SOLUTIONS & ANSWERS FOR JEE MAINS-2021
22nd July Shift 2
[PHYSICS, CHEMISTRY & MATHEMATICS]

PART – A – PHYSICS

Section A

Q.1 Intensity of sunlight is observed as 0.092 Wm^{-2} at a point in free space. What will be the peak value of magnetic field at that point ? ($\varepsilon_0 = 8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$)

Options
1. $5.88\ T$
2. $8.31\ T$
3. $1.96 \times 10^{-8}\ T$
4. $2.77 \times 10^{-8}\ T$

Ans: 2.77×10^{-8} T

Sol: $I_{av} = \dfrac{B_0^2 C}{2\mu_0}$

$I = \dfrac{B_0^2}{2} \varepsilon_0 C^3 \quad \left[\because \mu_0 = \dfrac{1}{\varepsilon_0 C^2}\right]$

$B_0 = \sqrt{\dfrac{2I}{\varepsilon_0 C^3}} = 2.77 \times 10^{-8} T$

Q.2 What will be the projection of vector $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ on vector $\vec{B} = \hat{i} + \hat{j}$?

Options
1. $\sqrt{2}\left(\hat{i} + \hat{j} + \hat{k}\right)$
2. $\left(\hat{i} + \hat{j}\right)$
3. $\sqrt{2}\left(\hat{i} + \hat{j}\right)$
4. $2\left(\hat{i} + \hat{j} + \hat{k}\right)$

Ans: $\left(\hat{i} + \hat{j}\right)$

Sol: $A \cos\theta\ \hat{B} = A\left(\dfrac{\overline{A} \cdot \overline{B}}{AB}\right)\hat{B} = \dfrac{\overline{A} \cdot \overline{B}}{B} \times \hat{B}$

$= \dfrac{2}{\sqrt{2}}\left(\dfrac{\hat{i} + \hat{j}}{\sqrt{2}}\right) = \hat{i} + \hat{j}$

Q.3 Consider a situation in which reverse biased current of a particular P-N junction increases when it is exposed to a light of wavelength $\leq$ 621 nm. During this process, enhancement in carrier concentration takes place due to generation of hole-electron pairs. The value of band gap is nearly.

Options
1. 2 eV
2. 1 eV
3. 0.5 eV
4. 4 eV

Ans: 2 eV

Sol: We have band gap = $\frac{hc}{\lambda_0}$

$$\frac{1242}{621} = 2\,\text{eV}$$

Q.4 T_0 is the time period of a simple pendulum at a place. If the length of the pendulum is reduced to $\frac{1}{16}$ times of its initial value, the modified time period is :

Options
1. $8\pi\ T_0$
2. $\frac{1}{4}\ T_0$
3. $4\ T_0$
4. T_0

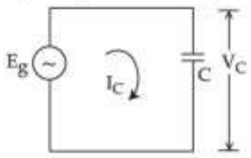
Ans: $\frac{1}{4}T_0$

Sol: $T_0 = 2\pi\sqrt{\frac{L}{g}}$

$$T_0' = 2\pi\sqrt{\frac{L/16}{g}} = \frac{2\pi}{4}\sqrt{\frac{L}{g}}$$

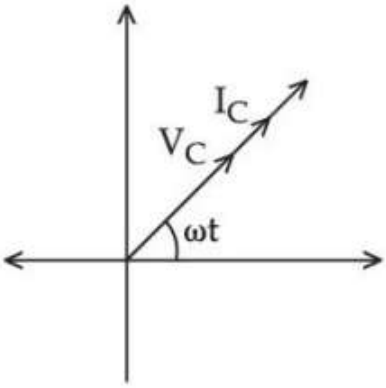
$$= \frac{\pi}{2}\sqrt{\frac{L}{g}} = \frac{T_0}{4}$$

Q.5 In a circuit consisting of a capacitance and a generator with alternating emf $E_g = E_{g_0} \sin\omega t$, V_C and I_C are the voltage and current. Correct phasor diagram for such circuit is :



Options

1.



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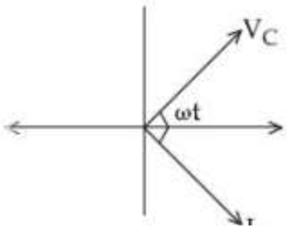

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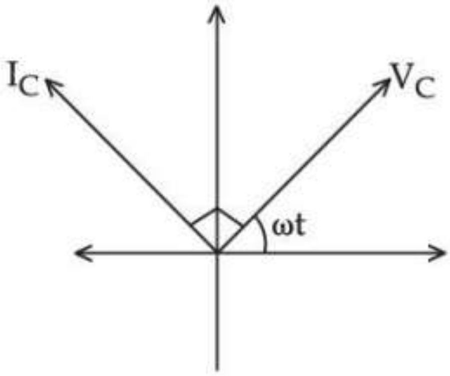

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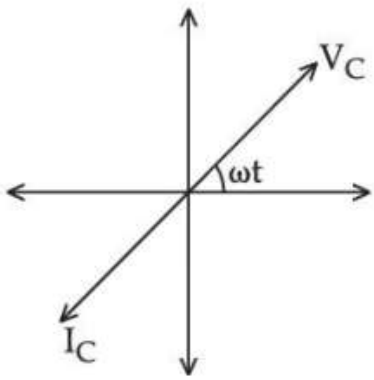
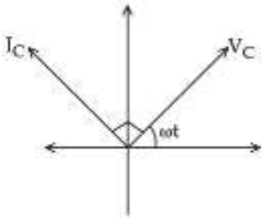


Ans:



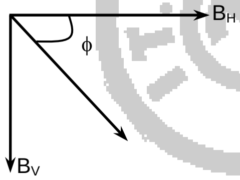
Sol: In a capacitor, current leads voltage by $\frac{\pi}{2}$. Hence option (3) is the correct answer.

Q.6 Choose the correct option :

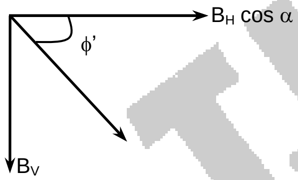
Options
1. True dip is less than the apparent dip.
2. True dip is always greater than the apparent dip.
3. True dip is not mathematically related to apparent dip.
4. True dip is always equal to apparent dip.

Ans: True dip is less than he apparent dip

Sol:



$\tan\phi = \frac{B_V}{B_H}$ (true dip circle)



$\tan\phi' = \frac{B_V}{B_H \cos\alpha}$ (apparent dip circle)

As $\cos\alpha < 1$

The apparent dip is greater than true dip

Q.7 Consider a situation in which a ring, a solid cylinder and a solid sphere roll down on the same inclined plane without slipping. Assume that they start rolling from rest and having identical diameter.

The **correct** statement for this situation is :

Options
1. All of them will have same velocity.
2. The sphere has the greatest and the ring has the least velocity of the centre of mass at the bottom of the inclined plane.
3. The ring has the greatest and the cylinder has the least velocity of the centre of mass at the bottom of the inclined plane.
4. The cylinder has the greatest and the sphere has the least velocity of the centre of mass at the bottom of the inclined plane.

Ans: The sphere has the greatest and the ring has the least velocity of the centre of mass at the bottom of the inclined plane

Sol: $a = \dfrac{g\sin\theta}{1 + \dfrac{I}{mR^2}}$

$I_{ring} > I_{\substack{soild \\ cylinder}} > I_{\substack{solid \\ sphere}}$

$\therefore\ a_{ring} < a_{solid\ cylinder} < a_{solid\ sphere}$

$\Rightarrow V_{ring} < V_{solid\ cylinder} < V_{solid\ sphere}$

Q.8 A body is projected vertically upwards from the surface of earth with a velocity sufficient enough to carry it to infinity. The time taken by it to reach height h is ________s.

Options

1. $\sqrt{\dfrac{2R_e}{g}}\left[\left(1 + \dfrac{h}{R_e}\right)^{3/2} - 1\right]$

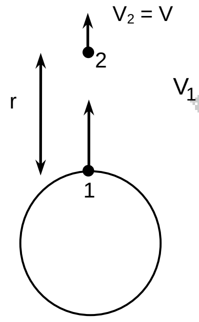
2. $\dfrac{1}{3}\sqrt{\dfrac{2R_e}{g}}\left[\left(1 + \dfrac{h}{R_e}\right)^{3/2} - 1\right]$

3. $\sqrt{\dfrac{R_e}{2g}}\left[\left(1 + \dfrac{h}{R_e}\right)^{3/2} - 1\right]$

4. $\dfrac{1}{3}\sqrt{\dfrac{R_e}{2g}}\left[\left(1 + \dfrac{h}{R_e}\right)^{3/2} - 1\right]$

Ans: $\dfrac{1}{3}\sqrt{\dfrac{2R_e}{g}}\left[\left(1 + \dfrac{h}{R_e}\right)^{3/2} - 1\right]$

Sol:



$V_1 = \sqrt{\dfrac{2GM}{R_e}}$ = Escape Velocity

According to energy conservation

$$\frac{1}{2}m\left(\frac{2GM}{R_e}\right) - \frac{GMm}{R_e} = \frac{1}{2}mV^2 - \frac{GMm}{R + r}$$

$$\frac{1}{2}mV^2 = \frac{GMm}{R + r}$$

$$V = \sqrt{\frac{2GM}{R+r}} = \frac{dr}{dt}$$

$$\sqrt{2GM}\int_0^t dt = \int_{R_e}^{R_e+h} \sqrt{R+r}\ dr$$

$$\sqrt{2GM}\ t = \frac{2}{3}\left[(R+r)^{3/2}\right]_{R_e}^{R_e+h}$$

$$t = \frac{2}{3}\sqrt{\frac{R_e{}^3}{2GM}}\left[\left(1+\frac{h}{R_e}\right)^{3/2} - 1\right]$$

But $g = \frac{GM}{R_e{}^2}$

i.e, $t = \frac{1}{3}\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2} - 1\right]$

Q.9 A bullet of '4 g' mass is fired from a gun of mass 4 kg. If the bullet moves with the muzzle speed of 50 ms^{-1}, the impulse imparted to the gun and velocity of recoil of gun are :

Options

1. $0.4\ kg\ ms^{-1}$, $0.1\ ms^{-1}$
2. $0.2\ kg\ ms^{-1}$, $0.05\ ms^{-1}$
3. $0.2\ kg\ ms^{-1}$, $0.1\ ms^{-1}$
4. $0.4\ kg\ ms^{-1}$, $0.05\ ms^{-1}$

Ans: $0.2\ kg\ ms^{-1}$, $0.05\ ms^{-1}$

Sol: By the conservation of momentum

$4 \times 10^{-3} (50 - V) - 4V = 0$

$$V = \frac{4\times10^{-3}\times50}{4+4\times10^{-3}} = 0.05\,ms^{-1}$$

i.e., Impulse J = mV = 4 × 0.05

= $0.2\ kg\ ms^{-1}$

Q.10 An electron of mass m_e and a proton of mass m_p are accelerated through the same potential difference. The ratio of the de-Broglie wavelength associated with the electron to that with the proton is :

Options

1. $\frac{m_e}{m_p}$
2. $\sqrt{\frac{m_p}{m_e}}$
3. $\frac{m_p}{m_e}$
4. 1

Ans: $\sqrt{\frac{m_p}{m_e}}$

Sol: KE = e ΔV

$$\lambda_e = \frac{h}{\sqrt{2m_e(e\Delta V)}}$$

$$\lambda_p = \frac{h}{\sqrt{2m_p(e\Delta V)}}$$

$$\therefore \frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}}$$

Q.11 A porter lifts a heavy suitcase of mass 80 kg and at the destination lowers it down by a distance of 80 cm with a constant velocity. Calculate the workdone by the porter in lowering the suitcase.
(take $g = 9.8\ ms^{-2}$)

Options
1. 784.0 J
2. +627.2 J
3. −627.2 J
4. −62720.0 J

Ans: -627.2 J

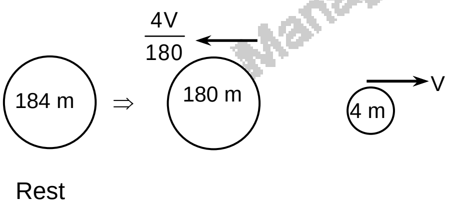
Sol: $W_{porter} + W_{mg} = \Delta KE = 0$
$W_{porter} = -W_{mg} = -mgh$
i.e., = -80 × 9.8 × 0.80 = -627.2 J

Q.12 A nucleus with mass number 184 initially at rest emits an α-particle. If the Q value of the reaction is 5.5 MeV, calculate the kinetic energy of the α-particle.

