SOLUTIONS & ANSWERS FOR JEE MAINS-2021
25th July Shift 1
[PHYSICS, CHEMISTRY & MATHEMATICS]

PART – A – PHYSICS

Section A

Q.1 A particle of mass 4M at rest disintegrates into two particles of mass M and 3M respectively having non zero velocities. The ratio of de-Broglie wavelength of particle of mass M to that of mass 3M will be :

Options

1. $3:1$
2. $1:3$
3. $1:\sqrt{3}$
4. $1:1$

Ans: 1 : 1

Sol: $\lambda = \frac{h}{P}$

Here the 4 M is at rest so the disintergraded particles should have equal momentum in opposite directions to satisfy conservation of linear momentum.

$\therefore |P_M| = |P_{3M}|$ = same momentum

$\therefore \frac{\lambda_M}{\lambda_{3M}} = 1$

Q.2 The half-life of ^{198}Au is 3 days. If atomic weight of ^{198}Au is 198 g/mol then the activity of 2 mg of ^{198}Au is [in disintegration/second] :

Options

1. 6.06×10^{18}
2. 2.67×10^{12}
3. 32.36×10^{12}
4. 16.18×10^{12}

Ans: 16.18×10^{12}

Sol: Number of nuclei = $\frac{m}{M}\times N_A$

$= \frac{2\times10^{-3}}{198}\times6.02\times10^{23}$

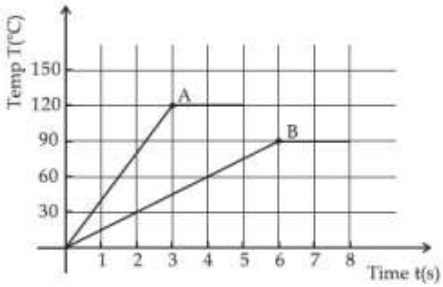
$\lambda = \frac{0.693}{T_{1/2}}$

$T_{1/2} = 3\,\text{days}$

$= 3 \times 24 \times 60 \times 60\text{s}$

$\therefore A_0 = \lambda N_0 = \frac{0.693}{3\times24\times60\times60}\times\frac{2\times10^{-3}}{198}\times6.02\times10^{23} = 16.18 \times 10^{12}$

Q.3 Two different metal bodies A and B of equal mass are heated at a uniform rate under similar conditions. The variation of temperature of the bodies is graphically represented as shown in the figure. The ratio of specific heat capacities is :



Options

1. $\frac{3}{8}$
2. $\frac{4}{3}$
3. $\frac{8}{3}$
4. $\frac{3}{4}$

Ans: $\frac{3}{8}$

Sol: $Q = nc\,\Delta t$

$$\frac{dQ}{dt} = nc\frac{dT}{dt}$$

Given both metals are heated at same rate

$$\therefore \left(\frac{dQ}{dt_B}\right)_A = \left(\frac{dQ}{dt}\right)_B$$

$$\Rightarrow n_A c_A \left(\frac{dT}{dt}\right)_A = n_A c_B \left(\frac{dT}{dt}\right)_B$$

Given same mass

$\therefore n_A = n_B$

$$\therefore \frac{c_A}{c_B} = \frac{\left(\frac{dT}{dt}\right)_B}{\left(\frac{dT}{dt}\right)_A} = \frac{\frac{90}{6}}{\frac{120}{3}} = \frac{90\times 3}{120\times 6}$$

$$\therefore \frac{c_A}{c_B} = \frac{3}{8}$$

Q.4 Some nuclei of a radioactive material are undergoing radioactive decay. The time gap between the instances when a quarter of the nuclei have decayed and when half of the nuclei have decayed is given as :
(where λ is the decay constant)

Options

1. $\frac{ln2}{\lambda}$
2. $\frac{ln\frac{3}{2}}{\lambda}$
3. $\frac{2ln2}{\lambda}$
4. $\frac{1}{2}\ \frac{ln2}{\lambda}$

Ans: $\frac{\ell n\frac{3}{2}}{\lambda}$

Sol: $N = N_0 e^{-\lambda t}$

$N_1 = N_0\ e^{-\lambda t_1}$ $N_1 = \frac{3N_0}{4}$

$\frac{3N_0}{4} = N_0 e^{-\lambda t_1}$

$t_1 = \frac{\ell n\left(4/3\right)}{\lambda}$ ----(1)

$N_2 = N_0\ e^{-\lambda t_2}$ $N_2 = \frac{N_0}{2}$

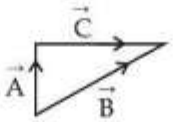
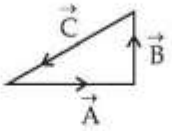
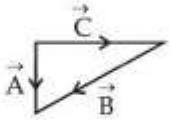
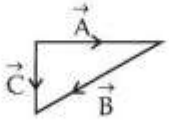
$\frac{N_0}{2} = N_0 e^{-\lambda t_2}$

$t_2 = \frac{\ell n2}{\lambda}$ -----(2)

$t_2 - t_1 = \frac{\ell n2}{\lambda} - \frac{\ell n\left(\frac{4}{3}\right)}{\lambda}$ $\ell n\ A - \ell n\ B =$

$\frac{\ell n\left(\frac{3}{2}\right)}{\lambda}$

Q.5 Match **List I** with **List II**.

List I	List II
(a) $\vec{C} - \vec{A} - \vec{B} = 0$	(i) 
(b) $\vec{A} - \vec{C} - \vec{B} = 0$	(ii) 
(c) $\vec{B} - \vec{A} - \vec{C} = 0$	(iii) 
(d) $\vec{A} + \vec{B} = -\vec{C}$	(iv) 

Choose the correct answer from the options given below :

Options

1. (a) → (iv), (b) → (i), (c) → (iii), (d) → (ii)
2. (a) → (i), (b) → (iv), (c) → (ii), (d) → (iii)
3. (a) → (iii), (b) → (ii), (c) → (iv), (d) → (i)
4. (a) → (iv), (b) → (iii), (c) → (i), (d) → (ii)

Ans: (a) → (iv), (b) → (iii), (c) → (i), (d) → (ii)

Sol: (a) → (iv), (b) → (iii), (c) → (i), (d) → (ii)

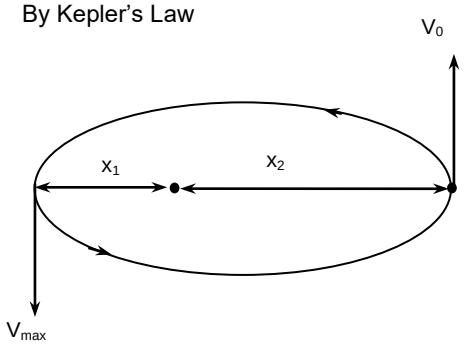
Q.6 The minimum and maximum distances of a planet revolving around the Sun are x_1 and x_2. If the minimum speed of the planet on its trajectory is v_0 then its maximum speed will be :

Options

1. $\frac{v_0 x_1^2}{x_2^2}$
2. $\frac{v_0 x_1}{x_2}$
3. $\frac{v_0 x_2}{x_1}$
4. $\frac{v_0 x_2^2}{x_1^2}$

Ans: $\frac{V_0 x_2}{x_1}$

Sol: By Kepler's Law



By conservation of angular momentum

$mV_{max}\, x_1 = mV_0 x_2$

$\therefore V_{max}\, x_1 = V_0 x_2$

$\therefore V_{max} = \frac{V_0 x_2}{x_1}$

Q.7 For a gas $C_P - C_V = R$ in a state P and $C_P - C_V = 1.10\ R$ in a state Q, T_P and T_Q are the temperatures in two different states P and Q respectively. Then

Options

1. $T_P = T_Q$
2. $T_P = 0.9\ T_Q$
3. $T_P < T_Q$
4. $T_P > T_Q$

Ans: $T_P > T_Q$

Sol: $T_P > T_Q$

Q.8 A linearly polarized electromagnetic wave in vacuum is

$$E = 3.1 \cos\left[(1.8)z - \left(5.4\times10^6\right)t\right]\hat{i}\ N/C$$

is incident normally on a perfectly reflecting wall at $z = a$. Choose the correct option

Options 1.

The reflected wave will be $3.1 \cos\left[(1.8)z + \left(5.4 \times 10^6\right)t\right]\hat{i}\ N/C$

2. The wavelength is 5.4 m

3.

The frequency of electromagnetic wave is 54×10^4 Hz.

4.

The transmitted wave will be $3.1 \cos\left[(1.8)z - \left(5.4 \times 10^6\right)t\right]\hat{i}\ N/C$

Ans: The reflected wave will be $3.1\cos\left[(1.8)z + (5.4\times10^6)t\right]\hat{i}\,N/C$

Sol: $w = 2\pi f = 5.4 \times 10^6$

$f = \frac{5.4\times10^6}{2\pi} = 8.59 \times 10^5$ Hz

$\therefore \lambda = \frac{c}{f} = \frac{3\times10^8}{8.59\times10^5} = 349.2\text{m}$

$\therefore$ Reflected wave 3.1 cos [1.8 z + 5.4 × 10^6 t] $\hat{i}$

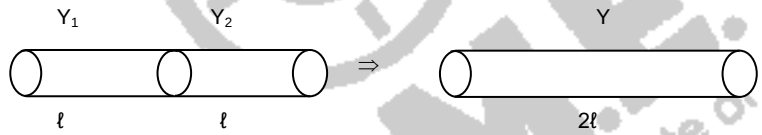
Q.9 Two wires of same length and radius are joined end to end and loaded. The Young's modulii of the materials of the two wires are Y_1 and Y_2. The combination behaves as a single wire then its Young's modulus is :

Options

1. $Y = \frac{2Y_1Y_2}{3(Y_1 + Y_2)}$
2. $Y = \frac{2Y_1Y_2}{Y_1 + Y_2}$
3. $Y = \frac{Y_1Y_2}{2(Y_1 + Y_2)}$
4. $Y = \frac{Y_1Y_2}{Y_1 + Y_2}$

Ans: $Y = \frac{2Y_1Y_2}{Y_1+Y_2}$

Sol:



$\therefore K = \frac{K_1K_2}{K_1+K_2}$

$\frac{YA}{2\ell} = \frac{\frac{Y_1A}{\ell}\times\frac{Y_2A}{\ell}}{\frac{Y_1A}{\ell}+\frac{Y_2A}{\ell}}$

$\therefore Y = \frac{2Y_1Y_2}{Y_1+Y_2}$

Q.10 In Amplitude Modulation, the message signal

$V_m(t) = 10\ \sin(2\pi\times10^5 t)$volts and

Carrier signal

$V_c(t) = 20\ \sin(2\pi\times10^7 t)$volts

The modulated signal now contains the message signal with lower side band and upper side band frequency, therefore the bandwidth of modulated signal is α kHz.

The value of α is :

Options

1. 0
2. 50 kHz
3. 200 kHz
4. 100 kHz

Ans: 200 kHz

Sol: Band with = 2 f_m = 2 ×10^5 Hyz = 200 kHz

Q.11 Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : Moment of inertia of a circular disc of mass 'M' and radius 'R' about X, Y axes (passing through its plane) and Z-axis which is perpendicular to its plane were found to be I_x, I_y & I_z respectively. The respective radii of gyration about all the three axes will be the same.

