$\Rightarrow 3(x^2 - 2x - 3) = 0$

x = –1 and x = 3

Local minimum at x = 3

So local minimum value = f(3)

$$= 3^3 - 3^3 - 3 \times \frac{6}{2} \times 3 + 0 = -27$$

Q.20 Let A, B and C be three events such that the probability that exactly one of A and B occurs is $(1-k)$, the probability that exactly one of B and C occurs is $(1-2k)$, the probability that exactly one of C and A occurs is $(1-k)$ and the probability of all A, B and C occur simultaneously is k^2, where $0 < k < 1$. Then the probability that at least one of A, B and C occur is :

Options

1. exactly equal to $\frac{1}{2}$
2. greater than $\frac{1}{4}$ but less than $\frac{1}{2}$
3. greater than $\frac{1}{2}$
4. greater than $\frac{1}{8}$ but less than $\frac{1}{4}$

Ans: greater than $\frac{1}{2}$

Sol: $P(A) + P(B) - 2P(A \cap B) = 1 - K$

$P(A) + P(C) - 2P(A \cap C) = 1 - 2K$

$P(B) + P(C) - 2P(B \cap C) = 1 - K$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= \frac{3 - 4k}{2} + k^2 = \frac{2k^2 - 4k + 3}{2}$

Now value of $2k^2 - 4k + 3$ is greater than 1.

$\therefore P(A \cup B \cup C) > \frac{1}{2}$

Section B

Q.1 The number of solutions of the equation $\log_{(x+1)}(2x^2 + 7x + 5) + \log_{(2x+5)}(x+1)^2 - 4 = 0$, $x > 0$, is ________.

Ans: 1

Sol: $\log_{(x+1)}((2x+5)(x+1)) + \log_{(2x+5)}(x+1)^2 = 4$

$1 + \log_{(x+1)}(2x+5) + 2\log_{(2x+5)}(x+1) = 4$

Put $\log_{(x+1)}(2x+5) = t$

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$\therefore 1+t+\frac{2}{t}=4$

$t^2+t+2-4t \Rightarrow t^2-3t+2-0$

$t = 1, t= 2$

For $t = 1$	For $t = 2$
$2x + 5 = x + 1$	$2x+5 = (x+1)^2$
$\Rightarrow x = -4$	$x = 2, x = -2$

Q.2 If $\lim\limits_{x\to 0} \dfrac{\alpha x e^x - \beta \log_e(1+x) + \gamma x^2 e^{-x}}{x \sin^2 x} = 10$, $\alpha, \beta, \gamma \in \mathbf{R}$, then the value of $\alpha+\beta+\gamma$ is __________.

Ans: 3

Sol: $\lim\limits_{x\to 0} \dfrac{\alpha x\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+...\right)-\beta\left(x-\frac{x^2}{2!}+\frac{x^3}{3!}+...\right)+\gamma x^2\left(1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+...\right)}{x^3}=10,$

$\Rightarrow \alpha-\beta=0, \Rightarrow \alpha=\beta$

$\Rightarrow \alpha+\frac{\beta}{2}+\gamma=0, \Rightarrow \gamma=-\frac{3\beta}{2}$

$\Rightarrow \frac{\alpha}{2}-\frac{\beta}{2}-\gamma=10, \Rightarrow \frac{3\beta-2\beta+9\beta}{6}=10$

$\because \beta=6, \alpha=6, \gamma=3$

Therefore $\alpha+\beta+\gamma=3$

Q.3 Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $a_1=1$, $a_2=1$ and $a_{n+2}=2a_{n+1}+a_n$ for all $n \geqslant 1$. Then the value of $47\sum\limits_{n=1}^{\infty}\frac{a_n}{2^{3n}}$ is equal to __________.

Ans: 7

Sol: Let $s=\sum\limits_{n-1}^{\infty}\frac{a_n}{2^{3n}}=\sum\limits_{n-1}^{\infty}\frac{a_n}{8^n}=\frac{a_1}{8^1}+\frac{a_2}{8^2}+\frac{a_3}{8^3}+..........(1)$

and $a_{n+2}=2a_{n+1}+a_n$

$=\sum\limits_{n-1}^{\infty}\left(\frac{a_{n+2}-2a_{n+1}}{8^n}\right)$

$=\sum\limits_{n-1}^{\infty}\left(\frac{8^2.a_{n+2}}{8^{n+2}}-\frac{2\times 8\times a_{n+1}}{8^{n+1}}\right)$

$s= =64\sum\limits_{n-1}^{\infty}\frac{a_{n+2}}{8^{n+2}}-16\sum\limits_{n-1}^{\infty}\frac{a_{n+1}}{8^{n+1}}$

$\therefore s=64\left(s-\frac{a_1}{8}+\frac{a_2}{64}\right)-16\left(s-\frac{a_1}{8}\right), from(1)$

$\because a_1=a_2=1$

$\Rightarrow (47)s=7$

Q.4 Consider a triangle having vertices $A(-2, 3)$, $B(1, 9)$ and $C(3, 8)$. If a line L passing through the circum-center of triangle ABC, bisects line BC, and intersects y-axis at point $\left(0, \frac{\alpha}{2}\right)$, then the value of real number α is __________.