Options
1. 5.38 MeV
2. 5.0 MeV
3. 0.12 MeV
4. 5.5 MeV

Ans: 5.38 MeV

Sol:



$$\frac{1}{2}(4m)V^2 + \frac{1}{2} \times 180m \times \left(\frac{4V}{180}\right)^2 = 5.5$$

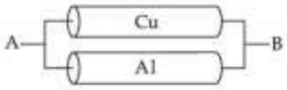
$$\frac{1}{2}4mV^2\left[1 + 45\left(\frac{4}{180}\right)^2\right] = 5.5$$

$$KE_\alpha = \frac{5.5}{1 + 45\left(\frac{4}{180}\right)^2} = 5.38\,MeV$$

Q.13 A Copper (Cu) rod of length 25 cm and cross-sectional area 3 mm^2 is joined with a similar Aluminium (Al) rod as shown in figure. Find the resistance of the combination between the ends A and B.

(Take Resistivity of Copper $= 1.7 \times 10^{-8}\ \Omega m$

Resistivity of Aluminium $= 2.6 \times 10^{-8}\ \Omega m$)



Options

1. $0.858\ m\Omega$
2. $1.420\ m\Omega$
3. $0.0858\ m\Omega$
4. $2.170\ m\Omega$

Ans: $0.858\ m\Omega$

Sol: $R = \frac{R_1R_2}{R_1 + R_2} = \frac{\ell}{A}\frac{\rho_1\rho_2}{\rho_1 + \rho_2}$

$$R = \frac{25 \times 10^{-2}}{3 \times 10^{-6}} \times \frac{1.7 \times 2.6 \times 10^{-16}}{4.3 \times 10^{-8}} = 0.858\,m\Omega$$

Q.14 **Statement I :** The ferromagnetic property depends on temperature. At high temperature, ferromagnet becomes paramagnet.

Statement II : At high temperature, the domain wall area of a ferromagnetic substance increases.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

Options

1. **Statement I** is false but **Statement II** is true
2. **Statement I** is true but **Statement II** is false
3. Both **Statement I** and **Statement II** are false
4. Both **Statement I** and **Statement II** are true

Ans: Statement I is true but statement II is false

Sol: As the T increases, ferromagnetism decreases and above curie temperature, it will become paramagnet.

Q.15 What will be the average value of energy for a monoatomic gas in thermal equilibrium at temperature T ?

Options

1. k_BT
2. $\frac{2}{3}k_BT$
3. $\frac{3}{2}k_BT$
4. $\frac{1}{2}\ k_BT$

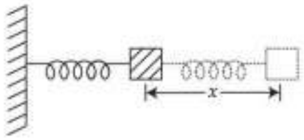
Ans: $\frac{3}{2}K_BT$

Sol: In monoatomic gas f = 3

$\therefore$ Average Energy = $3 \times \frac{1}{2}K_BT$

$= \frac{3}{2}K_BT$

Q.16 The motion of a mass on a spring, with spring constant K is as shown in figure.



The equation of motion is given by $x(t) = A\sin\omega t + B\cos\omega t$ with $\omega = \sqrt{\frac{K}{m}}$

Suppose that at time t = 0, the position of mass is $x(0)$ and velocity $v(0)$, then its displacement can also be represented as $x(t) = C\cos(\omega t - \phi)$, where C and ϕ are :

Options

1. $C = \sqrt{\frac{v(0)^2}{\omega^2} + x(0)^2}$, $\phi = \tan^{-1}\left(\frac{v(0)}{x(0)\omega}\right)$

2. $C = \sqrt{\frac{2v(0)^2}{\omega^2} + x(0)^2}$, $\phi = \tan^{-1}\left(\frac{x(0)\omega}{2v(0)}\right)$

3. $C = \sqrt{\frac{2v(0)^2}{\omega^2} + x(0)^2}$, $\phi = \tan^{-1}\left(\frac{v(0)}{x(0)\omega}\right)$

4. $C = \sqrt{\frac{v(0)^2}{\omega^2} + x(0)^2}$, $\phi = \tan^{-1}\left(\frac{x(0)\omega}{v(0)}\right)$

Ans: **1** $C = \sqrt{\frac{[V(0)]^2}{\omega^2} + [x(0)]^2}, \phi = \tan^{-1}\left(\frac{V(0)}{x(0)\omega}\right)$

Sol: x = A sin ωt + B cos ωt

$V = \frac{dx}{dt} = a\omega\cos\omega t - bw\sin\omega t$

At t = 0, x (0) = B

$\therefore$ V(0) = Aω

x = A sin ωt + B sin (ωt + 90)

$A_{net} = \sqrt{A^2 + B^2}$

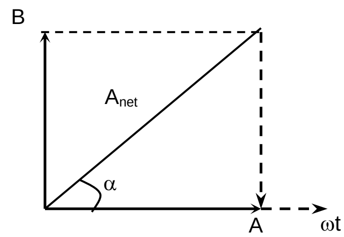
$\tan\alpha = \frac{B}{A} \Rightarrow \cot\alpha = \frac{A}{B}$

$x = \sqrt{A^2 + B^2}\sin(\omega t + \alpha)$

$x = \sqrt{A^2 + B^2}\cos(\omega t - (90 - \alpha))$

x = C cos (ωt - ϕ)

$C = \sqrt{A^2 + B^2}$



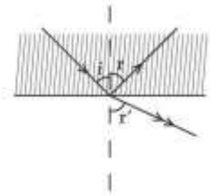
$$C = \sqrt{\frac{[V(0)]^2}{\omega^2} + [x(0)]^2}$$

$\phi = 90 - \alpha$

$\tan \alpha = \cos \alpha = \frac{A}{B} \Rightarrow \tan\phi = \frac{V(0)}{x(0)\omega}$

$\therefore \phi = \tan^{-1}\left(\frac{V(0)}{x(0)\omega}\right)$

Q.17 A ray of light passes from a denser medium to a rarer medium at an angle of incidence *i*. The reflected and refracted rays make an angle of 90° with each other. The angle of reflection and refraction are respectively r and r'. The critical angle is given by :



Options

1. $\sin^{-1}(\tan r')$
2. $\sin^{-1}(\tan r)$
3. $\sin^{-1}(\cot r)$
4. $\tan^{-1}(\sin i)$

Ans: $\sin^{-1}(\tan r)$

Sol: $r + r' + 90° = 180°$

$r' = 90 – r = 90 – i$

$n_1 \sin i = n_2 \sin r' = n_2 \sin (90 – i)$

$n_1 \sin i = n_2 \cos i$

$\tan i = \frac{n_2}{n_1}$

$\sin C = \frac{n_2}{n_1} = \tan i$

$C = \sin^{-1}(\tan i) = \sin^{-1}(\tan r)$

Q.18 Match **List - I** with **List - II** :

	List - I		List - II
(a)	$\omega L > \frac{1}{\omega C}$	(i)	Current is in phase with emf
(b)	$\omega L = \frac{1}{\omega C}$	(ii)	Current lags behind the applied emf
(c)	$\omega L < \frac{1}{\omega C}$	(iii)	Maximum current occurs
(d)	Resonant frequency	(iv)	Current leads the emf

Choose the **correct** answer from the options given below :

Options
1. (a)-(ii); (b)-(i); (c)-(iii); (d)-(iv)
2. (a)-(iv); (b)-(iii); (c)-(ii); (d)-(i)
3. (a)-(iii); (b)-(i); (c)-(iv); (d)-(ii)
4. (a)-(ii); (b)-(i); (c)-(iv); (d)-(iii)

Ans: (a) – (ii), (b) – (i), (c) – (iv), (d) – (iii)

Sol: (a) – (ii)
(b) – (i)
(c) – (iv)
(d) – (iii)

Q.19 What should be the height of transmitting antenna and the population covered if the television telecast is to cover a radius of 150 km ? The average population density around the tower is $2000/\text{km}^2$ and the value of $R_e = 6.5 \times 10^6$ m.

Options
1. Height = 1600 m
Population Covered $= 2 \times 10^5$
2. Height = 1731 m
Population Covered $= 1413 \times 10^5$
3. Height = 1241 m
Population Covered $= 7 \times 10^5$
4. Height = 1800 m
Population Covered $= 1413 \times 10^8$

Ans: Height = 1731 m
Population covered = 1413×10^5

Sol: Radius, $r = \sqrt{2RH}$

$150\,\text{km} = \sqrt{2 \times 6.5 \times 10^6 \times H}$

H = 1731 m

Population covered = $\pi r^2 \times 2000/\text{km}^2$

$= 3.14 \times (150)^2 \times 2000$

$= 1413 \times 10^5$

Q.20 An electric dipole is placed on x-axis in proximity to a line charge of linear charge density 3.0×10^{-6} C/m. Line charge is placed on z-axis and positive and negative charge of dipole is at a distance of 10 mm and 12 mm from the origin respectively. If total force of 4 N is exerted on the dipole, find out the amount of positive or negative charge of the dipole.

Options
1. 815.1 nC
2. 4.44 μC
3. 0.485 mC
4. 8.8 μC

Ans: 4.44 μC

Sol: Let r = 10 mm, x = 2

$$F_q = \frac{2k\lambda}{r} q$$

$$F_{-q} = \frac{2k\lambda}{r+x} q$$

$$F_{net} = \frac{2k\lambda q}{r} \; \frac{-2k\lambda q}{r+x} = \frac{2k\lambda qx}{r(r+x)}$$

$$4 = \frac{2 \times 9 \times 10^{9} \times 3 \times 10^{-6} \times q \times 2\,mm}{10\,mm \times 12\,mm}$$

q = 4.4 μC

Section B

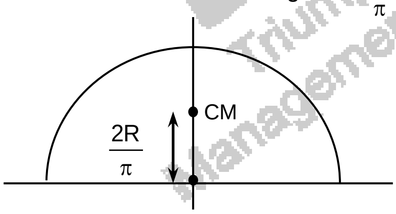
Q.1 The position of the centre of mass of a uniform semi-circular wire of radius 'R' placed in x-y plane with its centre at the origin and the line joining its ends as x-axis is given by $\left(0, \frac{xR}{\pi}\right)$.

Then, the value of $|x|$ is ________.

Given **0.67**
Answer :

Ans: 2.00

Sol: Center of mass of semi circular ring is at $\frac{2R}{\pi}$



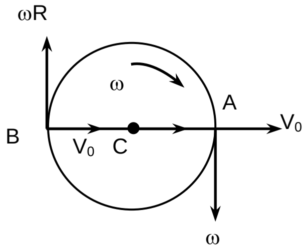
Distance from the centre ⇒ x = 2

Q.2 The centre of a wheel rolling on a plane surface moves with a speed v_0. A particle on the rim of the wheel at the same level as the centre will be moving at a speed $\sqrt{x}\, v_0$. Then the value of x is ________.

Given **2.00**
Answer :

Ans: 2.00

Sol:



For no slipping, $V_0 = \omega R$

$V_A = V_B = \sqrt{V_0^2 + (\omega R)^2} = \sqrt{2}V_0$

$\Rightarrow x = 2.00$

Q.3 A ray of light passing through a prism $(\mu = \sqrt{3})$ suffers minimum deviation. It is found that the angle of incidence is double the angle of refraction within the prism. Then, the angle of prism is _________ (in degrees).

Given --
Answer :

Ans: 60.00

Sol: For minimum deviation

$r_1 = r_2 = \frac{A}{2}$

given $i = 2r_1 = A$

$1 \sin i = \sqrt{3} \sin r_1$

$1 \sin A = \sqrt{3} \sin \frac{A}{2}$

$2 \sin \frac{A}{2} \cos \frac{A}{2} = \sqrt{3} \sin \frac{A}{2}$

$\cos \frac{A}{2} = \frac{\sqrt{3}}{2} \Rightarrow \frac{A}{2} = 30°$

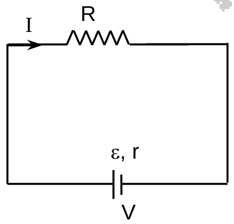
$A = 60°$

Q.4 In an electric circuit, a cell of certain emf provides a potential difference of 1.25 V across a load resistance of 5 Ω. However, it provides a potential difference of 1 V across a load resistance of 2 Ω. The emf of the cell is given by $\frac{x}{10}$ V. Then the value of x is _________.

Given --
Answer :

Ans: 15.00

Sol:



$V = IR = \frac{\varepsilon R}{R + r}$

1st case $\Rightarrow 1.25 = \frac{\varepsilon \times 5}{5 + r}$ ------(1)

2nd case $\Rightarrow 1 = \frac{\varepsilon \times 2}{2 + r}$ ------(2)

From (1) and (2)

$r = 1\ \Omega,\ \varepsilon = \frac{3}{2}V = \frac{15}{10}V$

i.e., x = 15

Q.5 The total charge enclosed in an incremental volume of 2×10^{-9} m³ located at the origin is _________ nC, if electric flux density of its field is found as

$D = e^{-x}\sin y\,\hat{i} - e^{-x}\cos y\,\hat{j} + 2z\hat{k}$ C/m².