Reason R : A rigid body making rotational motion has fixed mass and shape.

In the light of the above statements, choose the most appropriate answer from the options given below :

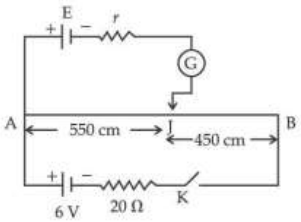
Options

1. Both **A** and **R** are correct and **R** is the correct explanation of **A**.
2. **A** is not correct but **R** is correct.
3. **A** is correct but **R** is not correct.
4. Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**.

Ans: **A** is not correct but **R** is correct

Sol: A is not correct but R is correct

Q.12 In the given figure, there is a circuit of potentiometer of length AB = 10 m. The resistance per unit length is 0.1 Ω per cm. Across AB, a battery of emf E and internal resistance 'r' is connected. The maximum value of emf measured by this potentiometer is :



Options

1. 6 V
2. 5 V
3. 2.25 V
4. 2.75 V

Ans: 5 V

Sol: $R_{AB} = 10 \times 0.1 \times 100 = 100\ \Omega$

$$\therefore V_{AB} = \frac{6}{20+100} \times 100 = 5V$$

Q.13 In the Young's double slit experiment, the distance between the slits varies in time as $d(t) = d_0 + a_0 \sin\omega t$; where d_0, ω and a_0 are constants. The difference between the largest fringe width and the smallest fringe width obtained over time is given as :

Options

1. $\dfrac{2\lambda D a_0}{\left(d_0^2 - a_0^2\right)}$
2. $\dfrac{2\lambda D (d_0)}{\left(d_0^2 - a_0^2\right)}$
3. $\dfrac{\lambda D}{d_0^2}\ a_0$
4. $\dfrac{\lambda D}{d_0 + a_0}$

Ans: $\frac{2\lambda D a_0}{\left(d_0^2 - a_0^2\right)}$

Sol: Fringe with $\beta = \frac{D\lambda}{d}$

β_{min} is when $\sin \omega t = +1$

β_{max} is when $\sin \omega t = -1$

$$\beta = \frac{D\lambda}{d_0 + a_0 \sin\omega}$$

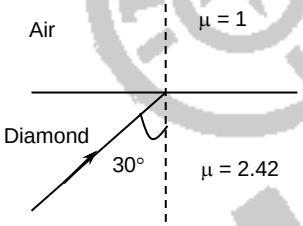
$$\therefore \beta_{max} - \beta_{min} = \frac{D\lambda}{d_0 - a_0} - \frac{D\lambda}{d_0 + a_0} = D\lambda\left[\frac{d_0 + a_0 - d_0 + a_0}{(d_0 - a_0)(d_0 + a_0)}\right] = \frac{2\lambda D a_0}{d_0^2 - a_0^2}$$

Q.14 A ray of laser of a wavelength 630 nm is incident at an angle of 30° at the diamond-air interface. It is going from diamond to air. The refractive index of diamond is 2.42 and that of air is 1. Choose the correct option.

Options
1. angle of refraction is 53.4°
2. angle of refraction is 30°
3. refraction is not possible
4. angle of refraction is 24.41°

Ans: refraction is not possible

Sol:



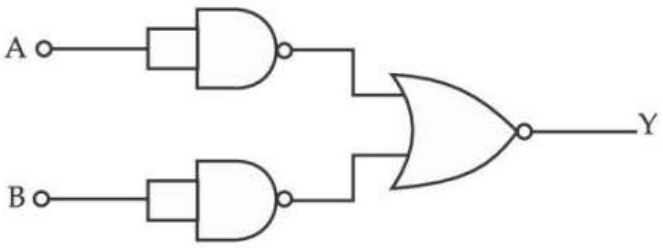
$\therefore$ Critical angle $c = \sin^{-1}\left(\frac{1}{2.42}\right) = 24.4°$

$\therefore$ Incident angle > 24.4°

$\therefore$ Ray undergo total internal reflection at the surface

$\therefore$ option (3) refraction is not possible

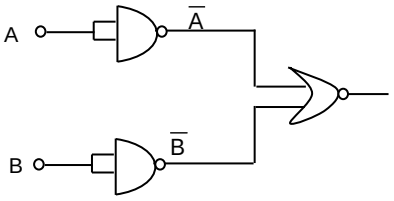
Q.15 Identify the logic operation carried out.



Options
1. AND
2. OR
3. NOR
4. NAND

Ans: AND

Sol:

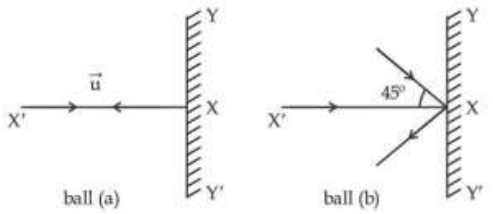


$Y = \overline{\overline{A} + \overline{B}}$

$= A \bullet B$

$\therefore$ option (1) AND

Q.16 Two billiard balls of equal mass 30 g strike a rigid wall with same speed of 108 kmph (as shown) but at different angles. If the balls get reflected with the same speed then the ratio of the magnitude of impulses imparted to ball 'a' and ball 'b' by the wall along 'X' direction is :



Options

1. $1 : \sqrt{2}$
2. $1 : 1$
3. $2 : 1$
4. $\sqrt{2} : 1$

Ans: $\sqrt{2} : 1$

Sol: Ratio of impulse

$$\frac{I_1}{I_2} = \frac{\Delta P_1}{\Delta P_2} = \frac{m(u+u)}{m\left(\frac{u}{\sqrt{2}} + \frac{u}{\sqrt{2}}\right)} = \sqrt{2} : 1$$

Q.17 Water droplets are coming from an open tap at a particular rate. The spacing between a droplet observed at 4^{th} second after its fall to the next droplet is 34.3 m. At what rate the droplets are coming from the tap ? (Take $g = 9.8\ m/s^2$)

Options

1. 1 drop/7 seconds
2. 3 drops/2 seconds
3. 2 drops/second
4. 1 drop/second

Ans: 1 drop / second

Sol: Distance travelled by 1^{st} drop in 4 second is $ut + \frac{1}{2}at^2$

$\therefore S_1 = \frac{1}{2}at^2 = \frac{1}{2} \times 9.8 \times 4^2 = 78.4$ m

Distance travelled by succeeding drop in (4 – t) second is

$S_2 = \frac{1}{2} \times 9.8\ (4 - t)^2 = 4.9\ (4 - t)^2$

$\therefore\ S_1 - S_2 = 34.3$

$78.4 - 4.9\ (4 - t)^2 = 34.3$

$(4 - t)^2 = 9 \Rightarrow 4 - t = 3$
$t = 1$ s

Q.18 A parallel plate capacitor with plate area 'A' and distance of separation 'd' is filled with a dielectric. What is the capacity of the capacitor when permittivity of the dielectric varies as :

$\varepsilon(x) = \varepsilon_0 + kx$, for $\left(0 < x \le \frac{d}{2}\right)$

$\varepsilon(x) = \varepsilon_0 + k(d - x)$, for $\left(\frac{d}{2} \le x \le d\right)$

Options

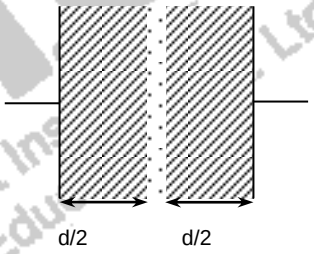
1. $\frac{kA}{2} \ln\left(\frac{2\varepsilon_0}{2\varepsilon_0 - kd}\right)$
2. 0
3. $\left(\varepsilon_0 + \frac{kd}{2}\right)^{2/kA}$
4. $\frac{kA}{2\ln\left(\frac{2\varepsilon_0 + kd}{2\varepsilon_0}\right)}$

Ans: $\frac{kA}{2\ell n\left(\frac{2\varepsilon_0 + kd}{2\varepsilon_0}\right)}$

Sol: $\frac{1}{C_{eq}} = \int \frac{dx}{A\varepsilon}$

$= \int_0^{d/2} \frac{dx}{A(\varepsilon_0 + Kx)} + \int \frac{dx}{A(\varepsilon_0 + K(d - x)}$

$= \frac{1}{A}\left\{\frac{1}{K}\left[\ell n(\varepsilon_0 + Kx)\right]_0^{d/2} + \frac{1}{K}\left[\varepsilon_0 + K(d - x)\right]_{d/2}^{d}\right\}$



$= \frac{1}{AK} \times 2\ell n \frac{\varepsilon_0 + K\,d/2}{\varepsilon_0}$

$\therefore C_{eq} = \frac{KA}{2\ell n\left(\frac{2\varepsilon_0 + Kd}{2\varepsilon_0}\right)}$

Q.19 What should be the order of arrangement of de-Broglie wavelength of electron (λ_e), an α-particle (λ_α) and proton (λ_p) given that all have the same kinetic energy ?

Options

1. $\lambda_e = \lambda_p > \lambda_\alpha$
2. $\lambda_e = \lambda_p = \lambda_\alpha$
3. $\lambda_e > \lambda_p > \lambda_\alpha$
4. $\lambda_e < \lambda_p < \lambda_\alpha$

Ans: $\lambda_e > \lambda_p > \lambda_\alpha$

Sol: $\lambda = \frac{h}{\sqrt{2mKE}} \lambda\alpha \frac{1}{\sqrt{m}}$

$\therefore m_\alpha > m_p > m_e$

$\therefore \lambda_e > \lambda_p > \lambda_\alpha$

Q.20 A monoatomic ideal gas, initially at temperature T_1 is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If l_1 and l_2 are the lengths of the gas column, before and after the expansion respectively, then the value of $\frac{T_1}{T_2}$ will be :

Options

1. $\left(\frac{l_1}{l_2}\right)^{\frac{2}{3}}$
2. $\left(\frac{l_2}{l_1}\right)^{\frac{2}{3}}$
3. $\frac{l_1}{l_2}$
4. $\frac{l_2}{l_1}$

Ans: $\left(\frac{\ell_2}{\ell_1}\right)^{\frac{2}{3}}$

Sol: For adiabatic process,

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{V_2}{V_1}\right)^{\nu-1}$$

$$\Rightarrow \left(\frac{V_2}{V_1}\right)^{\frac{5}{3}-1} = \left(\frac{\Delta\ell_2}{\Delta\ell_1}\right)^{\frac{2}{3}}$$

$$= \left(\frac{\ell_2}{\ell_1}\right)^{\frac{2}{3}}$$

Section B

Q.1 An inductor of 10 mH is connected to a 20 V battery through a resistor of 10 kΩ and a switch. After a long time, when maximum current is set up in the circuit, the current is switched off. The current in the circuit after 1 μs is $\frac{x}{100}$ mA. Then x is equal to _____.
(Take $e^{-1} = 0.37$)

Given --
Answer :

Ans: 74

Sol: $i_0 = \frac{20}{10} = 2A$

$i = i_0 e^{-Rt/L} = 2 \times e^{-\frac{10\times10^{-3}}{10\times10^{-3}}} = \frac{2}{e}$

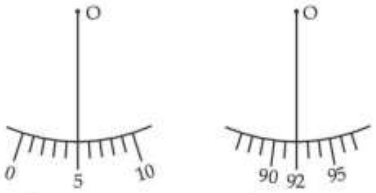
$\therefore \frac{x}{100} = \frac{2}{e} = 0.74$

$\therefore x = 74$

Q.2 Student A and Student B used two screw gauges of equal pitch and 100 equal circular divisions to measure the radius of a given wire. The actual value of the radius of the wire is 0.322 cm. The absolute value of the difference between the final circular scale readings observed by the students A and B is _____.