Ans: 9

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Sol: Midpoint of BC is (2, 17/2) and slope of BC is -1/2
Since a line passes through circumcentre of ΔABC and bisects the side BC is perpendicular bisector of side BC.
Equation of required line is

$$y = \frac{17}{2} = 2(x-2)$$

$$x - 2y + 9 = 0$$

It intersects the y-axis at $\left(0, \frac{\alpha}{2}\right)$

$$\Rightarrow -\alpha + 9 = 0$$
$$\Rightarrow \alpha = 9$$

Q.5 Let a function $g : [0, 4] \to \mathbf{R}$ be defined as $g(x) = \begin{cases} \max\limits_{0 \le t \le x} \{t^3 - 6t^2 + 9t - 3\}, & 0 \le x \le 3 \\ 4 - x & , 3 < x \le 4 \end{cases}$, then the number of points in the interval (0, 4) where $g(x)$ is NOT differentiable, is ________.

Ans: 1

Sol: $f(t) = t^3 - 6t^2 + 9t - 3$
$f'(t) = 3t^2 - 12t + 9 = 0$
$= t^2 - 4t + 3 = 0$
$t = 1, 3$
$f(1) = 1 \; f(3) = -3$

$$g(x) = \begin{cases} x^3 - 6x^2 + 9x - 3 & 0 \le x \le 1 \\ 1 & 1 \le x \le 3 \\ 4 - x & 3 < x \le 4 \end{cases}$$

Function is not differentiable at x = 3

Q.6 Let a curve $y = y(x)$ be given by the solution of the differential equation

$$\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx = \sqrt{e^{2x} - 1}\,dy$$

If it intersects y-axis at $y = -1$, and the intersection point of the curve with x-axis is $(\alpha, 0)$, then e^{α} is equal to ________.

Ans: 2

Sol: $\cos\left(\frac{1}{2}\cos^{-1}\left(e^{-x}\right)\right)dx = \sqrt{e^{2x} - 1}\,dy$

$$\int \frac{\cos\left(\frac{1}{2}\cos^{-1}\left(e^{-x}\right)\right)dx}{\sqrt{e^{2x} - 1}} = \int dy$$

Put $\cos^{-1}\left(e^{-x}\right) = t$

$$\frac{e^{-x}}{\sqrt{1 - e^{2x}}}dx = dt$$

$$2\sin\left(\frac{1}{2}\cos^{-1}\left(e^{-x}\right)\right) = y + c$$

At x= 0 $\Rightarrow$ y = –1
c = 1

$y = 2\sin\left(\frac{1}{2}\cos^{-1}\left(e^{-x}\right)\right)$

$y = 0$ then $x = \alpha$

$2\sin\left(\frac{1}{2}\cos^{-1}\left(e^{-x}\right)\right) = 1$

$e^{-x} = \frac{1}{2} \Rightarrow x = \ln 2 = \alpha$

So, the value of $e^{\alpha} = 2$

Q.7 For $p > 0$, a vector $\vec{v}_2 = 2\hat{i} + (p+1)\hat{j}$ is obtained by rotating the vector $\vec{v}_1 = \sqrt{3}\,p\hat{i} + \hat{j}$ by an angle θ about origin in counter clockwise direction. If $\tan\theta = \frac{(\alpha\sqrt{3}-2)}{(4\sqrt{3}+3)}$, then the value of α is equal to ________.