Given Answer : --

Ans: 4.00

Sol: Electric flux density = $\frac{Q}{4\pi r^2}\hat{r} = \varepsilon_0\left(\frac{Q\hat{r}}{4\pi\varepsilon_0 r^2}\right)$

$E = \frac{D}{\varepsilon_0} = \frac{e^{-x}\sin y\,\hat{i} - e^{-x}\cos y\,\hat{j} + 2z\,\hat{k}}{\varepsilon_0}$

By Gauss's law

$\frac{\rho}{\varepsilon_0} = \left(\frac{r}{rx}\hat{i} + \frac{r}{ry}\hat{j} + \frac{r}{rz}\hat{k}\right)\cdot E$

$= \left(\frac{r}{rx}\hat{i} + \frac{r}{ry}\hat{j} + \frac{r}{rz}\hat{k}\right)\cdot\frac{\vec{D}}{\varepsilon_0}$

$\Rightarrow \rho = \frac{r}{rx}\left(e^{-x}\sin y\right) + \frac{r}{ry}\left(-e^{-x}\cos y\right) + \frac{r}{rz}(2z)$

$\rho = -e^{-x}\sin y + e^{-x}\sin y + 2$

At origin $\rho = -e^{-0}\sin 0 + e^{-0}\sin 0 + 2$

$\therefore\ \rho = 2$ C/m³

Charge = $\rho \times V = 2 \times 2 \times 10^{-9} = 4 \times 10^{-9}$

= 4 nC

Q.6 In 5 minutes, a body cools from 75°C to 65°C at room temperature of 25°C. The temperature of body at the end of next 5 minutes is _________°C.

Given Answer : **55.00**

Ans: 57.00

Sol: $T_p - T_s = (T_f - T_s)\,e^{-Q}$

$65 - 25 = (75 - 25)\,e^{-5C}$

$\Rightarrow e^{-5C} = \frac{4}{5}$

$T - 25 = (65 - 25)\,e^{-5C}$

$\Rightarrow\ 40 \times \frac{4}{5} = 32$

i.e., T = 57°C

Q.7 Three particles P, Q and R are moving along the vectors $\vec{A} = \hat{i} + \hat{j}$, $\vec{B} = \hat{j} + \hat{k}$ and $\vec{C} = -\hat{i} + \hat{j}$ respectively. They strike on a point and start to move in different directions. Now particle P is moving normal to the plane which contains vector $\vec{A}$ and $\vec{B}$. Similarly particle Q is moving normal to the plane which contains vector $\vec{A}$ and $\vec{C}$. The angle between the direction of motion of P and Q is $\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)$. Then the value of x is _________.

Given Answer : **3.00**

Ans: 3.00

Sol: Direction of P $\hat{V}_1 = \pm \frac{\overline{A} \times \overline{B}}{|\overline{A} \times \overline{B}|}$

$= \pm \frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$

Direction of Q $\hat{V}_2 = \pm \frac{\overline{A} \times \overline{C}}{|\overline{A} \times \overline{C}|}$

$= \pm \frac{2\hat{k}}{2} = \pm\hat{k}$

Angle between $\hat{V}_1$ and $\hat{V}_2$

$\frac{\hat{V}_1 \bullet \hat{V}_2}{|\hat{V}_1||\hat{V}_2|} = \frac{\pm 1/\sqrt{3}}{1 \times 1} = \pm \frac{1}{\sqrt{3}}$

i.e., x = 3

Q.8 Three students S_1, S_2 and S_3 perform an experiment for determining the acceleration due to gravity (g) using a simple pendulum. They use different lengths of pendulum and record time for different number of oscillations. The observations are as shown in the table.

Student No.	Length of Pendulum (cm)	No. of oscillations (n)	Total time for n oscillations	Time period (s)
1	64.0	8	128.0	16.0
2	64.0	4	64.0	16.0
3	20.0	4	36.0	9.0

(Least count of length = 0.1 cm

least count for time = 0.1 s)

If E_1, E_2 and E_3 are the percentage errors in 'g' for students 1, 2 and 3 respectively, then the minimum percentage error is obtained by student no. _________.

Given **1.00**
Answer :

Ans: 1.00

Sol: $T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow g = \frac{4\pi^2\ell}{T^2}$

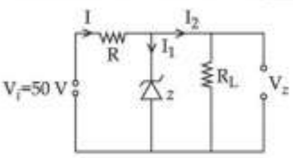
$\frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + \frac{2\Delta T}{T}$

$\frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + \frac{2\Delta T_0}{nT}$

As $\Delta \ell$ and ΔT_0 are same for all observations so $\frac{\Delta g}{g}$ is minimum for highest value of ℓ, n and T

$\Rightarrow$ Minimum percentage error in g is for student number -1

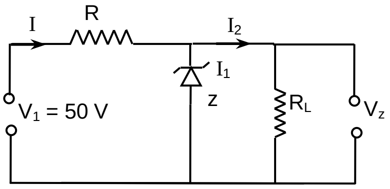
Q.9 In a given circuit diagram, a 5 V zener diode along with a series resistance is connected across a 50 V power supply. The minimum value of the resistance required, if the maximum zener current is 90 mA will be _________ Ω.



Given --
Answer :

Ans: 500.00

Sol:



$$I_2 = \frac{5}{R_L}$$

V across R = 50 – 5 = 45 volt

$$V = IR \Rightarrow \frac{V}{I} = \frac{45}{I_1 + I_2}$$

$$R = \frac{45}{90\,mA + \frac{5}{R_L}}$$

When $R_L \to \infty$, current in the zener diode will be maximum

i.e., $I_2 \to 0$, $I_1 \to 1$

So $R = \frac{45}{90} = 500\ \Omega$

Q.10 The area of cross-section of a railway track is 0.01 m^2. The temperature variation is 10°C. Coefficient of linear expansion of material of track is 10^{-5}/°C. The energy stored per meter in the track is _________ J/m.

(Young's modulus of material of track is 10^{11} Nm^{-2})

Given --
Answer :

Ans: 5.00

Sol: $E = \frac{y}{2}(\text{strain})^2 \times A \times L$

$$\frac{E}{L} = \frac{y}{2}(\text{strain})^2 \times A$$

$$= \frac{10^{11}}{2} \times \left(10^{-4}\right)^2 \times 10^{-2}$$

= 5 J/m

PART – B – CHEMISTRY

Section A

Q.1 Thiamine and pyridoxine are also known respectively as :

Options
1. Vitamin B_1 and Vitamin B_6
2. Vitamin E and Vitamin B_2
3. Vitamin B_6 and Vitamin B_2
4. Vitamin B_2 and Vitamin E

Ans: Vitamin B_1 and Vitamin B_6

Sol: Thiamine – Vitamin B_1
Pyridoxine – Vitamin B_6

Q.2 Which one of the following molecules does not show stereo isomerism ?

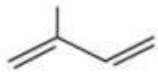
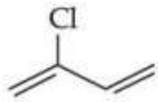
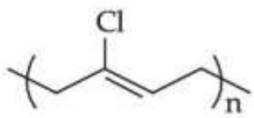
Options
1. 4-Methylhex-1-ene
2. 3,4-Dimethylhex-3-ene
3. 3-Ethylhex-3-ene
4. 3-Methylhex-1-ene

Ans: 3-Ethylhex-3-ene

Sol: $CH_3-CH_2\,C=C-CH_2-CH_3$
with CH_2CH_3 attached below

3-Ethylhex-3-ene does not show geometrical or optical isomerism.

Q.3 Match **List - I** with **List - II** :

	List - I		**List - II**
(a)	Chloroprene	(i)	
(b)	Neoprene	(ii)	
(c)	Acrylonitrile	(iii)	
(d)	Isoprene	(iv)	$CH_2=CH-CN$

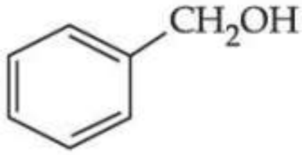
Choose the **correct** answer from the options given below :

Options
1. (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)
2. (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)
3. (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)
4. (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

Ans: (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

Sol: (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

Q.4 Which one of the following compounds does not exhibit resonance ?

Options
1. $CH_3CH_2OCH=CH_2$
2. 
3. $CH_3CH_2CH=CHCH_2NH_2$
4. $CH_3CH_2CH_2CONH_2$

Ans: $CH_3–CH_2CH=CHCH_2NH_2$

Sol: $CH_3–CH_2–CH=CH_2–NH_2$ has no conjugation between π-electrons and lone pair on nitrogen. Hence three will be no resonance

Q.5 The set having ions which are coloured and paramagnetic both is :

Options
1. Cu^{2+}, Cr^{3+}, Sc^{+}
2. Cu^{+}, Zn^{2+}, Mn^{4+}
3. Ni^{2+}, Mn^{7+}, Hg^{2+}
4. Sc^{3+}, V^{5+}, Ti^{4+}

Ans: Cu^{2+}, Cr^{3+}, Sc^{+}

Sol: Cu^{2+}, Sc^{+}, Cr^{3+} has 1, 2, 3 unpaired electron respectively. Hence it is paramagnetic and coloured.

Q.6 Which one of the following group-15 hydride is the strongest reducing agent ?

Options
1. PH_3
2. AsH_3
3. BiH_3
4. SbH_3

Ans: **BiH_3**

Sol: On moving down the group the E–H bond weakness and hence reducing property increases.

Q.7 Match **List - I** with **List - II** :

	List - I (Elements)		List - II (Properties)
(a)	Ba	(i)	Organic solvent soluble compounds
(b)	Ca	(ii)	Outer electronic configuration $6s^2$
(c)	Li	(iii)	Oxalate insoluble in water
(d)	Na	(iv)	Formation of very strong monoacidic base

Choose the **correct** answer from the options given below :

Options
1. (a)-(ii), (b)-(iii), (c)-(i) and (d)-(iv)
2. (a)-(iv), (b)-(i), (c)-(ii) and (d)-(iii)
3. (a)-(i), (b)-(iv), (c)-(ii) and (d)-(iii)
4. (a)-(iii), (b)-(ii), (c)-(iv) and (d)-(i)

Ans: (a)-(ii), (b)-(iii), (c)-(i) and (d)-(iv)

Sol: (a)-(ii), (b)-(iii), (c)-(i) and (d)-(iv)

Q.8 Sulphide ion is soft base and its ores are common for metals.

(a) Pb (b) Al (c) Ag (d) Mg

Choose the **correct** answer from the options given below :

Options
1. (c) and (d) only
2. (a) and (d) only
3. (a) and (c) only
4. (a) and (b) only

Ans: (a) and (c) only

Sol: The softer acid (Ag^+) and border line acid (Pb^{2+}) can be precipitated as sulphides from their aq.solutions. Since S^{2-} ion is a softer base

$$S^{2-} + Pb^{2+} \rightarrow \underset{\text{Black}}{PbS\downarrow}$$

$$S^{2-} + Ag^{+} \rightarrow \underset{\text{black}}{Ag_2S\downarrow}$$

Q.9 When silver nitrate solution is added to potassium iodide solution then the sol produced is :

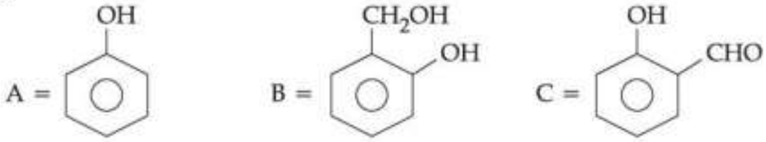
Options
1. AgI/I^-
2. AgI/Ag^+
3. KI/NO_3^-
4. $AgNO_3/NO_3^-$

Ans: AgI / I^-

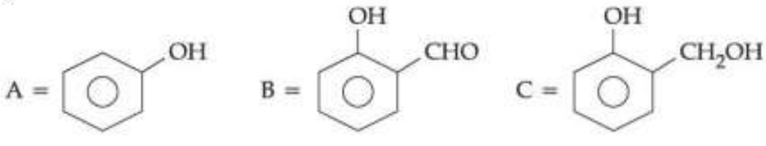
Sol: When silver nitrate solution is added to KI solution, the precipitated silver iodide absorbs iodide ions from the dispersión médium and negatively charged coloidal sol results

Q.10 An organic compound A (C_6H_6O) gives dark green colouration with ferric chloride. On treatment with $CHCl_3$ and KOH, followed by acidification gives compound B. Compound B can also be obtained from compound C on reaction with pyridinium chlorochromate (PCC). Identify A, B and C.

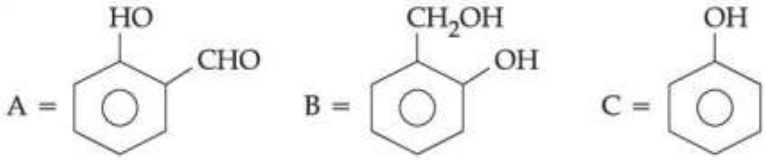
Options 1.



2.



3.