[Figure shows position of reference 'O' when jaws of screw gauge are closed]

Given pitch = 0.1 cm.



Screw gauge (A) Screw gauge (B)

Given --
Answer :

Ans: 13

Sol: For A Error

Reading = PSR + HSR + 5 x LC
0.322 = 0.300 + HSR + 0.005
CSR = 0.017
For B
Reading = PSR + HSR + Error
0.322 = 0.200 + HSR + 0.092
CSR = 0.030
Difference = 0.030 – 0.017 = 0.013 cm

In circular scale = $\frac{0.013}{0.001} = 13$

Q.3 A pendulum bob has a speed of 3 m/s at its lowest position. The pendulum is 50 cm long. The speed of bob, when the length makes an angle of 60° to the vertical will be ($g = 10\ m/s^2$) _____ m/s.

Given **0**
Answer :

Ans: 2

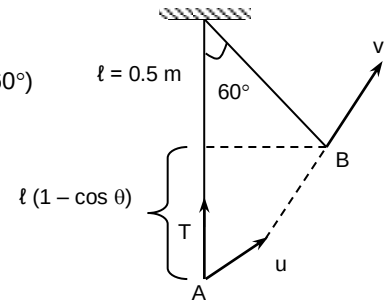
Sol: $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg\ell(1 - \cos 60°)$

$u^2 = v^2 + 2g\ell(1 - ½)$

$9 = v^2 + 20 \times 0.5 \times \frac{1}{2}$

$9 = v^2 + 5$

$v = 2$ m/s



Q.4 A particle of mass 'm' is moving in time 't' on a trajectory given by

$$\vec{r} = 10\,\alpha t^2\,\hat{i} + 5\beta(t-5)\,\hat{j}$$

Where α and β are dimensional constants.

The angular momentum of the particle becomes the same as it was for $t=0$ at time $t=$_____ seconds.

Given --
Answer :

Ans: 10

Sol: $\vec{r} = 10\alpha t^2\hat{i} + 5\beta(t-5)\hat{j}$

$\vec{v} = \dfrac{dr}{dt} = 20\alpha t\hat{i} + 5\beta\hat{j}$

$\vec{L} = m(\vec{r}\times\vec{v})$= m ([10$\alpha$r^2 $\hat{i}$ + 5β (t – 5) $\hat{j}$] ×[20αt $\hat{i}$ + 5β $\hat{j}$])

= m (t^2 50 $\alpha\beta$ $\hat{k}$ - 100 $\alpha\beta$t (t – 5) $\hat{k}$)

At t = 0 $\vec{L} = 0$

$\therefore$ 0 = t 50 $\alpha\beta\,\hat{k}$ (t – 2 (t – 5))

$\therefore$ t – 2t + 10 = 0

t = 10 sec

Q.5 A body of mass 2 kg moving with a speed of 4 m/s. makes an elastic collision with another body at rest and continues to move in the original direction but with one fourth of its initial speed. The speed of the two body centre of mass is $\frac{x}{10}$ m/s. Then the value of x is _____.

Given --
Answer :

Ans: 25

Sol: From conservation of linear momentum

2 × 4 + 0 = 2 ×1 + m_2V_2 ------(1)

By elastic collision

$V_2 - V_1 = e\,(U_1 - U_2)$

$V_2 - 1 = 1\,(4-0)$

$V_2 = 5$

$\therefore$ eq (1) $\Rightarrow$ 8 = 2 + m_2 ×5

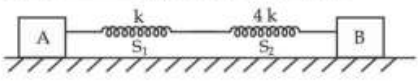
$\therefore m_2 = \dfrac{6}{5}$

$$V_{CH} = \frac{m_1V_1 + m_2V_2}{m_1 + m_2} = \frac{2\times4+0}{2+\frac{6}{5}} = 2.5\,m/s$$

$$\therefore \frac{x}{10} = 2.5\,m/s$$

x = 25

Q.6 In the reported figure, two bodies A and B of masses 200 g and 800 g are attached with the system of springs. Springs are kept in a stretched position with some extension when the system is released. The horizontal surface is assumed to be frictionless. The angular frequency will be _____ rad/s when k = 20 N/m.



Given **10**
Answer :

Ans: 10

Sol: Reduced mass $\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{200 \times 800}{200 + 800} = 0.16\,Kg$

$$K_{eq} = \frac{K_1 K_2}{K_1 + K_2} = \frac{4K \times K}{4K + K} = \frac{4}{5}K = \frac{4}{5} \times 20 = 16\,N/m$$

$$\therefore w = \sqrt{\frac{K_{eq}}{\mu}} = \sqrt{\frac{16}{0.16}} = 10\,rad/s$$

Q.7 The value of aluminium susceptibility is 2.2×10^{-5}. The percentage increase in the magnetic field if space within a current carrying toroid is filled with Aluminium is $\frac{x}{10^4}$. Then the value of x is _____.

Given --
Answer :

Ans: 22

Sol: $B = m\,(H + I) = mH\left(1 + \frac{I}{H}\right) = mH\,(1 + x) = B_0(1 + x)$

$\therefore B - B_0 = B_0 x$

$$\frac{B - B_0}{B_0} = x$$

$$\frac{B - B_0}{B_0} \times 100 = 100x$$

$$2.2 \times 10^{-3} = \frac{22}{10^4}$$

$\therefore x = 22$

Q.8 A circular conducting coil of radius 1 m is being heated by the change of magnetic field $\vec{B}$ passing perpendicular to the plane in which the coil is laid. The resistance of the coil is 2 μΩ. The magnetic field is slowly switched off such that its magnitude changes in time as

$$B = \frac{4}{\pi} \times 10^{-3} T \left(1 - \frac{t}{100}\right)$$

The energy dissipated by the coil before the magnetic field is switched off completely is E = _____ mJ.

Given --
Answer :

Ans: 80

Sol: $\varepsilon = \left| -A\frac{dB}{dt} \right|$ $\qquad B = \frac{4}{\pi} \times 10^{-3}\left(1 - \frac{t}{100}\right)$

$= \pi \times 1^2 \times \frac{4}{\pi} \times \frac{10^{-3}}{100}$ $\qquad \therefore \frac{dB}{dt} = \frac{4}{\pi} \times \frac{10^{-3}}{100} = 4 \times 10^{-5}$ V

When B = 0 t = 100

Energy $E = \frac{\varepsilon^2}{R} \times t = \frac{16 \times 10^{-10}}{2 \times 10^{-6}} \times 100 = \frac{16 \times 10^{-4}}{2} \times 100 = 8 \times 10^{-2}$ J = 80 mJ

Q.9 An electric bulb rated as 200 W at 100 V is used in a circuit having 200 V supply. The resistance 'R' that must be put in series with the bulb so that the bulb delivers the same power is _____ Ω.

Given --
Answer :

Ans: 50

Sol: Resistance of bulb

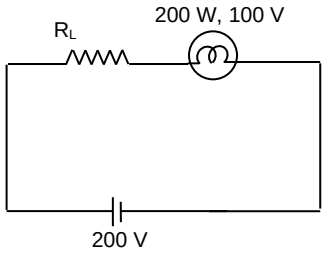
$$R_B = \frac{V^2}{P} = \frac{(100)^2}{200} = 50\Omega$$

$\therefore$ Current through bulb

$$i = \frac{V}{R_B} = \frac{100}{50} = 2A$$

$\therefore V_L = 100$

$$R_L = \frac{100}{2} = 50\Omega$$

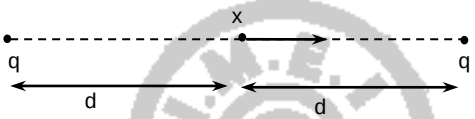


Q.10 A particle of mass 1 mg and charge q is lying at the mid-point of two stationary particles kept at a distance '2 m' when each is carrying same charge 'q'. If the free charged particle is displaced from its equilibrium position through distance 'x' (x << 1 m). The particle executes SHM. Its angular frequency of oscillation will be _____ $\times 10^5$ rad/s if $q^2 = 10$ C^2.

Given --
Answer :

Ans: 6000.00

Sol:



Net force on free charged particle

$$F = \frac{Kq^2}{(d+x)^2} - \frac{Kq^2}{(d-x)^2}$$

$$ma = -Kq^2\left[\frac{4dx}{(d^2 - x^2)^2}\right]$$

$$a = -\frac{4Kq^2 d}{m}\left(\frac{x}{d^4}\right)$$

if $d >> x$

$d^2 - x^2 \approx d^2$

$$a = -\left(\frac{4Kq^2}{md^3}\right)x$$

$$\therefore w = \sqrt{\frac{4Kq^2}{md^3}}$$

i.e., $a = -w^2x = \sqrt{\frac{4\times 9\times 10^9 \times 10}{1\times 10^{-6}\times 1^3}} = 6\times 10^8 \text{rad/sec}$

PART – B – CHEMISTRY

Section A

Q.1 At 298.2 K the relationship between enthalpy of bond dissociation (in kJ mol^{-1}) for hydrogen (E_H) and its isotope, deuterium (E_D), is best described by :

Options

1. $E_H \simeq E_D - 7.5$
2. $E_H = E_D$
3. $E_H = 2E_D$
4. $E_H = \frac{1}{2}E_D$

Ans: $E_H \cong E_D - 7.5$

Sol: Bond dissociation enthalpy of hydrogen is (in kJ mol^{-1}) is 435.88 kJ mol^{-1} and for Deuterium is 443.35 kJ mol^{-1}
$\therefore E_H \cong E_D - 7.5$

Q.2 Which one of the following species responds to an external magnetic field ?

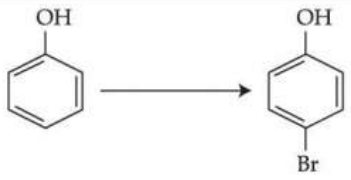
Options

1. $[Ni(CO)_4]$
2. $[Co(CN)_6]^{3-}$
3. $[Fe(H_2O)_6]^{3+}$
4. $[Ni(CN)_4]^{2-}$

Ans: $[Fe(H_2O)_6]^{3+}$

Sol: $[Fe(H_2O)_6]^{3+}$
Fe^{3+} $[Ar]3d^5$
High spin paramagnetic complex with five unpaired electron. So it is responds to an external magnetic field.

Q.3



(Major Product)

The given reaction can occur in the presence of :

(a) Bromine water
(b) Br_2 in CS_2, 273 K
(c) $Br_2/FeBr_3$
(d) Br_2 in $CHCl_3$, 273 K

Choose the **correct** answer from the options given below :

Options

1. (b) and (d) only
2. (a) and (c) only
3. (a), (b) and (d) only
4. (b), (c) and (d) only

Ans: (b), (c) and (d) only

Sol: Except bromine water, all other gives p-Bromophenol as the major product and with bromine water, its gives tribromo substituted product.