Ans: 6

Sol: $|v_1| = |v_2|$

$\Rightarrow 3p^2 + 1 = 4 + (P+1)^2$

Solving,

$\Rightarrow p = -1, 2$

$\Rightarrow p = 2$ (since $p>0$)

Let angle between $\vec{v}_1$ & $\vec{v}_2$ be θ

$\Rightarrow \cos\theta \frac{\vec{v}_1 . \vec{v}_2}{|\vec{v}_1||\vec{v}_2|} = \frac{(2\sqrt{3}\,\hat{i} + \hat{j})(2\hat{i} + 3\hat{j})}{\sqrt{13}\sqrt{13}}$

$\Rightarrow \cos\theta = \frac{4\sqrt{3}+3}{13}$

$\Rightarrow \tan\theta = \frac{\sqrt{112 - 24\sqrt{3}}}{4\sqrt{3}+3}$

$\Rightarrow \tan\theta = \frac{2\sqrt{28 - 6\sqrt{3}}}{4\sqrt{3}+3} = \frac{2(3\sqrt{3}-1)}{4\sqrt{3}+3}$

$= \frac{6\sqrt{3}-2}{4\sqrt{3}+3} \Rightarrow \alpha = 6$

Q.8 If the point on the curve $y^2 = 6x$, nearest to the point $\left(3, \frac{3}{2}\right)$ is (α, β), then $2(\alpha + \beta)$ is equal to ________.

Ans: 9

Sol: $y^2 = 6x$

$2yy' = 6$

$\frac{dy}{dx} = \frac{3}{\beta}$

$-\frac{\beta}{3} = \frac{\beta - 3/2}{\alpha - 3}$

$\alpha\beta = \frac{9}{2} \Rightarrow \beta = \frac{9}{2\alpha}$

$\therefore \beta^2 = 6\alpha$

$\frac{81}{4\alpha^2} = 6\alpha$

$$\alpha^3 = \frac{27}{8} \quad \alpha = \frac{3}{2}, \beta^2 = 9 \Rightarrow \beta = \pm 3$$

$$\alpha = \frac{3}{2}\ , \beta = 3$$

$$2(\alpha + \beta) = 9$$

Q.9 For $k \in \mathbf{N}$, let $\dfrac{1}{\alpha(\alpha+1)(\alpha+2)\ .....\ (\alpha+20)} = \sum_{k=0}^{20} \dfrac{A_k}{\alpha+k}$, where $\alpha > 0$. Then the value of $100\left(\dfrac{A_{14}+A_{15}}{A_{13}}\right)^2$ is equal to ________.

Ans: 9

Sol:
$$\frac{1}{\alpha(\alpha+1)(\alpha+2).........(\alpha+20)} = \sum_{k-0}^{20} \frac{A_k}{\alpha+k}$$

$$\Rightarrow \frac{1}{\alpha(\alpha+1)(\alpha+2).........(\alpha+20)} = \frac{A_0}{\alpha} + \frac{A_1}{\alpha+1} + \frac{A_2}{\alpha+2} + \frac{A_3}{\alpha+3} + ... \frac{A_{20}}{\alpha+20}$$

$$\Rightarrow 1 = A_0(\alpha+1)(\alpha+2).........(\alpha+20) + A_1(\alpha)(\alpha+2).........(\alpha+20)$$

$$+ ...... + A_{20}\alpha(\alpha+1)(\alpha+2)(\alpha+2).........(\alpha+19)$$

Comparing $A_{14} = \dfrac{1}{(-14)(-13)(-12)......(-1)(1)(2)....(6)}$

$$A_{15} = \frac{1}{(-15)(-14)(-13)......(-1)(1)(2)....(5)}$$

$$A_{13} = \frac{1}{(-13)(-12)......(-1)(1)(2)....(6)(7)}$$

$$\Rightarrow \left(\frac{A_{15}+A_{14}}{A_{13}}\right)^2 = \left(\frac{A_{15}}{A_{13}} + \frac{A_{14}}{A_{13}}\right)^2$$

$$= \left(\frac{6.7}{15.14} - \frac{7}{14}\right)^2 = \left(\frac{1}{5} - \frac{1}{2}\right)^2 \frac{9}{100}$$

$$\Rightarrow 100\left(\frac{A_{15}+A_{14}}{A_{13}}\right) = 9$$

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Q.10 Let $A = \{a_{ij}\}$ be a 3×3 matrix, where

$$a_{ij} = \begin{cases} (-1)^{j-i} & \text{if } i < j, \\ 2 & \text{if } i = j, \\ (-1)^{i+j} & \text{if } i > j, \end{cases}$$

then $\det(3\ \text{Adj}(2\ A^{-1}))$ is equal to ________.

Ans: 108

Sol:

|A|= 2(4–1) + 1(–2+1) +1 (1–2)
= 2(3) +1(–1) +1 (–1)
= 4

$|3\text{Adj}(2A^{-1})| = 3^3\ |\text{Adj}(2A^{-1})| = 3^3 \times |2A^{-1}|^2 = 108$

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