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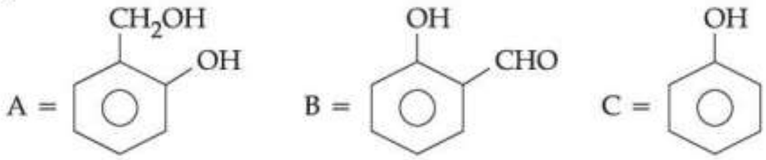
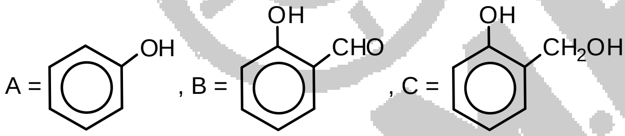
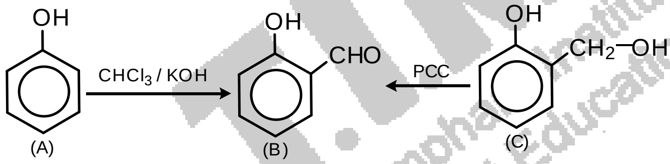


Ans:

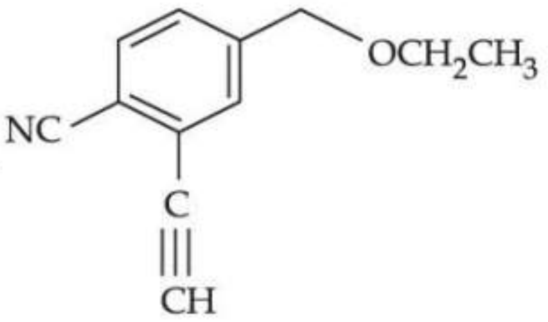


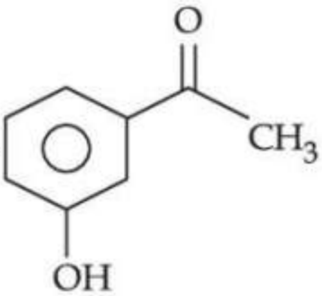
Sol:

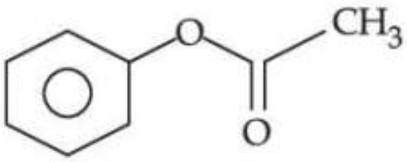


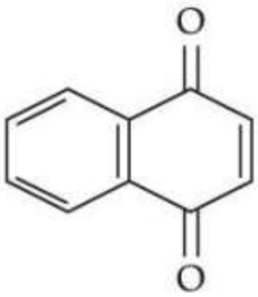
Q.11 Which one of the following compounds will provide a tertiary alcohol on reaction with excess of CH_3MgBr followed by hydrolysis ?

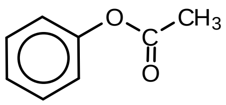
Options

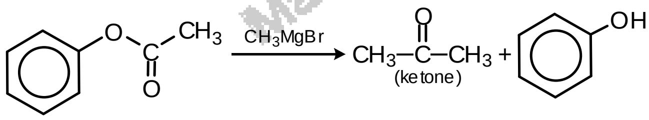
1. 

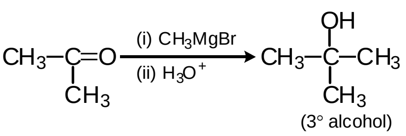
2. 

3. 

4. 

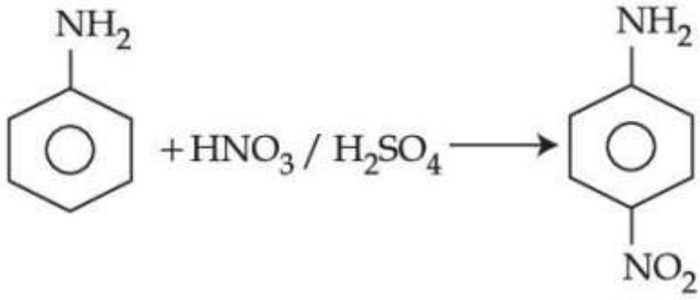
Ans: 

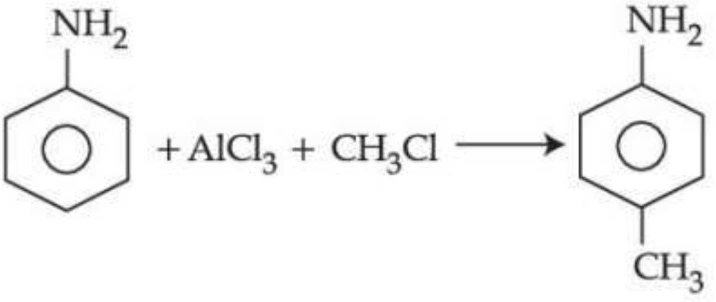
Sol: 

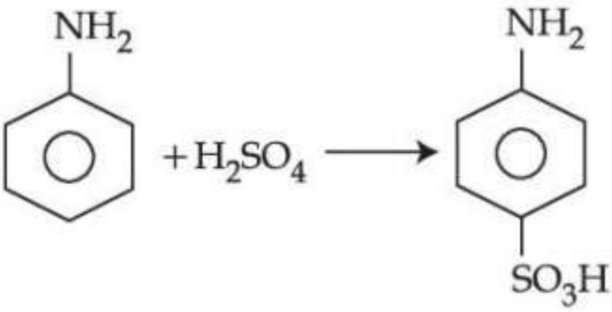


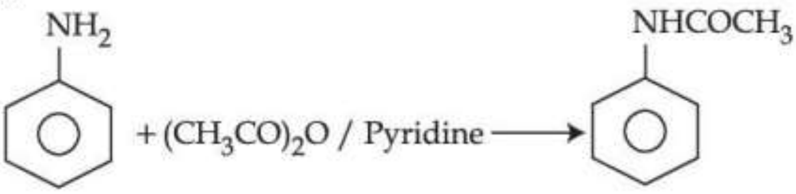
Q.12 Which one of the following reactions does not occur ?

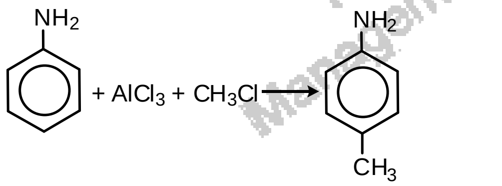
Options

1. 

2. 

3. 

4. 

Ans: 

Sol: Aniline does not undergo Friedel-Crafts reaction (alkylation and acetylation) due to salt formation with $AlCl_3$, the Lewis acid, which is used as a catalyst.

Q.13 Which one of the following statements for D.I. Mendeleeff, is **incorrect** ?

Options
1. Element with atomic number 101 is named after him.
2. At the time, he proposed Periodic Table of elements structure of atom was known.
3. He invented accurate barometer.
4. He authored the textbook - Principles of Chemistry.

Ans: At the time, he proposed Periodic Table of elements structure of atom was known

Sol: At the time, be prepared periodic table of elements structure of atom was known

Q.14 The water having more dissolved O_2 is :

Options
1. boiling water
2. water at 4°C
3. polluted water
4. water at 80°C

Ans: water at 4°C

Sol: Solubility of oxygen increases with decrease in temperature

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Q.15 Which one of the following 0.06 M aqueous solutions has lowest freezing point ?

Options
1. $Al_2(SO_4)_3$
2. KI
3. $C_6H_{12}O_6$
4. K_2SO_4

Ans: $Al_2(SO_4)_3$

Sol: $\Delta T_f = i\,K_f.m$, $\Delta T_f \ \alpha\ I$, $T_f = T_f^{\circ} - \Delta T_f$
Greater the i value lower will be the freezing point

Q.16 Match **List - I** with **List - II** :

	List - I (Species)		**List - II (Hybrid Orbitals)**
(a)	SF_4	(i)	sp^3d^2
(b)	IF_5	(ii)	d^2sp^3
(c)	NO_2^+	(iii)	sp^3d
(d)	NH_4^+	(iv)	sp^3
		(v)	sp

Choose the **correct** answer from the options given below :

Options
1. (a)-(ii), (b)-(i), (c)-(iv) and (d)-(v)
2. (a)-(iii), (b)-(i), (c)-(v) and (d)-(iv)
3. (a)-(iv), (b)-(iii), (c)-(ii) and (d)-(v)
4. (a)-(i), (b)-(ii), (c)-(v) and (d)-(iii)

Ans: (a)-(iii), (b)-(i), (c)-(v) and (d)-(iv)

Sol: SF_4 No. of hybrid orbitals $= \frac{6+4}{2} = 5$ sp^3d

IF_4 No. of hybrid orbitals $= \frac{7+5}{2} = 6$ sp^3d^2

NO_2^+ No. of hybrid orbitals $= \frac{5-1}{2} = 2$ sp

NH_4^+ No. of hybrid orbitals $= \frac{5+4-1}{2} = 4$ sp^3

Q.17 Isotope(s) of hydrogen which emits low energy β^- particles with $t_{1/2}$ value > 12 years is/are :

Options
1. Protium
2. Deuterium and Tritium
3. Deuterium
4. Tritium

Ans: Tritium

Sol: Only tritium is radioactive and emits low energy β^- particles with $t_{1/2}$ value 12.33 yrs

Q.18 Given below are the statements about diborane.

(a) Diborane is prepared by the oxidation of $NaBH_4$ with I_2.

(b) Each boron atom is in sp^2 hybridized state.

(c) Diborane has one bridged 3 centre-2-electron bond.

(d) Diborane is a planar molecule.

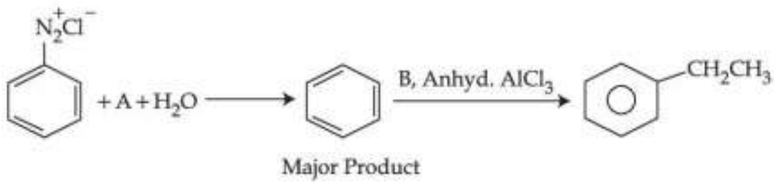
The option with **correct** statement(s) is :

Options
1. (a) only
2. (c) and (d) only
3. (c) only
4. (a) and (b) only

Ans: (a) only

Sol: $2NaBH_4 + I_2 \xrightarrow{\text{ether}} B_2H_6 + 2NaI + H_2$

Q.19

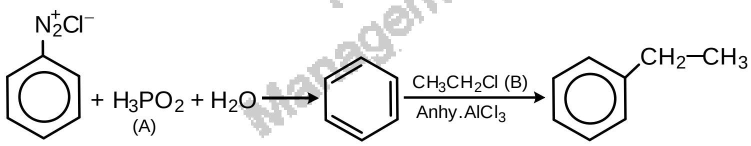


In the chemical reactions given above A and B respectively are :

Options
1. H_3PO_2 and CH_3CH_2Cl
2. H_3PO_2 and CH_3CH_2OH
3. CH_3CH_2OH and H_3PO_2
4. CH_3CH_2Cl and H_3PO_2

Ans: H_3PO_2 and CH_3CH_2Cl

Sol:



Q.20 Which purification technique is used for high boiling organic liquid compound (decomposes near its boiling point) ?

Options
1. Steam distillation
2. Reduced pressure distillation
3. Simple distillation
4. Fractional distillation

Ans: Reduced pressure distillation

Sol: This process is used if the liquid has a tendency to decompose near its boiling point

Section B

Q.1 Number of electrons that Vanadium (Z = 23) has in p-orbitals is equal to ________.

Given **12.00**
Answer :

Ans: 12

Sol: $_{23}V \rightarrow 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3d^2\ 4s^2$

No. of electrons in p-orbitals = 6 + 6 = 12

Q.2 If the concentration of glucose ($C_6H_{12}O_6$) in blood is 0.72 g L^{-1}, the molarity of glucose in blood is ________ $\times 10^{-3}$ M. (Nearest integer)

(Given : Atomic mass of C = 12, H = 1, O = 16 u)

Given **4.00**
Answer :

Ans: 4

Sol: $\text{Molarity} = \frac{W_B}{M_B V_{(L)}} = \frac{0.72}{180 \times 1} = 0.004 = 4 \times 10^{-3}$

Q.3 $N_2O_{5(g)} \rightarrow 2NO_{2(g)} + \frac{1}{2} O_{2(g)}$

In the above first order reaction the initial concentration of N_2O_5 is 2.40×10^{-2} mol L^{-1} at 318 K. The concentration of N_2O_5 after 1 hour was 1.60×10^{-2} mol L^{-1}. The rate constant of the reaction at 318 K is ________ $\times 10^{-3}$ min^{-1}. (Nearest integer)

[Given : log 3 = 0.477, log 5 = 0.699]

Given --
Answer :

Ans: 7

Sol: $K = \frac{2.303}{t} \log \frac{[R]_0}{[R]} = \frac{2.303}{60} \log \frac{2.4 \times 10^{-2}}{1.6 \times 10^{-2}}$

$= \frac{2.303}{60} \log \frac{3}{2} = \frac{2.303}{60} \times [0.477 - 0.3010]$

$= \frac{2.303}{60} \times 0.176 = 0.006755 = 6.75 \times 10^{-3}$

Q.4 Methylation of 10 g of benzene gave 9.2 g of toluene. Calculate the percentage yield of toluene ________. (Nearest integer)

Given **99.2**
Answer :

Ans: 78

Sol: $\underset{\text{(Benzene)}}{C_6H_6} \xrightarrow{\text{Methylation}} \underset{\text{(Toluene)}}{C_6H_5CH_3}$

78 g $\rightarrow$ 92 g

10 g $\rightarrow$ x

(Theoretical wt), $x = \frac{92 \times 10}{78} = 11.79\,g$

Actual weight of Toluene = 9.2 g

$\%\ \text{yield} = \frac{W_{actual}}{W_{theoretical}} \times 100 = \frac{9.2}{11.79} \times 100 = 78.03$

Q.5 The total number of unpaired electrons present in $[Co(NH_3)_6]Cl_2$ and $[Co(NH_3)_6]Cl_3$ is __________.