Q.4 Sodium stearate $CH_3(CH_2)_{16}COO^-Na^+$ is an anionic surfactant which forms micelles in oil. Choose the **correct** statement for it from the following :

Options 1.
It forms non-spherical micelles with $CH_3(CH_2)_{16}-$ group pointing towards the centre.

2.
It forms spherical micelles with $CH_3(CH_2)_{16}-$ group pointing outwards on the surface of sphere.

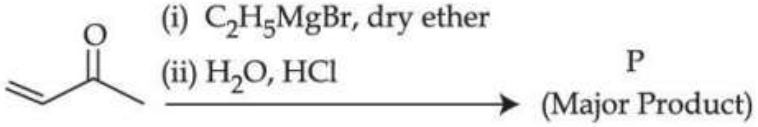
3.
It forms spherical micelles with $CH_3(CH_2)_{16}-$ group pointing towards the centre of sphere.

4.
It forms non-spherical micelles with $-COO^{\ominus}$ group pointing outwards on the surface.

Ans: It forms spherical micelles with $CH_3(CH_2)_{16}-$ group pointing towards the centre of sphere

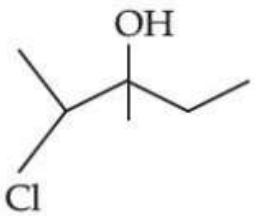
Sol: sodium stearate forms spherical micelles with hydrocarbon part pointing towards the centre of sphere

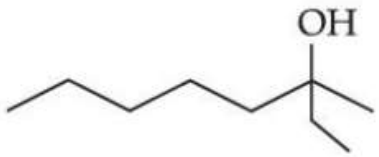
Q.5

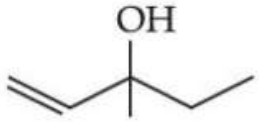


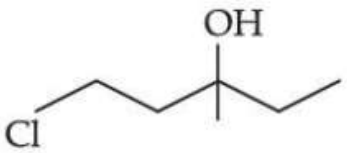
Consider the above reaction, the major product 'P' is :

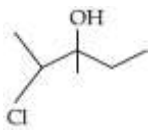
Options

1.


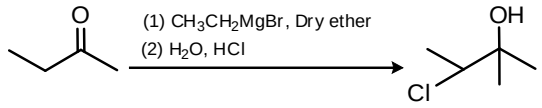
2.


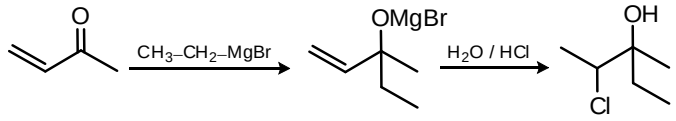
3.


4.


Ans:


Sol:





Q.6 The correct order of following 3d metal oxides, according to their oxidation numbers is :

(a) CrO_3
(b) Fe_2O_3
(c) MnO_2
(d) V_2O_5
(e) Cu_2O

Options

1. (a) > (c) > (d) > (b) > (e)
2. (a) > (d) > (c) > (b) > (e)
3. (d) > (a) > (b) > (c) > (e)
4. (c) > (a) > (d) > (e) > (b)

Ans: (a) > (d) > (c) > (b) > (e)

Sol: $\underset{+6}{\underline{Cr}O_3}$ $\underset{+3}{Fe_2O_3}$ $\underset{+4}{\underline{Mn}O_2}$ $\underset{+5}{V_2O_5}$ $\underset{+1}{Cu_2O}$

$CrO_3 > V_2O_5 > MnO_2 > Fe_2O_3 > Cu_2O$

a > d > c > b > e

Q.7 The ionic radii of K^+, Na^+, Al^{3+} and Mg^{2+} are in the order :

Options

1. $Al^{3+} < Mg^{2+} < Na^+ < K^+$
2. $Na^+ < K^+ < Mg^{2+} < Al^{3+}$
3. $K^+ < Al^{3+} < Mg^{2+} < Na^+$
4. $Al^{3+} < Mg^{2+} < K^+ < Na^+$

Ans: $Al^{3+} < Mg^{2+} < Na^+ < K^+$

Sol: For isoelectronic species such as Al^{3+}, Mg^{2+}, Na^+ as positive charge increases, size of the ion decreases. And in the case of K^+ and Na^+ the order of is $Na^+ < K^+$

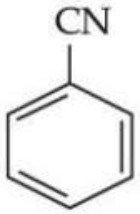
$\therefore Al^{3+} < Mg^{2+} < Na^+ < K^+$

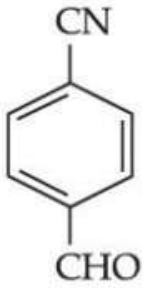
Fraction of total ion present as $Fe^{3+} = \dfrac{[Fe^{3+}]}{[Fe^{2+}]+[Fe^{3+}]}$

$= \dfrac{1}{1+\sqrt{10}} = \dfrac{1}{4.16} = 24\times10^{-2}$

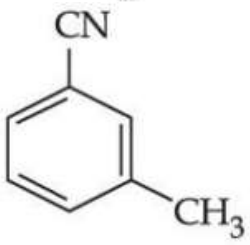
Q.8 Which one of the products of the following reactions **does not** react with Hinsberg reagent to form sulphonamide ?

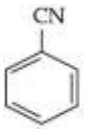
Options

1.  $+ SnCl_2 + HCl \longrightarrow$

2.  $+ LiAlH_4 \xrightarrow{H_3O^{\oplus}}$

3.  $+ Na/Hg \xrightarrow{C_2H_5OH}$

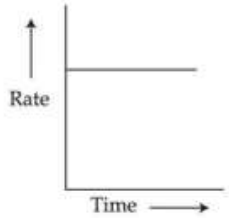
4.  $+ H_2/Ni \longrightarrow$

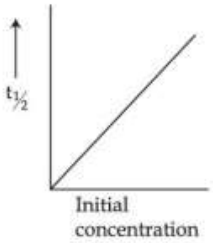
Ans:  $+ SnCl_2 + HCl \longrightarrow$

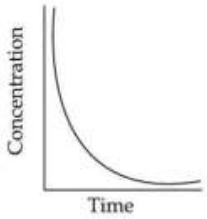
Sol: Benzonitrile when reacts with $SnCl_2$ / HCl form bezaldehyde. –CHO group does not reacts with Hinsberg's reagent

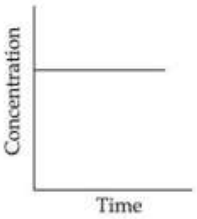
Hinsberg's reagent used to distinguish 1°, 2° and 3°

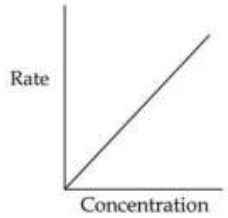
Q.9 For the following graphs,

(a) 

(b) 

(c) 

(d) 

(e) 

Choose from the options given below, the **correct** one regarding order of reaction is :

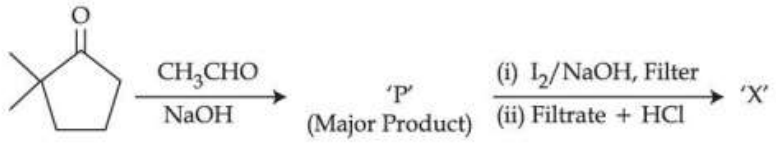
Options

1. (a) and (b) Zero order
   (c) and (e) First order
2. (b) Zero order
   (c) and (e) First order
3. (a) and (b) Zero order
   (e) First order
4. (b) and (d) Zero order
   (e) First order

Ans: (b) zero order
(c) and (e) first order

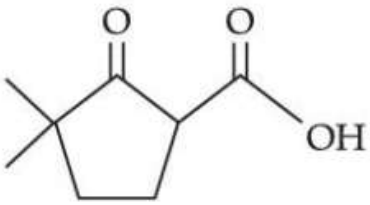
Sol: a, b – zero order reaction
e – first order reaction

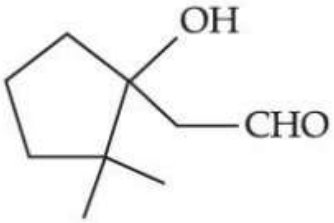
Q.10

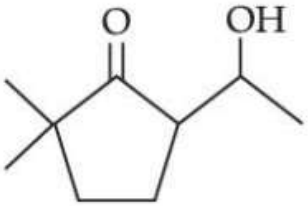


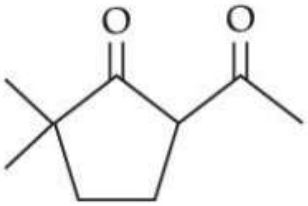
Consider the given reaction, the product 'X' is :

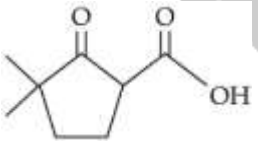
Options

1. 

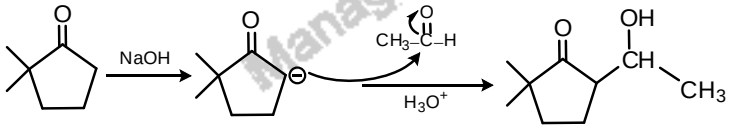
2. 

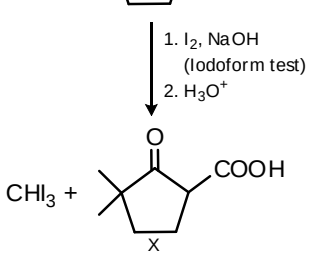
3. 

4. 

Ans: 

Sol:





Q.11 Given below are two statements, one is labelled as **Assertion (A)** and other is labelled as **Reason (R)**.

Assertion (A) : Gabriel phthalimide synthesis cannot be used to prepare aromatic primary amines.

Reason (R) : Aryl halides do not undergo nucleophilic substitution reaction.

In the light of the above statements, choose the **correct** answer from the options given below :

Options

1. Both **(A)** and **(R)** are true and **(R)** is correct explanation of **(A)**.
2. **(A)** is true but **(R)** is false.
3. **(A)** is false but **(R)** is true.
4. Both **(A)** and **(R)** are true but **(R)** is not the correct explanation of **(A)**.

Ans: Both **(A)** and **(R)** are correct and **(R)** is correct explanation of **(A)**

Sol: Both (A) and (R) are correct and (R) is correct explanation of (A)
Aromatic 1° amine such as aniline cannot be prepared by Gabriel's phthalimide synthesis because aryl halide not easily undergo nucleophilic substitution with phthalimide ion.