Given **3.00**
Answer :

Ans: 3

Sol: $[Co(NH_3)_6]Cl_2$
Co^{2+} : [Ar] $3d^7\ 4s^0\ 4p^0$
No pairing of electrons takes place
Hence, no. of unpaired electrons = 3
$[Co(NH_3)_6]Cl_3$
Pairing of electrons takes place
$\therefore$ Hence no. of unpaired electrons = 0
$\therefore$ Total no. of unpaired electrons = 3 + 0 = 3

Q.6 The number of acyclic structural isomers (including geometrical isomers) for pentene are __________.

Given **8.00**
Answer :

Ans: 6

Sol: $CH_3–CH_2–CH_2–CH=CH_2$ — (1)
$CH_3–CH_2–CH=CH–CH_3$ — (2) [1 cis and 1 trans]

$CH_3–CH_2–C–CH_2$ — (1)
(with CH_3 on C)

$CH_3–CH–CH=CH_2$ — (1)
(with CH_3 on CH)

$CH_3–C=CH–CH_3$ — (1)
(with CH_3 on C)

Total = 6

Q.7 Value of K_p for the equilibrium reaction $N_2O_{4(g)} \rightleftharpoons 2NO_{2(g)}$ at 288 K is 47.9. The K_c for this reaction at same temperature is __________. (Nearest integer)
($R = 0.083\ L\ bar\ K^{-1}\ mol^{-1}$)

Given --
Answer :

Ans: 2

Sol: $K_p = K_c(RT)^{\Delta n_g}$ $\qquad \Delta n_g = 2 - 1 = 1$
$47.9 = K_c\ (0.083 \times 288)^1$
$K_c = \dfrac{47.9}{0.083 \times 288} = 2.033$

Q.8 Assume a cell with the following reaction
$Cu_{(s)} + 2Ag^+(1 \times 10^{-3}\ M) \rightarrow Cu^{2+}(0.250\ M) + 2\ Ag_{(s)}$
$E^{\ominus}_{cell} = 2.97\ V$
E_{cell} for the above reaction is __________ V. (Nearest integer)
[Given : log 2.5 = 0.3979, T = 298 K]

Given --
Answer :

Ans: 3

Sol: $E_{cell} = E^{\circ}_{cell} - \frac{0.06}{2}\log\frac{[Cu^{2+}]}{[Ag^{+}]^2}$

$= 2.97 - \frac{0.06}{2}\log\frac{0.250}{(1\times10^{-3})^2}$

$= 2.97 - 0.03\ \log\frac{0.250}{10^{-6}}$

$= 2.97 - 0.03 \log 0.250 \times 10^6$

$= 2.97 - 0.03 \log 2.5 \times 10^5$

$= 2.97 - 0.03 [\log 2.5 + 5 \log 10]$

$= 2.97 - 0.03 \times 5.3979$

$= 2.97 - 0.1619 = 2.808$

Q.9 A copper complex crystallising in a CCP lattice with a cell edge of 0.4518 nm has been revealed by employing X-ray diffraction studies. The density of a copper complex is found to be 7.62 g cm^{-3}. The molar mass of copper complex is ________ g mol^{-1}. (Nearest integer) [Given : $N_A = 6.022\times10^{23}$ mol^{-1}]

Given -- Answer :

Ans: 106

Sol: $d = \frac{ZM}{a^3N_A}$, $M = \frac{da^3N_A}{Z} = \frac{7.62\times(0.4518\times10^{-7})^3\times6.023\times10^{23}}{4} = 105.7$

Q.10 If the standard molar enthalpy change for combustion of graphite powder is -2.48×10^2 kJ mol^{-1}, the amount of heat generated on combustion of 1 g of graphite powder is ________ kJ. (Nearest integer)

Given -- Answer :

Ans: 21

Sol: $\underset{(graphite)}{C} + O_2 \rightarrow CO_2$ $\quad \Delta H = -2.48\times10^2 kg/mol$

$12\ g \rightarrow 2.48 \times 10^2\ kg$

$1\ g \rightarrow x$

$x = \frac{-2.48\times10^2}{12} = 0.2066\times10^2 = 20.66$

PART – C – MATHEMATICS

Section A

Q.1 Let $A = [a_{ij}]$ be a real matrix of order 3×3, such that $a_{i1} + a_{i2} + a_{i3} = 1$, for $i = 1, 2, 3$. Then, the sum of all the entries of the matrix A^3 is equal to :

Options
1. 3
2. 1
3. 2
4. 9

Ans: 3

Sol: A matrix which satisfies the given condition is ,

$$A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I$$

$$\Rightarrow A^3 = I^3 = I = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Sum of all elements =3

Q.2 Let the circle $S: 36x^2+36y^2-108x+120y+C=0$ be such that it neither intersects nor touches the co-ordinate axes. If the point of intersection of the lines, $x-2y=4$ and $2x-y=5$ lies inside the circle S, then :

Options

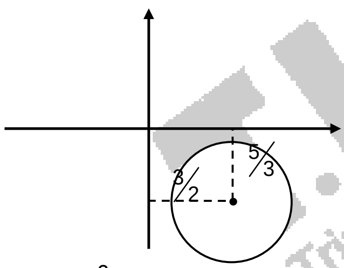
1. $81 < C < 156$
2. $\frac{25}{9} < C < \frac{13}{3}$
3. $100 < C < 156$
4. $100 < C < 165$

Ans: 100 < C < 156

Sol: $S: x^2+y^2-3x+\frac{10}{3}y+\frac{k}{36}=0$ [C is denoted by k to avoid confusion]

$2g=-3$, $2f=\frac{10}{3}$, $C=\frac{k}{36}$

Centre: (-g , -f) = $\left(\frac{3}{2}, -\frac{5}{3}\right)$



Clearly $r < \frac{3}{2}$

$\Rightarrow g^2+f^2-c < \frac{9}{4}$

$\frac{9}{4}+\frac{25}{9}-\frac{k}{36} < \frac{9}{4}$

$\frac{25}{9} < \frac{k}{36} \Rightarrow k > 100$ —(1)

Point of intersection of x-2y=4 and 2x-y=5 is (2 , -1) [After solving the equations]

$\Rightarrow S_{(2,-1)} < 0$ ($\because$ Point (2, -1) is inside the circle S=0)

$\Rightarrow 2^2+(-1)^2-3(2)+\frac{10}{3}(-1)+\frac{k}{36} < 0$

$\Rightarrow 5-6-\frac{10}{3}+\frac{k}{36} < 0$

$\Rightarrow -\frac{13}{3}+\frac{k}{36} < 0$

$\Rightarrow \frac{k}{36} < \frac{13}{3} \Rightarrow k < 156$ —(2)

From (1) and (2) , $100<k<156$

ie; $100<C<156$

Q.3 Let S_n denote the sum of first n-terms of an arithmetic progression. If $S_{10}=530$, $S_5=140$, then $S_{20}-S_6$ is equal to :

Options
1. 1852
2. 1862
3. 1842
4. 1872

Ans: 1862

Sol: $S_{10}=530 \Rightarrow \frac{10}{2}(2a+9d)=530 \Rightarrow 2a+9d=106$ —(1)

$S_5=140 \Rightarrow \frac{5}{2}(2a+4d)=140 \Rightarrow 2a+4d=56$ —(2)

(1)—(2) $\Rightarrow 5d=50 \Rightarrow d=10$ and

$2a+90=106 \Rightarrow 2a=16 \Rightarrow a=8$

$S_{20}-S_6=\frac{20}{2}(2a+19d)-\frac{6}{2}(2a+5d)$ = 10(16+190)-3(16+50) = 1862

Q.4

Let $f:\mathbf{R}\to\mathbf{R}$ be defined as $f(x)=\begin{cases}\dfrac{x^3}{(1-\cos 2x)^2}\log_e\left(\dfrac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right), & x\neq 0\\ \alpha, & x=0\end{cases}$.

If f is continuous at $x=0$, then α is equal to :

Options
1. 3
2. 0
3. 1
4. 2

Ans: 1

Sol: $\lim\limits_{x\to 0} f(x)=\lim\limits_{x\to 0}\frac{x^3}{(2\sin^2 x)^2}.\left[\log(1+2xe^{-2x})-2\log(1-xe^{-x})\right]$

$=\frac{1}{4}\lim\limits_{x\to 0}\left[\left(\frac{x^4}{\sin^4 x}\right)\left\{\frac{\log(1+2xe^{-2x})-2\log(1-xe^{-x})}{x}\right\}\right]$ —(1)

Where $\lim\limits_{x\to 0}\left(\frac{x^4}{\sin^4 x}\right)=\lim\limits_{x\to 0}\left[\frac{1}{\left(\frac{\sin x}{x}\right)^4}\right]=\frac{1}{1}=1$

$\therefore (1) \Rightarrow \frac{1}{4}.\lim\limits_{x\to 0}\left[\frac{\log(1+2xe^{-2x})-2\log(1-xe^{-x})}{x}\right]$

$=\frac{1}{4}\lim\limits_{x\to 0}\left[\frac{1}{(1+2xe^{-2x})}.2\left\{xe^{-2x}.(-2)+e^{-2x}\right\}-\frac{2(-1)}{(1-xe^{-x})}\left\{xe^{-x}(-1)+e^{-x}\right\}\right]$

$=\frac{1}{4}\times 4=1$ (Using L'Hospitals rule)

Q.5 Let $f: \mathbf{R} \rightarrow \mathbf{R}$ be defined as

$$f(x) = \begin{cases} -\dfrac{4}{3}x^3 + 2x^2 + 3x, & x > 0 \\ 3xe^x \quad , & x \leq 0 \end{cases}$$

Then f is increasing function in the interval.

Options
1. $(0,\ 2)$
2. $(-3,\ -1)$
3. $\left(-1,\ \dfrac{3}{2}\right)$
4. $\left(-\dfrac{1}{2},\ 2\right)$

Ans: $\left(-1, \dfrac{3}{2}\right)$

Sol: $f(x) = \begin{cases} -\frac{4}{3}x^3 + 2x^2 + 3x, x > 0 \\ 3xe^x, x \leq 0 \end{cases}$

Case 1 (x>0)

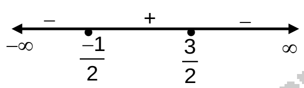
$y = -\dfrac{4}{3}x^2 + 2x^2 + 3x$

$\dfrac{dy}{dx} = -4x^2 + 4x + 3 = 0 \text{ (say)}$

$\Rightarrow 4x^2 - 4x - 3 = 0$

$\Rightarrow x = \dfrac{4 \pm \sqrt{16 + 48}}{8} = \dfrac{4 \pm 8}{8}$

$\Rightarrow x = \dfrac{12}{8} \text{ or } -\dfrac{4}{8} \Rightarrow x = \dfrac{3}{2} \text{ or } -\dfrac{1}{2}$

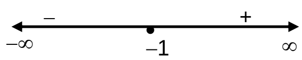


Hence if x>0 , f(x) is increasing on $\left(0, \dfrac{3}{2}\right)$ — (1)

Case 2 (x<0)

$y = 3xe^x$

$\dfrac{dy}{dx} = 3\left[xe^x + e^x\right] = 3e^x(x + 1)$



Hence if x<0, f(x) is increasing on [-1 , 0) — (2)
Hence from (1) and (2),

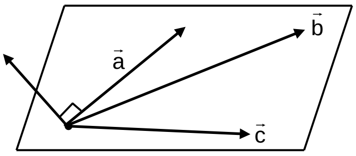
$x \in \left(-1, \dfrac{3}{2}\right)$

Q.6 Let a vector $\vec{a}$ be coplanar with vectors $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$. If $\vec{a}$ is perpendicular to $\vec{d} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, and $|\vec{a}| = \sqrt{10}$. Then a possible value of $[\vec{a}\ \vec{b}\ \vec{c}] + [\vec{a}\ \vec{b}\ \vec{d}] + [\vec{a}\ \vec{c}\ \vec{d}]$ is equal to :

Options
1. -29
2. -38
3. -42
4. -40

Ans: -42

Sol:



$\vec{a}$ is parallel to $\vec{d} \times (\vec{b} \times \vec{c}) = (\vec{d}.\vec{c})\vec{b} - (\vec{d}.\vec{b})\vec{c}$

$= (3 - 2 + 6)\vec{b} - (6 + 2 + 6)\vec{c}$

$= 7\vec{b} - 14\vec{c}$

$= 7[(2\hat{i} + \hat{j} + \hat{k}) - 2(\hat{i} - \hat{j} + \hat{k})]$

$= 7(0\hat{i} + 3\hat{j} - \hat{k})$

$\therefore \vec{a} || 3\hat{j} - \hat{k}$

since $|\vec{a}| = \sqrt{10}$,

$\bar{a} = 3\hat{j} - \hat{k}$

$[\vec{a}\vec{b}\vec{c}] = 0$ since $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar

$[\vec{a}\vec{b}\vec{d}] + [\vec{a}\vec{c}\vec{d}] = [\vec{a}\ \vec{b} + \vec{c}\ \vec{d}]$

$$= \begin{vmatrix} 0 & 3 & -1 \\ 3 & 0 & 2 \\ 3 & 2 & 6 \end{vmatrix} = -3(18 - 6) - 1(6) = -36 - 6 = -42$$

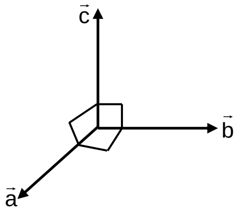
Q.7 Let three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ be such that $\vec{a} \times \vec{b} = \vec{c}$, $\vec{b} \times \vec{c} = \vec{a}$ and $|\vec{a}| = 2$. Then which one of the following is **not** true ?

Options
1. $[\vec{a}\ \vec{b}\ \vec{c}] + [\vec{c}\ \vec{a}\ \vec{b}] = 8$
2. $|3\vec{a} + \vec{b} - 2\vec{c}|^2 = 51$
3. Projection of $\vec{a}$ on $(\vec{b} \times \vec{c})$ is 2
4. $\vec{a} \times ((\vec{b} + \vec{c}) \times (\vec{b} - \vec{c})) = \vec{0}$

Ans: $|3\vec{a} + \vec{b} - 2\vec{c}|^2 = 51$

Sol: $\vec{a}\times\vec{b}=\vec{c}\Rightarrow\vec{c}$ is perpendicular to both $\vec{a}$ and $\vec{b}$

$\vec{b}\times\vec{c}=\vec{a}\Rightarrow\vec{a}$ is perpendicular to both $\vec{b}$ and $\vec{c}$



$|\vec{a}\times\vec{b}|=|\vec{c}|$

$\Rightarrow |\vec{a}||\vec{b}||\sin 90^\circ|=|\vec{c}|$

$\Rightarrow 2|\vec{b}|=|\vec{c}| \quad -(1)$

$|\vec{b}\times\vec{c}|=|\vec{a}|$

$\Rightarrow |\vec{b}||\vec{c}|\sin 90^\circ=|\vec{a}|$

$\Rightarrow |\vec{b}||\vec{c}|=2 \quad -(2)$

$\therefore |\vec{b}|.2|\vec{b}|=2\Rightarrow|\vec{b}|=1$

and $|\vec{c}|=2$

$[\vec{a}\,\vec{b}\,\vec{c}]+[\vec{c}\,\vec{a}\,\vec{b}]=[\vec{a}\,\vec{b}\,\vec{c}]+[\vec{a}\,\vec{b}\,\vec{c}]$

$=2[\vec{a}\,\vec{b}\,\vec{c}]=2|\vec{a}||\vec{b}||\vec{c}|$

$=2\times2\times1\times2=8$

(1) is true.

$|3\vec{a}+\vec{b}-2\vec{c}|^2=9\vec{a}^2+\vec{b}^2+4\vec{c}^2$

$=9(2)^2+(1)^2+4(2)^2$

$=36+1+16=53$

Q.8 Let n denote the number of solutions of the equation $z^2+3\bar{z}=0$, where z is a complex number. Then the value of $\sum_{k=0}^{\infty}\frac{1}{n^k}$ is equal to :

Options
1. 2
2. 1
3. $\frac{4}{3}$
4. $\frac{3}{2}$

Ans: $\frac{4}{3}$

Sol: $z^2=-3\bar{z}$

$\Rightarrow (x+iy)^2=-3(x-iy);\ x,y\in R$

$(x^2-y^2)+i(2xy)=-3x+3yi$

$\Rightarrow x^2-y^2=-3x-(1)$

$2xy=3y-(2)$

Case 1: y=0

$x^2-y^2+3x=0$

So, x =0 and x=-3

Therefore z=0 and z=-3

Case 2: $x=\frac{3}{2}$

$x^2-y^2+3x=0$

$y = \frac{3\sqrt{3}}{2}$ *and* $y = -\frac{3\sqrt{3}}{2}$

$\therefore n = 2$

$\therefore \sum_{k=0}^{\infty} \frac{1}{n^k} = \frac{1}{n^0} + \frac{1}{n^1} + \frac{1}{n^2} + \ldots\ldots\ldots \ldots\text{to}\infty = 1 + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \ldots\ldots\ldots \text{to}\infty = \frac{1}{\left(1-\frac{1}{4}\right)} = \frac{4}{3}$

Q.9 If $\int_0^{100\pi} \frac{\sin^2 x}{e^{\left(\frac{x}{\pi} - \left[\frac{x}{\pi}\right]\right)}} dx = \frac{\alpha\pi^3}{1+4\pi^2}$, $\alpha \in \mathbf{R}$,

where $[x]$ is the greatest integer less than or equal to x, then the value of α is :

Options
1. $150\ (e^{-1} - 1)$
2. $50\ (e - 1)$
3. $200\ (1 - e^{-1})$
4. $100\ (1 - e)$

Ans: $200(1 - e^{-1})$

Sol: Both $\sin^2 x$ and $e^{\frac{x}{\pi} - \left[\frac{x}{\pi}\right]}$ are periodic functions with period π

$$\therefore I = \int_0^{100\pi} \frac{\sin^2 x}{e^{\left(\frac{x}{\pi} - \left[\frac{x}{\pi}\right]\right)}} dx = 100\int_0^{\pi} \frac{\sin^2 x}{e^{\left(\frac{x}{\pi} - \left[\frac{x}{\pi}\right]\right)}} dx$$

If $0 < x < \pi$ then $0 < \frac{x}{\pi} < 1 \Rightarrow \left[\frac{x}{\pi}\right] = 0$

$$\therefore I = 100\int_0^{\pi} \frac{\sin^2 x}{e^{\left(\frac{x}{\pi} - \left[\frac{x}{\pi}\right]\right)}} dx = 100\int_0^{\pi} e^{-\frac{1}{\pi}x} \cdot \left(\frac{1-\cos 2x}{2}\right) dx$$

$$= 50\left[\int_0^{\pi} e^{-x/\pi} dx - \int_0^{\pi} e^{-\frac{1}{\pi}x} \cos 2x dx\right] = 50\left[\left[\frac{e^{-x/\pi}}{-1/\pi}\right]_0^{\pi} - \int_0^{\pi} e^{-\frac{x}{\pi}} \cos 2x dx\right]$$

$$= 50\left[-\pi\left\{e^{-1} - e^0\right\} - \int_0^{\pi} e^{-\frac{x}{\pi}} . \cos 2x dx\right]$$

$$= 50\pi\left(1 - e^{-1}\right) - 50 . \int_0^{\pi} e^{-\frac{x}{\pi}} . \cos 2x dx$$

$$= 50\pi\left(1 - e^{-1}\right) - \frac{50}{\left(-\frac{1}{\pi}\right)^2 + 2^2}\left[e^{-\frac{x}{\pi}}\left(-\frac{1}{\pi}\cos 2x + 2\sin 2x\right)\right]_0^{\pi}$$

$$\left[\because \int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2+b^2}\left(a\cos bx + b\sin bx\right) + c\right]$$

$$= 50\pi\left(1-e^{-1}\right) - \frac{50}{\left(\frac{1}{\pi^2}+4\right)}\left[e^{-1}\left(-\frac{1}{\pi}+0\right) - 1\left(-\frac{1}{\pi}\right)\right]$$

$$= 50\pi\left(1-e^{-1}\right) - \frac{50\pi^2}{\left(4\pi^2+1\right)} \times \frac{1}{\pi}(1-e^{-1}) = 50\pi\left(1-e^{-1}\right)\left[1-\frac{1}{4\pi^2+1}\right]$$

$$= 50\pi\left(1-e^{-1}\right) \cdot \frac{4\pi^2}{4\pi^2+1}$$

$$= \frac{200\pi^3(1-e^{-1})}{1+4\pi^2}$$

$$\Rightarrow \alpha = 200(1-e^{-1})$$

Q.10 Let $y=y(x)$ be the solution of the differential equation

$\operatorname{cosec}^2 x \, dy + 2dx = (1 + y\cos 2x)\operatorname{cosec}^2 x \, dx$, with $y\left(\frac{\pi}{4}\right) = 0$. Then, the value of $(y(0)+1)^2$ is equal to :

Options

1. e^{-1}
2. $e^{-1/2}$
3. e
4. $e^{1/2}$

Ans: e^{-1}

Sol: $\operatorname{cosec}^2 x dy + 2dx = (1 + y\cos 2x)\operatorname{cosec}^2 x . dx$

$$\Rightarrow \frac{1}{\sin^2 x}dy + 2dx = \left[\frac{1}{\sin^2 x} + \frac{y(1-2\sin^2 x)}{\sin^2 x}\right]dx$$

$$\Rightarrow dy + 2\sin^2 x dx = \left[1 + y - 2y\sin^2 x\right]dx$$

$$\Rightarrow \frac{dy}{dx} + 2\sin^2 x = 1 + y - 2y\sin^2 x.$$

$$\Rightarrow \frac{dy}{dx} = 1 + y - 2y\sin^2 x - 2\sin^2 x$$

$$= (1+y) - 2\sin^2 x(1+y)$$

$$\Rightarrow \frac{dy}{dx} = (1+y)(1-2\sin^2 x)$$

$$\Rightarrow \frac{dy}{dx} = (1+y).\cos 2x$$

$$\Rightarrow \int \frac{dy}{1+y} = \int \cos 2x . dx$$

$$\log(1+y) = \frac{\sin 2x}{2} + C$$
$$\Rightarrow 1+y = e^{\left(\frac{\sin 2x}{2}\right)}.e^{C}$$
$$\Rightarrow y = k.e^{\frac{1}{2}\sin 2x} - 1$$
$$y\left(\pi/4\right) = 0 \Rightarrow 0 = k.e^{\frac{1}{2}} - 1$$
$$\Rightarrow k.e^{\frac{1}{2}} = 1 \Rightarrow k = e^{-\frac{1}{2}}$$
$$y(0) = ke^{0} - 1 = k - 1$$
$$\Rightarrow [y(0)+1]^2 = k^2 = \left(e^{-\frac{1}{2}}\right)^2 = e^{-1}$$

Q.11 The number of solutions of $\sin^7 x + \cos^7 x = 1$, $x \in [0, 4\pi]$ is equal to :

Options
1. 9
2. 11
3. 7
4. 5

Ans: 5

Sol: $\sin^7 x + \cos^7 x = 1$

$\sin^7 x \le \sin^2 x$ and $\cos^7 x \le \cos^2 x$

$\therefore \sin^7 x + \cos^7 x \le \sin^2 x + \cos^2 x$

$\Rightarrow \sin^7 x + \cos^7 x \le 1$

$\therefore \sin x = 1$ or $\cos x = 1$

$\Rightarrow x = \frac{x}{2}, 2\pi + \frac{\pi}{2}, 0, 2\pi, 4\pi$

$\therefore$ Number of solution $= 5$

Q.12 The values of λ and μ such that the system of equations $x+y+z=6$, $3x+5y+5z=26$, $x+2y+\lambda z=\mu$ has no solution, are :

Options
1. $\lambda=3, \mu=5$
2. $\lambda=2, \mu\neq 10$
3. $\lambda\neq 2, \mu=10$
4. $\lambda=3, \mu\neq 10$

Ans: $\lambda = 2, \mu \neq 10$

Sol: $x+y+z=6$ (1)

$3x+5y+5z=26$ (2)

$x+2y+\lambda z=\mu$ (3)

$(3)-(1) \Rightarrow y + (\lambda-1)z = \mu - 6 - (A)$

$(2) - 3\times(1) \Rightarrow 2y + 2z = 8 \Rightarrow y + z = 4$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{1}{1} = \frac{\lambda - 1}{1} \neq \frac{\mu - 6}{4}$

$\Rightarrow \lambda - 2 and \mu - 6 \neq 4$

$\Rightarrow \lambda = 2 and \mu \neq 10$

Q.13 Which of the following Boolean expressions is **not** a tautology ?