Q.12 The water soluble protein is :

Options

1. Collagen
2. Fibrin
3. Albumin
4. Myosin

Ans: Albumin

Sol: Albumin is a water soluble protein

Q.13 In the leaching of alumina from bauxite, the ore expected to leach out in the process by reacting with NaOH is :

Options

1. TiO_2
2. Fe_2O_3
3. ZnO
4. SiO_2

Ans: SiO_2

Sol: SiO_2 is acidic oxide
$2NaOH + SiO_2 \rightarrow Na_2SiO_3 + H_2O$

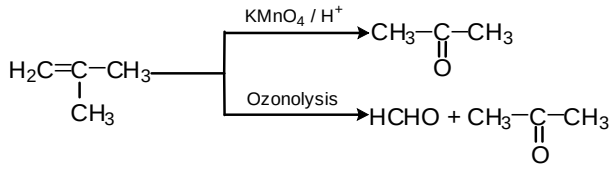
Q.14 An Organic compound 'A' C_4H_8 on treatment with $KMnO_4/H^+$ yields compound 'B' C_3H_6O. Compound 'A' also yields compound 'B' an ozonolysis. Compound 'A' is :

Options

1. But-2-ene
2. 1-Methylcyclopropane
3. 2-Methylpropene
4. Cyclobutane

Ans: 2-Methyl propane

Sol:



Both on ozonolysis and treatment with acidified $KMnO_4$, 2-methylpropene is converted into the same product acetone

Q.15 Given below are two statements :

Statement I : None of the alkaline earth metal hydroxides dissolve in alkali.

Statement II : Solubility of alkaline earth metal hydroxides in water increases down the group.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

Options

1. **Statement I** is correct but **Statement II** is incorrect.
2. **Statement I** and **Statement II** both are correct.
3. **Statement I** and **Statement II** both are incorrect.
4. **Statement I** is incorrect but **Statement II** is correct.

Ans: Statement I is incorrect but Statement II is correct

Sol: Statement I incorrect
$Be(OH)_2$ is amphoteric and hence its get it is dissolved in alkali
Statement II correct

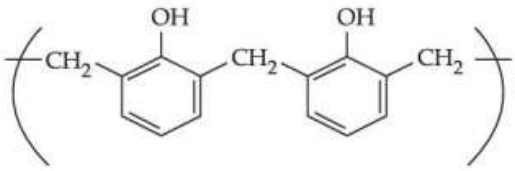
Q.16 Which one of the following compounds of Group-14 elements is **not** known ?

Options

1. $[SiCl_6]^{2-}$
2. $[GeCl_6]^{2-}$
3. $[Sn(OH)_6]^{2-}$
4. $[SiF_6]^{2-}$

Ans: $[SiCl_6]^{2-}$

Sol: Due to steric factors, $[SiCl_6]^{2-}$ does not exist

Q.17  is a repeating unit for :

Options

1. Acrilan
2. Neoprene
3. Buna-N
4. Novolac

Ans: Novolac

Sol: When phenol is treated with formaldehyde in presence of acid or base catalyst, the initial product will be linear polymer and is called Novolac.

Q.18 Which one of the following chemical agent is **not** being used for dry-cleaning of clothes ?

Options

1. $Cl_2C=CCl_2$
2. Liquid CO_2
3. CCl_4
4. H_2O_2

Ans: H_2O_2

Sol: hydrogen peroxide is used as bleaching agent.

Q.19 Which one of the following compounds will liberate CO_2, when treated with $NaHCO_3$?

Options

1. $CH_3-\overset{\overset{O}{\|}}{C}-NH_2$
2. CH_3NH_2
3. $(CH_3)_4\overset{\oplus}{N}\overset{\ominus}{O}H$
4. $(CH_3)_3\overset{\oplus}{N}H\overset{\ominus}{Cl}$

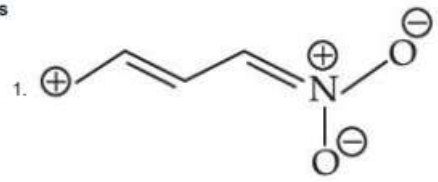
Ans: $(CH_3)_2\overset{+}{N}HCl^-$

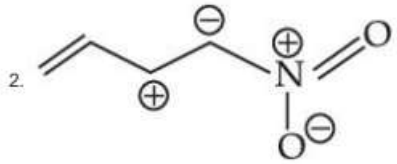
Sol: $(CH_3)_3\overset{+}{N}H_4Cl^- + NaHCO_3 \rightarrow H_2CO_3 + (CH_3)_3N + HCl$

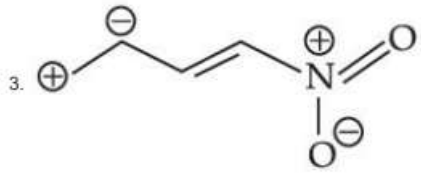
H_2CO_3 ion liberate CO_2

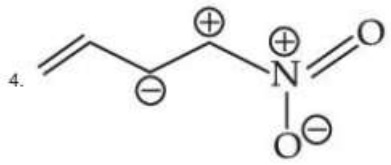
Q.20 Which one among the following resonating structures is **not** correct ?

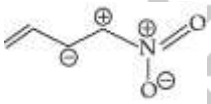
Options

1. 

2. 

3. 

4. 

Ans: 

Sol: Due to same charge on adjacent atom, that resonating structure is not correct

Section B

Q.1 For the reaction

$A + B \rightleftharpoons 2C$

the value of equilibrium constant is 100 at 298 K. If the initial concentration of all the three species is 1 M each, then the equilibrium concentration of C is $x \times 10^{-1}$ M. The value of x is _____. (Nearest integer)

Given –
Answer :

Ans: 25

Sol:

A	+ B	$\rightleftharpoons$	2C
1	1		1
1-α	1-α		1+2α

$$K_c = \frac{[1+2x]^2}{(1-x)(1-x)}$$

$$100 = \frac{(1+2x)^2}{(1-x)^2}$$

$$10 = \frac{1+2x}{1-x}$$

$$x = \frac{9}{12} = \frac{3}{4}$$

$$[C] = 1 + 2x = 1 + 2 \times \frac{3}{4} = 2.5$$

$x = 25 \times 10^{-1}$ M

$x = 25 \times 10^{-1}$ M

Q.2 A source of monochromatic radiation of wavelength 400 nm provides 1000 J of energy in 10 seconds. When this radiation falls on the surface of sodium, $x \times 10^{20}$ electrons are ejected per second. Assume that wavelength 400 nm is sufficient for ejection of electron from the surface of sodium metal. The value of x is _____. (Nearest integer)
$(h = 6.626 \times 10^{-34}$ Js)

Given --
Answer :

Ans: 464

Sol: $C_4H_{10} + \frac{13}{2}O_2 \rightarrow 4CO_2 + 5H_2O$

58 g $C_4H_{10} \rightarrow$ 90 g H_2O

x g $C_4H_{10} \rightarrow$ 72 g H_2O

$x = \frac{58 \times 72}{90} = 464 \times 10^{-1}$

Q.3 Consider the complete combustion of butane, the amount of butane utilized to produce 72.0 g of water is _____ $\times 10^{-1}$ g. (in nearest integer)

Given --
Answer :

Ans: 2.01

Sol: Energy in $J/s = \frac{1000}{10} = 100\ Js^{-1}$

$E = \frac{nhc}{\lambda}$

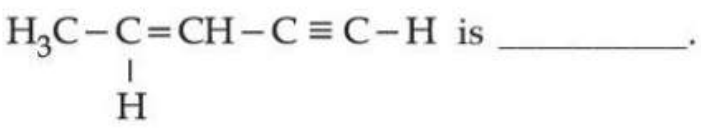
$n = \frac{E\lambda}{hc}$

$= \frac{100 \times 400 \times 10^{-9}}{6.626 \times 10^{-34} \times 3 \times 10^{8}}$

$= 2.01 \times 10^{+20}$

$x = 2.01$

Q.4 The number of sigma bonds in

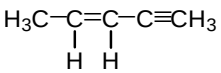


is _____.

Given 8
Answer :

Ans: 10

Sol:



Number of sigma bond = 10

Q.5 Three moles of AgCl get precipitated when one mole of an octahedral co-ordination compound with empirical formula $CrCl_3.3NH_3.3H_2O$ reacts with excess of silver nitrate. The number of chloride ions satisfying the secondary valency of the metal ion is _____.

Given --
Answer :

Ans: 0

Sol: The number of moles of AgCl precipitated gives the number of chloride ion outside the complex.
3 moles AgCl precipitated and hence the formula of the complex is $[Cr(H_2O)_3(NH_3)_3]Cl_3$
Here, none of the chloride ion are present inside the coordination sphere. Therefore, number of Cl^- ion satisfying secondary valency is zero

Q.6 Consider the cell at 25°C
$Zn|Zn^{2+}(aq)$, (1 M) || $Fe^{3+}(aq)$, $Fe^{2+}(aq)|Pt(s)$
The fraction of total iron present as Fe^{3+} ion at the cell potential of 1.500 V is $x \times 10^{-2}$. The value of x is _____. (Nearest integer)
(Given : $E^0_{Fe^{3+}/Fe^{2+}} = 0.77$ V, $E^0_{Zn^{2+}/Zn} = -0.76$ V)

Given –
Answer :

Ans: 3

Sol: Overall reaction

$Zn / Zn^{2+}_{(1M)} // Fe^3 / Fe^{2+}$ is

$$Zn + 2Fe^{3+} \rightarrow Zn^{2+} + 2Fe^{2+}$$

$$E^\circ_{cell} = E^\circ_{Fe^{3+}/Fe^{2+}} - E^\circ_{Zn^{2+}/Zn} = +0.77 - (-0.76) = 1.53$$

$$1.5 = 1.53 - \frac{0.06}{2}\log\frac{[Zn^{2+}][Fe^{2+}]}{[Fe^{3+}]}$$

$$\because [Zn^{2+}] = 1$$

$$0.03 = 0.03 \log\frac{[Fe^{2+}]^2}{[Fe^{3+}]^2}$$

$$\log\left[\frac{Fe^{2+}}{Fe^{3+}}\right]^2 = 1$$

$$\left[\frac{Fe^{2+}}{Fe^{3+}}\right] = 10$$

$$\frac{[Fe^{2+}]}{[Fe^{3+}]} = \sqrt{10}$$

Q.7 When 10 mL of an aqueous solution of Fe^{2+} ions was titrated in the presence of dil H_2SO_4 using diphenylamine indicator, 15 mL of 0.02 M solution of $K_2Cr_2O_7$ was required to get the end point. The molarity of the solution containing Fe^{2+} ions is $x \times 10^{-2}$ M. The value of x is _____. (Nearest integer)

Given –
Answer :

Ans: 18

Sol: $$6Fe^{2+} + Cr_2O_7^{2-} + 14H^+ \rightarrow 6Fe^{2+} + 2Cr^{3+} + 7H_2O$$

$N_1V_1 = N_2V_2$

$N_1 \times 10 = (0.02 \times 6) \times 15$

$N_1 = 18 \times 10^{-5}$

Q.8 CO_2 gas is bubbled through water during a soft drink manufacturing process at 298 K. If CO_2 exerts a partial pressure of 0.835 bar then x m mol of CO_2 would dissolve in 0.9 L of water. The value of x is _____. (Nearest integer)
(Henry's law constant for CO_2 at 298 K is 1.67×10^3 bar)

Given –
Answer :

Ans: 25

Sol: $P_i = K_H X_i$

$$0.835 = 1.67 \times 10^{-3} \times \frac{n_{CO_2}}{\frac{900}{18}}$$

$$n_{CO_2} = 0.025 = 25 \times 10^{-3} \text{ moles}$$

Q.9 A home owner uses 4.00×10^3 m^3 of methane (CH_4) gas, (assume CH_4 is an ideal gas) in a year to heat his home. Under the pressure of 1.0 atm and 300 K, mass of gas used is $x \times 10^5$ g. The value of x is _____. (Nearest integer)
(Given R = 0.083 L atm K^{-1} mol^{-1})

Given –
Answer :

Ans: 26

Sol: $PV = nRT$ $\quad V = 4 \times 10^3$ m^3

$V = 4 \times 10^6$ L

$1 \times 4 \times 10^3 \times 10^3 = n \times 0.083 \times 300$

$$n_{CH_4} = \frac{4 \times 10^6}{0.083 \times 300} = \frac{4 \times 10^4}{0.249} = 16 \times 10^4$$

Weight of $CH_4 = 16 \times 16 \times 10^4$

$= 256 \times 10^4 = 25.6 \times 10^4 \approx 26 \times 10^4$

Q.10 At 298 K, the enthalpy of fusion of a solid (X) is 2.8 kJ mol^{-1} and the enthalpy of vaporisation of the liquid (X) is 98.2 kJ mol^{-1}. The enthalpy of sublimation of the substance (X) in kJ mol^{-1} is _____. (in nearest integer)

Given –
Answer :

Ans: 101

Sol: $X_{solid} \xrightarrow[\Delta H = 2.8 \text{ kJ mol}^{-1}]{} X_{liquid} \xrightarrow[\Delta H = 98.2 \text{ kJ mol}^{-1}]{} X_{gas}$

$\Delta H = \Delta H_{fusion} + \Delta H_{vap}$

$= 2.8 + 9.2$

$= 101$ kJ mol^{-1}

PART – C – MATHEMATICS

Section A

Q.1 The area (in sq. units) of the region, given by the set $\{(x, y) \in \mathbf{R} \times \mathbf{R} \mid x \geq 0, 2x^2 \leq y \leq 4 - 2x\}$ is :

Options

1. $\frac{17}{3}$
2. $\frac{8}{3}$
3. $\frac{13}{3}$
4. $\frac{7}{3}$

Ans: $\frac{7}{3}$

Sol: $2x^2 = 4 - 2x$

$x^2 + x - 2 = 0$

Solving we get, $x = -2, x = 1$

Required area = $\frac{1}{2}(2+4)\times 1 - \int_0^1 2x^2 dx = 3 - \frac{2}{3} = \frac{7}{3}$ squareunits

Q.2

Let $f(x) = 3\sin^4 x + 10\sin^3 x + 6\sin^2 x - 3,\ x \in \left[-\frac{\pi}{6}, \frac{\pi}{2}\right]$. Then, f is :

Options

1. decreasing in $\left(0, \frac{\pi}{2}\right)$
2. increasing in $\left(-\frac{\pi}{6}, \frac{\pi}{2}\right)$
3. decreasing in $\left(-\frac{\pi}{6}, 0\right)$
4. increasing in $\left(-\frac{\pi}{6}, 0\right)$

Ans: Decreasing in $x \in \left(\frac{-\pi}{6}, 0\right)$

Sol: $f'(x) = 12\sin^3 x \cos x + 30\sin^2 x \cos x + 12 \sin x \cos x$

$f'(x) = 0$

$\Rightarrow 6 \sin x \cos x (2\sin^2 x + 5\sin x + 2) = 0$

$\sin 2x = 0$ or $2\sin^2 x + 5\sin x + 2 = 0$

$x = 0$ $(2\sin x + 1)$ [$(\sin x + 2) = 0$ is impossible]

$\sin x = \frac{-1}{2}$

$x = \frac{-\pi}{6}\ x \in \left(\frac{-\pi}{6}, \frac{\pi}{2}\right)$

Now, we can see that the function is decreasing in $x \in \left(\frac{-\pi}{6}, 0\right)$

Q.3 A spherical gas balloon of radius 16 meter subtends an angle 60° at the eye of the observer A while the angle of elevation of its center from the eye of A is 75°. Then the height (in meter) of the top most point of the balloon from the level of the observer's eye is :

Options

1. $8(\sqrt{6} + \sqrt{2} + 2)$
2. $8(\sqrt{6} - \sqrt{2} + 2)$
3. $8(\sqrt{2} + 2 + \sqrt{3})$
4. $8(2 + 2\sqrt{3} + \sqrt{2})$

Ans: $8(\sqrt{6} + \sqrt{2} + 2)$

Sol: In triangle ABC

$AC=\frac{16}{\tan 30^o}=16\sqrt{3}$

$CD = 16\sqrt{3}$

Now in Δ DFC′

$C'D = 6\sqrt{3}\sin 45°$

$=16\sqrt{3}.\frac{1}{\sqrt{2}}=8\sqrt{6}$

$EF = 8\sqrt{6}$

In Δ^{le} OED

$OE = 16\cos 45°=\frac{16}{\sqrt{2}}=8\sqrt{2}$

Height of top most point $=8\sqrt{6}+8\sqrt{2}+16=8(\sqrt{6}+\sqrt{2}+2)$

Q.4 Let the vectors

$(2+a+b)\hat{i}+(a+2b+c)\hat{j}-(b+c)\hat{k},$

$(1+b)\hat{i}+2b\hat{j}-b\hat{k}$ and $(2+b)\hat{i}+2b\hat{j}+(1-b)\hat{k}$, $a, b, c \in \mathbf{R}$

be co-planar. Then which of the following is true ?

Options

1. $3c=a+b$
2. $a=b+2c$
3. $2a=b+c$
4. $2b=a+c$

Ans: 2b = a + c

Sol: Three vectors are coplanar

$$\Rightarrow \begin{vmatrix} 2+a+b & a+2b+c & -(b+c) \\ 1+b & 2b & -b \\ 2+b & 2b & 1-b \end{vmatrix}=0$$

$C_2 \to C_2 + C_3$

$$\Rightarrow \begin{vmatrix} 2+a+b & a+b & -(b+c) \\ 1+b & b & -b \\ 2+b & 1+b & 1-b \end{vmatrix}=0$$

$C_1 \to C_1 - C_2$

$$\Rightarrow \begin{vmatrix} 2 & a+b & -(b+c) \\ 1 & b & -b \\ 2 & 1+b & 1-b \end{vmatrix}=0$$

Expanding along first row, we get

$\Rightarrow$ 2b = a + c

Q.5 The value of the definite integral

$$\int_{\pi/24}^{5\pi/24} \frac{dx}{1+\sqrt[3]{\tan 2x}} \text{ is :}$$

Options

1. $\frac{\pi}{18}$
2. $\frac{\pi}{3}$
3. $\frac{\pi}{12}$
4. $\frac{\pi}{6}$

Ans: $\frac{\pi}{12}$

Sol: $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$$I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{dx}{1+\sqrt[3]{\tan 2x}} \text{ ..... .(1)}$$

Also, $I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{dx}{1+\sqrt[3]{\cot 2x}}$

$$I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{\sqrt[3]{\tan 2x}dx}{1+\sqrt[3]{\tan 2x}} \text{ ..... .(2)}$$

From (1) & (2)

$$2I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{1+\sqrt[3]{\tan 2x}dx}{1+\sqrt[3]{\tan 2x}}$$

$$2I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} dx = \frac{\pi}{6}$$

$$\therefore I = \frac{\pi}{12}$$

Q.6 The values of a and b, for which the system of equations

$$2x + 3y + 6z = 8$$
$$x + 2y + az = 5$$
$$3x + 5y + 9z = b$$

has no solution, are :

Options

1. $a \neq 3, b \neq 13$
2. $a \neq 3, b = 3$
3. $a = 3, b = 13$
4. $a = 3, b \neq 13$

Ans: $a = 3, b \neq 13$

Sol: By cramers rule, $\Delta = 0$ and any one of Δ_1, Δ_2 and Δ_3 should not be equal to zero

$$\Delta = \begin{vmatrix} 2 & 3 & 6 \\ 1 & 2 & a \\ 3 & 5 & 9 \end{vmatrix} = 0$$

Solving, we get $a = 3$

$$\Delta_1 = \begin{vmatrix} 8 & 3 & 6 \\ 5 & 2 & 3 \\ b & 5 & 9 \end{vmatrix} = 8[18 - 15] - 3[45 - 3b] + 6[25 - 2b] = 3(13 - b)$$

$$\Delta_2 = \begin{vmatrix} 2 & 8 & 6 \\ 1 & 5 & 3 \\ 3 & b & 9 \end{vmatrix} = 3\begin{vmatrix} 2 & 8 & 2 \\ 1 & 5 & 1 \\ 3 & b & 3 \end{vmatrix} = 0$$

$$\Delta_3 = \begin{vmatrix} 2 & 3 & 8 \\ 1 & 2 & 5 \\ 3 & 5 & b \end{vmatrix} = 2[2b - 25] - 3[b - 15] + 8[5 - 6] = b - 13$$

If $b = 13$, then Δ_1, Δ_2, and Δ_3 all will be zero.
$\therefore b \neq 13$

Q.7 The Boolean expression

$(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to :

Options

1. $\sim p$
2. p
3. $\sim q$
4. q

Ans: $\sim p$

Sol: $= (p \rightarrow q) \wedge (p \rightarrow \sim q)$
$= (\sim p \vee q) \wedge (\sim p \vee \sim q)$
$= \sim p \vee (q \wedge \sim q)$
$= \sim p \vee f$
$= \sim p$

Q.8 Let $y = y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} = 1 + x e^{y-x},\ -\sqrt{2} < x < \sqrt{2},\ y(0) = 0$$

then, the minimum value of $y(x), x \in (-\sqrt{2}, \sqrt{2})$ is equal to :

Options

1. $(1 + \sqrt{3}) - \log_e(\sqrt{3} - 1)$
2. $(1 - \sqrt{3}) - \log_e(\sqrt{3} - 1)$
3. $(2 - \sqrt{3}) - \log_e 2$
4. $(2 + \sqrt{3}) + \log_e 2$

Ans: $\left(1-\sqrt{3}\right)-\log_e\left(\sqrt{3}-1\right)$

Sol: $\frac{dy}{dx} = 1 + xe^{y-x}$ ..... (1)

$e^{-y}\frac{dy}{dx} = e^{-y} + xe^{-x}$

Put $e^{-y} = t \Rightarrow e^{-y}\frac{dy}{dx} = -\frac{dt}{dx}$

$\frac{dt}{dx} + t = -xe^{-x}$ ..... (2)

$I.F = e^{\int 1.dx} = e^x$

Solution of equation (2) is

$te^x = \int(-xe^{-x}).e^x dx + c$

$te^x = -\frac{x^2}{2} + c$

$e^{x-y} = -\frac{x^2}{2} + c$.....(3)

$\therefore y(0) = 0 \Rightarrow 1 = c \Rightarrow e^{x-y} = \left(\frac{2-x^2}{2}\right)$

$x - y = \ln\left(\frac{2-x^2}{2}\right)$

$y = x - \ln\left(\frac{2-x^2}{2}\right)$

Now, $\frac{dy}{dx} = 1 + x\left(\frac{2-x^2}{2}\right)$

$\Rightarrow \left(\frac{2-x^2+2x}{2-x^2}\right) = 0$

$\Rightarrow -\left(\frac{x^2-2x-2x}{2-x^2}\right) = 0$

$x = 1 \pm \sqrt{3}$

$\Rightarrow y_{min}$ at $x = 1-\sqrt{3}$ $\Rightarrow y_{min} = (1-\sqrt{3}) - \ln(\sqrt{3}-1)$