Options
1. $(p \Rightarrow \sim q) \vee (\sim q \Rightarrow p)$
2. $(p \Rightarrow q) \vee (\sim q \Rightarrow p)$
3. $(q \Rightarrow p) \vee (\sim q \Rightarrow p)$
4. $(\sim p \Rightarrow q) \vee (\sim q \Rightarrow p)$

Ans: $(\sim p \Rightarrow q) \vee (\sim q \Rightarrow p)$

Sol: we know that $p \Rightarrow q = p' \vee q$ or $p'+q$ (in Boolean algebra)

(1) $\Rightarrow (p'+q') + [(q')'+p] = p'+q'+q+p = (q+q') + p + p' \Rightarrow$ Tautology $(\because q \vee \sim q = T)$

(2) $\Rightarrow (p'+q) + [(q')'+p] = p'+q+q+p = (p'+p) + q + q \Rightarrow$ Tautology

(3) $\Rightarrow (q'+p) + [(q')'+p] = q'+p+q+p \Rightarrow$ Tautology.

(4). $\Rightarrow [(p')'+q] + [(q')'+p] = p+q+q+p = (p+q)+(p+q)$

$= p+q = p \vee q \Rightarrow$ Not a tautology

Q.14 If the shortest distance between the straight lines $3(x-1) = 6(y-2) = 2(z-1)$ and $4(x-2) = 2(y-\lambda) = (z-3)$, $\lambda \epsilon \mathbf{R}$ is $\frac{1}{\sqrt{38}}$, then the integral value of λ is equal to :

Options
1. -1
2. 2
3. 5
4. 3

Ans: 3

Sol: $L_1 : \frac{(x-1)}{1/3} = \frac{y-2}{1/6} = \frac{z-1}{1/2} \Rightarrow \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-1}{3}$

$L_2 : \frac{x-2}{1/4} = \frac{y-\lambda}{1/2} = \frac{z-3}{1} \Rightarrow \frac{x-2}{1} = \frac{y-\lambda}{2} = \frac{z-3}{4}$

$\vec{a}_1 :(1,2,1)$; $\vec{b}_1 :(2,1,3)$; $\vec{a}_2 :(2,\lambda,3)$; $\vec{b}_2 :(1,2,4)$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{vmatrix} = -2\hat{i} - 5\hat{j} + 3\hat{k}$$

$\vec{a}_2 - \vec{a}_1 = \hat{i} + (\lambda - 2)\hat{j} + 2\hat{k}$

$$S.D = \left| \frac{(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right| = \frac{|-2 - 5\lambda + 10 + 6|}{\sqrt{4+25+9}} = \frac{|14 - 5\lambda|}{\sqrt{38}} = \frac{1}{\sqrt{38}}$$

$\Rightarrow 14 - 5\lambda = \pm 1 \Rightarrow 5\lambda = 14 \pm 1 \Rightarrow 5\lambda = 15$ or 13

Integral value of $\lambda = 3$

Q.15 Let $[x]$ denote the greatest integer less than or equal to x. Then, the values of $x \in \mathbf{R}$ satisfying the equation $[e^x]^2 + [e^x + 1] - 3 = 0$ lie in the interval :

Options
1. $[\log_e 2, \log_e 3)$
2. $[0, \log_e 2)$
3. $[1, e)$
4. $[0, 1/e)$

Ans: $[0, \log_e 2)$

Sol: $[e^x]^2 + [e^x + 1] - 3 = 0$

$\Rightarrow [e^x]^2 + [e^x] + 1 - 3 = 0$

$\Rightarrow [e^x]^2 + [e^x] - 2 = 0$

$\Rightarrow ([e^x] - 1)([e^x] + 2) = 0$

$\Rightarrow [e^x] = 1$ or $[e^x] = -2$

Since $e^x > 0$, $[e^x] = -2$ is not possible.

$\therefore [e^x] = 1 \Rightarrow 1 \le e^x < 2 \Rightarrow \log 1 \le \log(e^x) < \log 2$

$\Rightarrow 0 \le x < \log_e 2 \Rightarrow x \in [0, \log_e 2)$

Q.16 If the domain of the function $f(x) = \dfrac{\cos^{-1}\sqrt{x^2 - x + 1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$ is the interval $(\alpha, \beta]$, then $\alpha + \beta$ is equal to :

Options
1. 1
2. $\frac{3}{2}$
3. 2
4. $\frac{1}{2}$

Ans: $\frac{3}{2}$

Sol: $f(x) = \dfrac{\cos^{-1}(\sqrt{x^2 - x + 1})}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$

For denominator,

$0 < \frac{2x-1}{2} \le 1$

$\Rightarrow 0 < 2x - 1 \le 2$

$\Rightarrow 1 < 2x \le 3$

$\Rightarrow \frac{1}{2} < x \le \frac{3}{2} \quad -(1)$

For Numerator,

$o \le \sqrt{x^2 - x + 1} \le 1$

$\left(\because \text{"}\sqrt{\ }\text{" represents positive root}\right)$

$\Rightarrow 0 \le x^2 - x + 1 \le 1$

Since D = b^2-4ac<0 and a>0,

x^2-x+1≥0

$\therefore x^2 - x + 1 \le 1 \Rightarrow x^2 - x \le 0$

$\Rightarrow x(x-1) \le 0 \Rightarrow x \in [0,1] \quad -(2)$

Intersection of (1) and (2) is $\left(\frac{1}{2},1\right]$

$\therefore \alpha + \beta = \frac{3}{2}$

Q.17 Let a line L : $2x + y = k$, $k > 0$ be a tangent to the hyperbola $x^2 - y^2 = 3$. If L is also a tangent to the parabola $y^2 = \alpha x$, then α is equal to :

Options

1. 24
2. 12
3. −12
4. −24

Ans: –24

Sol: y=-2x+k —(1)

$\Rightarrow m = -2 \text{ and } c = k$

Hyperbola: $\frac{x^3}{3} - \frac{y^2}{3} = 1$—(2)

$\Rightarrow a^2 = 3, \; b^2 = 3$

Since (1) is tangent to (2),

$c^2 = a^2m^2 - b^2$

$\Rightarrow k^2 = 3(-2)^2 - 3 = 9$

$\Rightarrow k = 3 \quad (\because k > 0)$

$\therefore c = k = 3$

Parabola: $y^2 = 4\left(\frac{\alpha}{4}\right)x$

$\Rightarrow a = \frac{\alpha}{4}, c = 3, m = -2$

$c = \frac{a}{m}$ (Condition for tangency)

$\Rightarrow 3 = \frac{\alpha/4}{-2} = \frac{\alpha}{-8}$

$\Rightarrow \alpha = -24$

Q.18

Let $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, a > b. Let E_2 be another ellipse such that it touches the end points of major axis of E_1 and the foci of E_2 are the end points of minor axis of E_1. If E_1 and E_2 have same eccentricities, then its value is :

Options

1. $\frac{-1+\sqrt{8}}{2}$

2. $\frac{-1+\sqrt{3}}{2}$

3. $\frac{-1+\sqrt{5}}{2}$

4. $\frac{-1+\sqrt{6}}{2}$

Ans: $\frac{-1+\sqrt{5}}{2}$

Sol: For E_1

$e_1 = \sqrt{1-\frac{b^2}{a^2}}$

For E_2

Let E_2 : $\frac{x^2}{B^2} + \frac{y^2}{A^2} = 1$

Where B= a —(1)

Focii: (0,±c)=(0,±b)

$\Rightarrow c = b \Rightarrow \sqrt{A^2 - B^2} = b$

$\Rightarrow A^2 - B^2 = b^2 \Rightarrow A^2 = a^2 + b^2$

$e_2 = \sqrt{1-\frac{B^2}{A^2}}; \, since \, e_1 = e_2,$

$\frac{b^2}{a^2} = \frac{B^2}{A^2} = \frac{a^2}{a^2+b^2} \Rightarrow \frac{b^2}{a^2} = \frac{1}{1+\frac{b^2}{a^2}}$

$1 - e^2 = \frac{1}{1+1-e^2}$

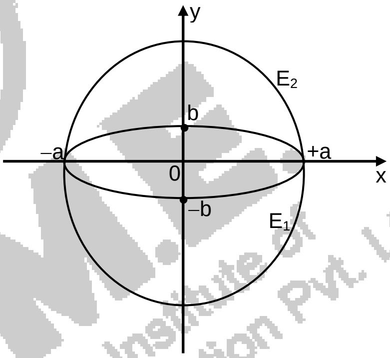
$Let 1 - e^2 = k$

$\Rightarrow k = \frac{1}{1+k} \Rightarrow k^2 + k = 1$

$\Rightarrow k^2 + k - 1 = 0 \Rightarrow k = \frac{-1+\sqrt{1+4}}{2} (\because 1 - e^2 > 0)$

$\therefore 1 - e^2 = \frac{\sqrt{5}-1}{2} \Rightarrow e^2 = 1 - \left(\frac{\sqrt{5}-1}{2}\right) = \frac{2-\sqrt{5}+1}{2} = \frac{3-\sqrt{5}}{2} = \left(\frac{-1+\sqrt{5}}{2}\right)^2$

$\Rightarrow e = \frac{-1+\sqrt{5}}{2}$

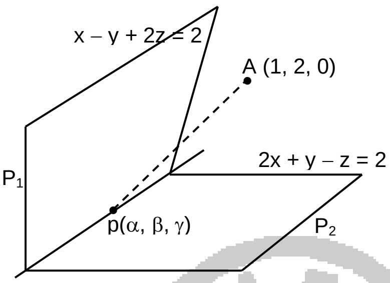


Q.19 Let L be the line of intersection of planes $\vec{r}\cdot(\hat{i}-\hat{j}+2\hat{k})=2$ and $\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})=2$. If P(α, β, γ) is the foot of perpendicular on L from the point (1, 2, 0), then the value of $35(\alpha+\beta+\gamma)$ is equal to :

Options
1. 119
2. 143
3. 134
4. 101

Ans: 119

Sol:



The vector normal to p_2 is $2\hat{i}+\hat{j}-\hat{k}$ $(=\vec{n}_2)$

$\therefore$ DR of L are given by $\vec{n}_1\times\vec{n}_2$

$$\vec{n}_1\times\vec{n}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 2\\ 2 & 1 & -1\end{vmatrix}=-\hat{i}+5\hat{j}+3\hat{k}$$

The vector along L is $\vec{b}=-\hat{i}+5\hat{j}+3\hat{k}$

(α,β,γ) satisfy the equation of the planes

$\therefore \alpha-\beta+2\gamma=2 \quad -(1)$

$2\alpha+\beta-\gamma=2 \quad -(1)$

$\vec{b}.\overrightarrow{Ap}=0$

$\Rightarrow(\alpha-1)(-1)+(\beta-2)(5)+(\gamma)(3)=0$

$\Rightarrow-\alpha+1+5\beta-10+3\gamma=0$

$\Rightarrow-\alpha+5\beta+3\gamma=9 \quad -(3)$

$(1)+(3)\Rightarrow 4\beta+5\gamma=11 \quad -(A)$

$(2)-2\times(1)\Rightarrow 3\beta-5\gamma=-2 \quad -(B)$

$(A)+(B)\Rightarrow 7\beta=9\Rightarrow\beta=\frac{9}{7}$

$(A)\Rightarrow\frac{36}{7}+5\gamma=11\Rightarrow 5\gamma=11-\frac{36}{7}=\frac{41}{7}\Rightarrow\gamma=\frac{41}{35}$

$(1)\Rightarrow\alpha-\frac{9}{7}+\frac{82}{35}=2\Rightarrow\alpha+\frac{-45+82}{35}=2$

$\Rightarrow\alpha+\frac{37}{35}=2\Rightarrow\alpha=2-\frac{37}{35}=\frac{33}{35}$

$\therefore 35(\alpha+\beta+\gamma)=35\left(\frac{33}{35}+\frac{9}{7}+\frac{41}{35}\right)=33+45+41=119$

Q.20 Four dice are thrown simultaneously and the numbers shown on these dice are recorded in 2×2 matrices. The probability that such formed matrices have all different entries and are non-singular, is :

Options

1. $\frac{23}{81}$
2. $\frac{43}{162}$
3. $\frac{45}{162}$
4. $\frac{22}{81}$

Ans: $\frac{43}{162}$

Sol: Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where $a \neq b \neq c \neq d$ and $ad - bc \neq 0$

Ie; $ad \neq bc$

Let us consider the cases in which ad = bc and a≠b≠c≠d

(1 , 6 ; 2 , 3) $\Rightarrow$ 4 Cases.

(2 , 3 ; 1 , 6) $\Rightarrow$ 4 Cases.

(2 , 6 ; 4 , 3) $\Rightarrow$ 4 Cases.

(3 , 4 ; 2 , 6) $\Rightarrow$ 4 Cases.