Q.9 Let 9 distinct balls be distributed among 4 boxes, B_1, B_2, B_3 and B_4. If the probability that B_3 contains exactly 3 balls is $k\left(\frac{3}{4}\right)^9$ then k lies in the set :

Options

1. $\{x \in \mathbf{R} : |x-3| < 1\}$
2. $\{x \in \mathbf{R} : |x-1| < 1\}$
3. $\{x \in \mathbf{R} : |x-2| \leq 1\}$
4. $\{x \in \mathbf{R} : |x-5| \leq 1\}$

Ans: $\{x \in \mathbf{R} : |x-3| < 1\}$

Sol: The numbers of ways of distributing 9 distinct balls in 4 boxes is = 4^9 When box 3, contains exactly 3 balls then number of ways = ${}^9C_3 \times (3)^6$

Required Probability = ${}^9C_3 \times \frac{3^6}{4^9} = \frac{28}{9}\left(\frac{3}{4}\right)^9$

Hence $k = \frac{28}{9} = 3\frac{1}{9}$

Now $2 < k < 4 \Rightarrow k \in \{x \in R : |x-3| < 1\}$

Q.10 If b is very small as compared to the value of a, so that the cube and other higher powers of $\frac{b}{a}$ can be neglected in the identity

$$\frac{1}{a-b} + \frac{1}{a-2b} + \frac{1}{a-3b} + \ldots + \frac{1}{a-nb} = \alpha n + \beta n^2 + \gamma n^3,$$

then the value of γ is :

Options

1. $\frac{a^2+b}{3a^3}$
2. $\frac{b^2}{3a^3}$
3. $\frac{a+b}{3a^2}$
4. $\frac{a+b^2}{3a^3}$

Ans: $\frac{b^2}{3a^3}$

Sol:

$$\frac{1}{a-b} + \frac{1}{a-2b} + \frac{1}{a-3b} + \ldots\ldots + \frac{1}{a-nb}$$

$$= \frac{1}{a}\left[\left\{1 + \left(\frac{b}{a}\right) + \left(\frac{b}{a}\right)^2 + \ldots.\right\} + \left\{1 + \left(\frac{2b}{a}\right) + \left(\frac{2b}{a}\right)^2 + \ldots\ldots\right\} + \left\{1 + \left(\frac{nb}{a}\right) + \left(\frac{nb}{a}\right)^2 + \ldots\right\}\right]$$

$$= \frac{1}{a}\left[n + \frac{n(n+1)}{2}\frac{b}{a} + \frac{n(n+1)(2n+1)}{6}\frac{b^2}{a^2}\right]$$

$$= n\left(\frac{1}{a} + \frac{b}{2a^2} + \frac{b^2}{6a^3}\right) + \left(\frac{b}{2a^2} + \frac{b^2}{6a^3}\right)n^2 + \frac{b^2}{3a^3}n^3$$

by comparing this result to $\alpha n + \beta n^2 + \gamma n^2$

we get $\gamma = \frac{b^2}{3a^3}$

Q.11 Let an ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a^2 > b^2$, passes through $\left(\sqrt{\frac{3}{2}}, 1\right)$ and has eccentricity $\frac{1}{\sqrt{3}}$. If a circle, centered at focus $F(\alpha, 0)$, $\alpha > 0$, of E and radius $\frac{2}{\sqrt{3}}$, intersects E at two points P and Q, then PQ^2 is equal to :

Options

1. 3
2. $\frac{16}{3}$
3. $\frac{4}{3}$
4. $\frac{8}{3}$

Ans: $\frac{16}{3}$

Sol: Let equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \ (a > b)$

Since it passes through $\left(\sqrt{\frac{3}{2}}, 1\right) \Rightarrow \frac{3}{2a^2} + \frac{1}{b^2} = 1$..... .(1)

Given $e = \frac{1}{\sqrt{3}} \Rightarrow b^2 = a^2(1-e^2) = \frac{2}{3}a^2$..... ..... (2)

Solving (1) & (2) we get $a^2 = 3$, $b^2 = 2$

$\therefore$ Ellipse is $\frac{x^2}{3} + \frac{y^2}{2} = 1$..... (3)

Focus $(\pm ae, 0) = \left(\pm\sqrt{3}.\frac{1}{\sqrt{3}}, 0\right) \equiv (\pm 1, 0)$

Therefore, circle is $(x-1)^2 + y^2 = \left(\frac{2}{\sqrt{3}}\right)^2 = \frac{4}{3}$ .....(4)

Solving (3) & (4)

$2x^2 + 3\left(\frac{4}{3} - (x-1)^2\right) = 6$

$x = 1, 5$

When $x = 1 \Rightarrow \frac{1}{3} + \frac{y^2}{2} = 1 \Rightarrow y^2 = \frac{4}{3} \Rightarrow y = \pm\frac{2}{\sqrt{3}}$

Hence $P(1, \frac{2}{\sqrt{3}})$, $Q(1, -\frac{2}{\sqrt{3}}) \Rightarrow PQ^2 = \frac{16}{3}$

when $x=5$, y^2 is negative and hance not acceptable

Q.12 Let a parabola P be such that its vertex and focus lie on the positive x-axis at a distance 2 and 4 units from the origin, respectively. If tangents are drawn from O(0, 0) to the parabola P which meet P at S and R, then the area (in sq. units) of ΔSOR is equal to :

Options

1. $8\sqrt{2}$
2. 32
3. 16
4. $16\sqrt{2}$

Ans: 16

Sol:

Equation of parabola
$(y-0)^2 = 4(2)(x-2)$
origin lies on the directrix $x = 0$
so ΔSOR is right angle triangle.
Equation of chord of contact SR is $x = 4$
It is latus rectum of parabola

Therefore area of SOR $= \frac{1}{2} \times 4 \times 8 = 16$ square units

Q.13 Let S_n be the sum of the first n terms of an arithmetic progression. If $S_{3n} = 3S_{2n}$, then the value of $\frac{S_{4n}}{S_{2n}}$ is :

Options

1. 6
2. 4
3. 8
4. 2

Ans: 6

Sol: $\frac{s_{3n}}{s_{2n}} = \frac{\frac{3n}{2}[2a+(3n-1)d]}{\frac{2n}{2}[2a+(2n-1)d]} = 3$

$\Rightarrow 2a+(3n-1)d = 2[2a + (2n-1)d]$
$\Rightarrow 2a+(n-1)d = 0$ ……(1)

Now $\frac{s_{4n}}{s_{2n}} = \frac{\frac{4n}{2}[2a+(4n-1)d]}{\frac{2n}{2}[2a+(2n-1)d]} = \frac{2[2a+(3n-1)d]}{[2a+(2n-1)d]}$

From (1), $2a = -(n-1)d$, substituting we get, $\frac{s_{4n}}{s_{2n}} = \frac{2[3nd]}{nd]} = 6$

Q.14 The sum of all values of x in $[0, 2\pi]$, for which $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$, is equal to :

Options

1. $11\ \pi$
2. $9\ \pi$
3. $12\ \pi$
4. $8\ \pi$

Ans: 9π

Sol: $(\sin x + \sin 4x) + (\sin 2x + \sin 3x) = 0$

$\Rightarrow 2\sin\frac{5x}{2}\cos\frac{3x}{2} + 2\sin\frac{5x}{2}\cos\frac{x}{2} = 0$

$\Rightarrow 2\sin\frac{5x}{2}\left(\cos\frac{3x}{2} + \cos\frac{x}{2}\right) = 0$

$\Rightarrow 4\sin\frac{5x}{2}\cos x\cos\frac{x}{2} = 0$

$\Rightarrow \sin\frac{5x}{2} = 0$ or $\cos x = 0$ or $\cos\frac{x}{2} = 0$

$\Rightarrow \frac{5x}{2} = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$ or $x = \frac{\pi}{2}, \frac{3\pi}{2}$ or $\frac{x}{2} = \frac{\pi}{2}$

$\Rightarrow x = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, \frac{10\pi}{5}$ or $x = \frac{\pi}{2}, \frac{3\pi}{2}$ or $x = \pi$

$\Rightarrow x = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, 2\pi, \frac{\pi}{2}, \frac{3\pi}{2}, \pi$

Hence sum of all solutions = 9π

Q.15 Let $f : [0, \infty) \to [0, \infty)$ be defined as

$f(x) = \int_0^x [y]\,dy$

where $[x]$ is the greatest integer less than or equal to x. Which of the following is true ?

Options 1.

f is continuous at every point in $[0, \infty)$ and differentiable except at the integer points.

2.

f is both continuous and differentiable except at the integer points in $[0, \infty)$.

3. f is differentiable at every point in $[0, \infty)$.

4.

f is continuous everywhere except at the integer points in $[0, \infty)$.

Ans: f is continuous at every point [0, ∞) and differentiable except at the integer points

Sol: $f(x) = \int_0^x [y]dy$

Let $n \le x < n + 1$, $n \in I$

$= \int_0^x (0)dy + \int_1^2 (1)dy + \int_2^3 (2)dy + ... + \int_{n-1}^n (n-1)dy + \int_n^x (n)dy$

$= 0 + 1 + 2 + 3 + ... + (n-1) + n(x-n)$

$\frac{n(n-1)}{2} = n^2 + nx$

$\Rightarrow f(x) = \frac{-n-n^2}{2} + nx = nx - \frac{n(n+1)}{2} \Rightarrow f(x) = n\left(x - \frac{n+1}{2}\right)$

$\Rightarrow f(x) = \begin{cases} x\left(\frac{x-1}{2}\right) & ; \quad x = n(\text{integer}) \\ [x]\left(\frac{2x-[x]-1}{2}\right) & ; x \notin \text{integer} \end{cases}$

Since $\lim_{x\to n} f(x) = \frac{n(n-1)}{2} = f(n)$

Therefore, f(x) is continuous at all integers.

But f'(x) = [x] is discontinuous at all integers.

Q.16 The number of real roots of the equation $e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^{x} + 1 = 0$ is :

Options

1. 6
2. 2
3. 1
4. 4

Ans: 2

Sol: $e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^{x} + 1 = 0$ is

$\Rightarrow (e^{3x} - 1)^2 - e^{x}(e^{3x} - 1) - 12e^{2x} = 0$

$\Rightarrow e^{3x} - 1 = 4e^{x}$ or $(e^{3x} - 1) \cdot e^{x} = 0$

Case-1 $e^{3x} - 1 = 4e^{x}$

It can be seen that there is only 1 solution to this.