$\therefore$ **Total = 16 Cases**

a≠b≠c≠d $\Rightarrow$ 6×5×4×3 = 360 Cases

$$\therefore P=\frac{360-16}{6\times6\times6\times6}=\frac{344}{6\times6\times6\times6}=\frac{43}{162}$$

Section B

Q.1 If the constant term, in binomial expansion of $\left(2x^r+\frac{1}{x^2}\right)^{10}$ is 180, then r is equal to _________.

Given Answer : **0.00**

Ans: 8.00

Sol: $(2x^r + x^{-2})^{10}$

$T_{k+1} = {}^{10}C_k (2x^r)^{10-k}.(x^{-2})^k = {}^{10}C_k 2^{10-k}.x^{10r-kr}.x^{-2k}$

$= {}^{10}C_k.2^{10-k}.x^{(10r-kr-2k)}$

For constant term, $10r - kr - 2k = 0$

$\Rightarrow k(r+2) = 10r \Rightarrow k = \frac{10r}{r+2}$ where $k \in \omega, k \le 10$

$180 = 45 \times 2^2 = {}^{10}C_k.2^{10-k}$

Clearly k=8

Q.2 The area (in sq. units) of the region bounded by the curves $x^2+2y-1=0$, $y^2+4x-4=0$ and $y^2-4x-4=0$, in the upper half plane is ________.

Given --
Answer :

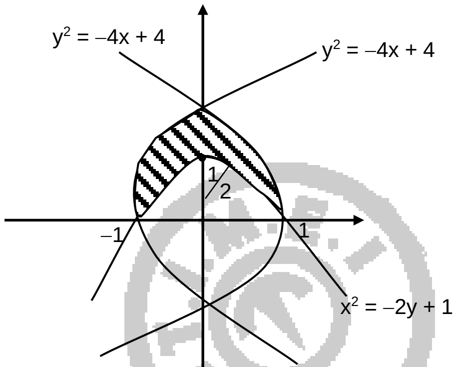
Ans: 2.00

Sol: $x^2 = -2y+1 = -2\left(y-\frac{1}{2}\right) = -4\left(\frac{1}{2}\right)\left(y-\frac{1}{2}\right)$

$y^2 = -4x+4 = -4(x-1) = -4(1)(x-1)$

$y^2 = 4x+4 \Rightarrow 4(1)(x+1)$

$$\text{Area} = 2\times A_2 = 2\int_0^1\left[\sqrt{-4x+4}-\frac{(1-x^2)}{2}\right]$$



$$= 2\int_0^1 2\sqrt{1-x}\,dx - 2\frac{1}{2}\int_0^1(1-x^2)dx$$

$$= 4\left[\frac{(1-x)^{3/2}}{3/2(-1)}\right]_0^1 - \left[x-\frac{x^3}{3}\right]_0^1$$

$$= \frac{8}{3}[1-0] - \left(1-\frac{1}{3}\right)$$

$$= \frac{8}{3}-\frac{2}{3} = \frac{6}{3} = 2$$

Q.3 The number of elements in the set $\{n\in\{1, 2, 3, \ldots\ldots, 100\} \mid (11)^n > (10)^n + (9)^n\}$ is ________.

Given 0.00
Answer :

Ans: 96

Sol: $11^n > 10^n + 9^n$ —(1)

$\Rightarrow \left(\frac{11}{10}\right)^n > 1+\left(\frac{9}{10}\right)^n$

$\Rightarrow (1.1)^n > 1+(0.9)^n$ –(2)

will not be satisfied for n=1,2,3,4.

For n=5,(1) is true.

As n increases from 5,(2) holds good.

$\therefore n \geq 5$

∴ Possible values of n are 5,6,7,8,……,100

⇒ Number of values = 96

Q.4 If the digits are not allowed to repeat in any number formed by using the digits 0, 2, 4, 6, 8, then the number of all numbers greater than 10,000 is equal to ________.

Given 96.00
Answer :

Ans: 96.00

Sol: Given digits : 0,2,4,6,8

□	□	□	□	□
4	4	3	2	1

Answer = $4\times 4 \times 3 \times 2 \times 1$=96

Q.5 Let $y = y(x)$ be the solution of the differential equation

$$\left((x+2)\,e^{\left(\frac{y+1}{x+2}\right)} + (y+1)\right)dx = (x+2)\,dy,\quad y(1) = 1.$$

If the domain of $y = y(x)$ is an open interval (α, β), then $|\alpha + \beta|$ is equal to ________.

Given --
Answer :

Ans: 4

Sol: $\left[(x+2)e^{\left(\frac{y+1}{x+2}\right)} + (y+1)\right]dx = (x+2).dy$

$$\frac{dy}{dx} = e^{\left(\frac{y+1}{x+2}\right)} + \frac{y+1}{x+2}$$

Let y+1=Y and x+2=X

$\Rightarrow dy = dY$ and $x + 2 = X$

$$\Rightarrow \frac{dy}{dx} = \frac{dY}{dX}$$

$$\therefore \frac{dY}{dX} = e^{Y/X} + \frac{Y}{X}$$

$$Y = VX \Rightarrow \frac{dY}{dX} = V + X\frac{dV}{dX}$$

$$V + X\frac{dV}{dX} = e^{V} + V$$

$$\Rightarrow X\frac{dV}{dX} = e^{V}$$

$$\int e^{-V}dV = \int \frac{dX}{X}$$

$$\Rightarrow -e^{-V} = \log X + C$$

$$\Rightarrow -e^{-Y/X} = \log X + C$$

$$\Rightarrow -e^{-\left(\frac{y+1}{x+2}\right)} = \log(x+2) + C$$

$$y(1) = 1 \Rightarrow -e^{-2/3} = \log 3 + C \Rightarrow C = -\log 3 - e^{-2/3}.$$

$$e^{-\left(\frac{y+1}{x+2}\right)} = -\log(x+2) + \log 3 + e^{-2/3} = \log\left(\frac{3}{x+2}\right) + e^{-2/3}$$

$$-\left(\frac{y+1}{x+2}\right) = \log\left[\log\left(\frac{3}{x+2}\right) + e^{-2/3}\right]$$

$$y + 1 = -(x+2).\log\left[\log\left(\frac{3}{x+2}\right) + e^{-2/3}\right]$$

Clearly x>-2 and $|\alpha + \beta| = 4$

Q.6 The sum of all the elements in the set $\{n \in \{1, 2, \ldots\ldots, 100\} \mid$ H.C.F. of n and 2040 is 1$\}$ is equal to __________.

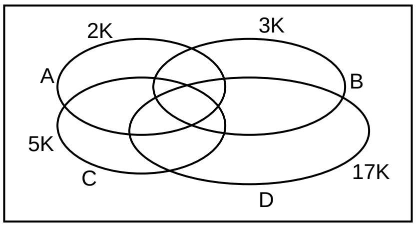
Given **30.00**
Answer :

Ans: 3804

Sol: $2040=2^3 \times 3 \times 5 \times 17$

n does not have factors 2,3,5 and 17

n(1)=100



S(2k)=2+4+6+…….+100=2(1+2+…..+50)= $\frac{2}{2}$ 50(50+1)=2550

S(3k)=3(1+2+…..+33)= $\frac{3}{2} \times 33 \times 34 = 1683$

S(5k)=5(1+2+…..+20)= $\frac{5}{2} \times 20 \times 21 = 1050$

S(17k)=17(1+2+….5)= $\frac{17}{2} \times 5 \times 6 = 255$

S(6k)=6(1+2+…..+16)= $\frac{6}{2} \times 16 \times 17 = 816$ 2,3,5,17.

S(10k)=10(1+2+…..+10)= $\frac{10}{2} \times 10 \times 11 = 550$

S(34k)= $34(1+2) = 102$

S(15k)=15(1+2+…..+6)= $\frac{15}{2} \times 6 \times 7 = 315$

S(51k)=51

S(85k)=85

S(30k)=30(1+2+3)=180

Ans=(2550+1683+1050+255)-(816+550+102+315+51+85)+(180)=3804

Q.7

Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Then the number of 3×3 matrices B with entries from the set {1, 2, 3, 4, 5} and satisfying AB=BA is ________.

Given --
Answer :

Ans: 3125

Sol: B= $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

AB = BA $\Rightarrow$ b=d,a=e,c=f and g=h

Number of ways $=5^5=3125$

Q.8

Let $f: \mathbf{R} \to \mathbf{R}$ be a function defined as $f(x) = \begin{cases} 3\left(1 - \frac{|x|}{2}\right) & \text{if } |x| \le 2 \\ 0 & \text{if } |x| > 2 \end{cases}$

Let $g: \mathbf{R} \to \mathbf{R}$ be given by $g(x) = f(x+2) - f(x-2)$. If n and m denote the number of points in **R** where g is not continuous and not differentiable, respectively, then n + m is equal to ________.

Given --
Answer :

Ans: 4.00

Sol: $f(x)=\begin{cases} 3\left(1+\frac{x}{2}\right) & \text{if } -2\le x<0 \\ 3\left(1-\frac{x}{2}\right) & \text{if } 0\le x\le 2 \\ 0 & \text{if } x<-2 \text{ or } x>2 \end{cases}$

$$f(x+2)=\begin{cases} 3\left(1+\frac{x+2}{2}\right) & \text{if } -2\le x+2<0 \\ 3\left(1-\frac{x+2}{2}\right) & \text{if } 0\le x+2\le 2 \\ 0 & \text{if } x+2<-2 \text{ or } x+2>2 \end{cases}$$

$$\Rightarrow f(x+2)=\begin{cases} 3\left(\frac{x+4}{2}\right) & \text{if } -4\le x<-2 \\ -3\frac{x}{2} & \text{if } -2\le x\le 0 \\ 0 & \text{if } x<-4 \text{ or } x>0 \end{cases}$$

$$f(x-2)=\begin{cases} 3\left(1+\frac{x-2}{2}\right) & \text{if } -2<x-2<0 \\ 3\left(1-\frac{x-2}{2}\right) & \text{if } 0\le x-2\le 2 \\ 0 & \text{if } x-2<-2 \text{ or } x-2>2 \end{cases}$$

$$\Rightarrow f(x-2)=\begin{cases} 3\frac{x}{2} & \text{if } 0<x<2 \\ 3\frac{(4-x)}{2} & \text{if } 2\le x\le 4 \\ 0 & \text{if } x<0 \text{ or } x>4 \end{cases}$$

$$g(x)=\begin{cases} 0 & \text{if } x<-4 \\ \frac{3}{2}(x+4) & \text{if } -4\le x\le -2 \\ -\frac{3}{2}x & \text{if } -2<x<0 \\ -\frac{3}{2}x & \text{if } 0\le x\le 2 \\ -3\frac{(4-x)}{2} & \text{if } 2<x\le 4 \\ 0 & \text{if } x>4 \end{cases}$$

$$\Rightarrow g'(x)=\begin{cases} 0 & \text{if } x<-4 \\ \frac{3}{2} & \text{if } -4\le x\le -2 \\ -\frac{3}{2} & \text{if } -2<x<0 \\ -\frac{3}{2} & \text{if } 0\le x\le 2 \\ \frac{3}{2} & \text{if } 2<x\le 4 \\ 0 & \text{if } x>4 \end{cases}$$

Clearly g(x) is continuous for all $x \in R$

g'(x) is not defined at x = –4, x = –2, x = 2, x = 4

$\therefore$ n + m = 4

Q.9 Consider the following frequency distribution :

Class :	0 - 6	6 - 12	12 - 18	18 - 24	24 - 30
Frequency :	a	b	12	9	5

If mean $= \frac{309}{22}$ and median = 14, then the value $(a-b)^2$ is equal to ________.

Given --
Answer :

Ans: 4.00

Sol:

Class	f_i	x_i	$f_i x_i$
0-6	A	3	3a
6-12	B	9	9b
12-18	12	15	180
18-24	9	21	189
24-30	5	27	135

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{3a+9b+504}{26+a+b} = \frac{309}{22}$$

$$\Rightarrow 8034 + 309a + 309b = 66a + 198b + 11088$$

$$\Rightarrow 243a + 111b = 3054 \Rightarrow 81a + 37b = 1018 \qquad -(1)$$

If b=10 , a=8 (a possible value)
We can see that in this case median =14

(Using the formula $m = l + \frac{\left(\frac{N}{2} - c\right)}{f}.h$)

$$\therefore (a-b)^2 = 4$$

Q.10 Let $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$. Then the number of bijective functions $f : A \to A$ such that $f(1) + f(2) = 3 - f(3)$ is equal to ________.

Given **7.00**
Answer :

Ans: 720

Sol: A={0, 1, 2, 3, 4, 5, 6, 7}
f(1) + f(2) + f(3) = 3
Case 1
f(1) = 0, f(2) = 1, f(3) = 2
Case 2
f(1) = 1,f(2) = 0, f(3) = 2 etc.
(6 Cases)
Case 1
Numbers of bijective function (8 – 3)!=5!
∴ Total= 5! × 6 = 6! = 720