Case –2 $(e^{3x} - 1)\, e^{x} = 0$

$\Rightarrow e^{3x} = 1$

$\Rightarrow x = 0$ (one solution)

$\therefore$ Total 2 roots

Q.17 The locus of the centroid of the triangle formed by any point P on the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$, and its foci is :

Options

1. $9x^2 - 16y^2 + 36x + 32y - 36 = 0$
2. $16x^2 - 9y^2 + 32x + 36y - 144 = 0$
3. $16x^2 - 9y^2 + 32x + 36y - 36 = 0$
4. $9x^2 - 16y^2 + 36x + 32y - 144 = 0$

Ans: $16x^2 - 9y^2 + 32x + 36y - 36 = 0$

Sol: Given $\frac{(x+1)^2}{9} - \frac{(y-2)^2}{16} = 1$

$a = 3, b = 4$

$b^2 = a^2 (e^2 - 1) \Rightarrow e = \frac{5}{3}$

Focus $(\pm ae, 0) \Rightarrow X = \pm ae, Y = 0$

$x + 1 = \pm 5, y - 2 = 0$

$x = -6, 4, y = 2$

Hence focus S(–6, 2), S′(4, 2)

Any point on hyperbola is given by $(-1 + 3\sec\theta, 2 + 4\tan\theta)$

Hence centroid is $= \left(\frac{-6 + 4 - 1 + 3\sec\theta}{3}, \frac{2 + 2 + 2 + 4\tan\theta}{3} \right)$

$h = \frac{-3 + 3\sec\theta}{3} \Rightarrow \sec\theta = \frac{3h + 3}{3}$

$\Rightarrow \tan\theta = \frac{3k - 6}{4}$

$\sec^2\theta - \tan^2\theta = 1$

$\left(\frac{3h + 3}{3} \right)^2 - \frac{(3k - 6)^2}{16} = 1$

$\Rightarrow 16x^2 - 9y^2 + 32x + 36y - 36 = 0$

Q.18

Let the foot of perpendicular from a point P(1, 2, −1) to the straight line $L: \frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$ be N. Let a line be drawn from P parallel to the plane $x + y + 2z = 0$ which meets L at point Q. If α is the acute angle between the lines PN and PQ, then $\cos\alpha$ is equal to _____.

Options

1. $\frac{1}{\sqrt{3}}$
2. $\frac{\sqrt{3}}{2}$
3. $\frac{1}{2\sqrt{3}}$
4. $\frac{1}{\sqrt{5}}$

Ans: $\frac{1}{\sqrt{3}}$

Sol: Foot of the perpendicular from (1, 2, – 1) on line $\frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$ is N (1, 0, – 1)

Now equation of line passes through P(1, 2, –1) is

$$\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+1}{c} \text{..... ..(1)}$$

This line is parallel to plane x + y + 2z = 0

$\Rightarrow$ a + b + 2c = 0 …… (2)

Any point on the line L: $\frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$ is

Q(r, 0 , – r) lies on line (1)

$$\Rightarrow \quad \frac{r-1}{a} = \frac{0-2}{b} = \frac{1-r}{c}$$

$\Rightarrow$ c = – a

From (2) we have b = a

Hence a : b : c = a : a : – a

So, line PQ is $\frac{x-1}{a} = \frac{y-2}{a} = \frac{z+1}{a}$

And directions line PN is (0, 2, 0)

Acute angle between PQ and PN is 'α' then

$$\cos\alpha = \left|\frac{0+2a+0}{\sqrt{3}a.2}\right| = \frac{1}{\sqrt{3}}$$

Q.19 Let $g : \mathbf{N} \to \mathbf{N}$ be defined as

$g(3n+1) = 3n+2,$

$g(3n+2) = 3n+3,$

$g(3n+3) = 3n+1,$ for all $n \geq 0.$

Then which of the following statements is true ?

Options 1.

There exists a function $f : \mathbf{N} \to \mathbf{N}$ such that $gof = f$

2. $gogog = g$

3.

There exists a one-one function $f : \mathbf{N} \to \mathbf{N}$ such that $fog = f$

4.

There exists an onto function $f : \mathbf{N} \to \mathbf{N}$ such that $fog = f$

Ans: There exists an onto function f:N→ N such that fog=f

Sol: If $f : N \to N$, f = 3n + 1 = 3n + 2 = 3n + 3
So, g(3n + 1) = 3n + 2, g(3n + 2) = 3n + 3, g(3n + 3) = 3n + 1
So $g(f(x)) \neq f(x)$
Therefore, statement (1) is not true
g(3n + 1) = 3n + 2
g(3n + 2) = 3n + 3
g(3n + 3) = 3n +1, $n \geq 0$
For x = 3n + 1
gogog (3n + 1) 3n + 1
Similarly
gogog (3n + 2) = 3n + 2
gogog (3n + 3) = 3n + 3
Therefore gogog (x) = x $\forall$ x $\in$ N
Therefore (2) is not true

(f $f : N \to N$ and f is a one-one function such that f(g(x)) = f(x) then
g(x) = x
but $g(x) \neq x$
Therefore, statement 3 is not true
If $f : N \to N$ and f is an onto function such that f(g(x)) = f(x) then
One of its possibilities is by taking f(x) as onto function

$$f(x) = \begin{cases} a & x = 3n+1 \\ a & x = 3n+2, \quad a \in N \\ a & x = 3n+3 \end{cases}$$

$\Rightarrow$ f(g(x)) = f(x) $\forall$ x $\in$ N
Therefore, statement 4 is true

Q.20 Let $f : \mathbf{R} \to \mathbf{R}$ be defined as

$$f(x) = \begin{cases} \dfrac{\lambda|x^2 - 5x + 6|}{\mu(5x - x^2 - 6)}, & x < 2 \\ e^{\frac{\tan(x-2)}{x-[x]}}, & x > 2 \\ \mu, & x = 2 \end{cases}$$

where $[x]$ is the greatest integer less than or equal to x. If f is continuous at $x = 2$, then $\lambda + \mu$ is equal to :

Options 1. $e(e-2)$

2. 1

3. $e(-e+1)$

4. $2e-1$

Ans: e(-e+1)

Sol: RHL= $\lim\limits_{x\to 2^+} e^{\frac{\tan(x-2)}{x-[x]}} = \lim\limits_{x\to 2^+} e^{\frac{\tan(x-2)}{(x-2)}} = e$

LHL= $\lim\limits_{x\to 2^-} \frac{\lambda}{\mu}\frac{|x^2-5x+6|}{5x-6-x^2}$

For $x < 2$, $|x^2 - 5x + 6| = x^2 - 5x + 6$

$\therefore$ LHL = $\lim\limits_{x\to 2^-} \frac{\lambda}{\mu}\frac{|x^2-5x+6|}{(5x-6-x^2)} = -\frac{\lambda}{\mu}$

Also, $f(2) = \mu$

For f(x) to be continuous at x = 2,

RHL = LHL = f(2)

$\therefore e = -\frac{\lambda}{\mu} = \mu$

$\Rightarrow \mu = e$ and $\lambda = -e^2$

$\therefore \lambda + \mu = -e^2+e$

Section B

Q.1 Let $y = y(x)$ be solution of the following differential equation

$$e^y\frac{dy}{dx} - 2e^y\sin x + \sin x\cos^2 x = 0,\ y\left(\frac{\pi}{2}\right) = 0.$$

If $y(0) = \log_e(\alpha + \beta e^{-2})$, then $4(\alpha+\beta)$ is equal to _____.

Given --
Answer :

Ans: 4.00

Sol: $e^y\frac{dy}{dx} - 2e^y\sin x = -\sin x\cos^2 x$

Put $e^y = t$

$e^y\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} - 2t\sin x = -\sin x\cos^2 x$

I.F $= e^{-\int 2\sin x dx} = e^{2\cos x}$

$e^y.e^{2\cos x} = \int e^{2\cos x}(-\sin x - \cos^2 x)dx$

$\Rightarrow e^y = \frac{1}{4}(2\cos^2 x - 2\cos x + 1) + c$

At $x = \frac{\pi}{2}$ $y = 0$.

$\Rightarrow 1 = \frac{1}{4} + c \Rightarrow c = \frac{3}{4}$

$\Rightarrow e^y = \frac{1}{4}(2\cos^2 x - 2\cos x + 1) + \frac{3}{4}e^{-2\cos x}$

$\Rightarrow y(0) = \ln\left(\frac{1}{4} + \frac{3}{4}e^2\right)$

$\Rightarrow y(0) = \ln\left(\alpha + \beta e^{-2}\right)$

$\Rightarrow \alpha = \frac{1}{4}, \beta = \frac{3}{4}$

$\Rightarrow 4(\alpha+\beta) = 4$

Q.2 The ratio of the coefficient of the middle term in the expansion of $(1+x)^{20}$ and the sum of the coefficients of two middle terms in expansion of $(1+x)^{19}$ is _____.

Given **5**
Answer :

Ans: 1.00

Sol: $$\frac{^{20}C_{10}}{^{19}C_9 + {}^{19}C_{10}} = \frac{^{20}C_{10}}{^{20}C_{10}} = 1$$

Q.3 Let $M = \left\{A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \{\pm 3, \pm 2, \pm 1, 0\}\right\}$. Define $f: M \to \mathbf{Z}$, as $f(A) = \det(A)$, for all $A \in M$, where $\mathbf{Z}$ is set of all integers. Then the number of $A \in M$ such that $f(A) = 15$ is equal to _____.

Given --
Answer :

Ans: 16.00

Sol: $|A| = (ad - bc) = 15$
where $a, b, c, d \in \{\pm 1, \pm 2, \pm 3\}$
$ad = 9$ & $bc = -6$
$ad \equiv (3,3)$ or $(-3, -3)$ $bc \equiv (2, -3), (-2, 3), (-3,2), (3, -2)$
number of matrices $= 2 \times 4 = 8$ matrix
$ad = 6$ and $bc = -9$
Number of matrices $= 4 \times 2 = 8$ matrix
Total matrices $= 8 + 8 = 16$ matrix

Q.4 There are 5 students in class 10, 6 students in class 11 and 8 students in class 12. If the number of ways, in which 10 students can be selected from them so as to include at least 2 students from each class and at most 5 students from the total 11 students of class 10 and 11 is 100 k, then k is equal to _____.

Given **6**
Answer :

Ans: 238

Sol: $$\text{Total number of ways} = {}^5C_2 \times 8({}^6C_3 + {}^6C_2) \times {}^5C_2 \times {}^6C_2 \times {}^8C_6$$
$$= 23800$$

Q.5 The term independent of 'x' in the expansion of $\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10}$, where $x \neq 0, 1$ is equal to _____.

Given --
Answer :

Ans: 210

Sol: $$\left((x^{1/3} + 1) - \left(\frac{\sqrt{x} + 1}{\sqrt{x}}\right)\right)^{10}$$
$$(x^{1/3} - x^{-1/2})^{10}$$
$$T_{r+1} = {}^{10}C_r (x^{1/3})^{10-r} (-x^{-1/2})^r$$
$$\frac{10 - r}{3} - \frac{r}{2} = 0 \Rightarrow 20 - 2r - 3r = 0$$
$$\Rightarrow \quad r = 4$$
$$T_5 = {}^{10}C_4 = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$$

Q.6 Let $S=\left\{n \in \mathbf{N} \middle| \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^n \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \forall\, a, b, c, d \in \mathbf{R}\right\}$, where $i=\sqrt{-1}$. Then the number of 2-digit numbers in the set S is _____.

Given –
Answer :

Ans: 11.00

Sol: Let $A=\begin{bmatrix} 0 & i \\ 1 & 0 \end{bmatrix}^n$ and $B=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

AB=IB
(A-I)B=0
A=I

$\begin{bmatrix} 0 & i \\ 1 & 0 \end{bmatrix}^n = I$

$A^8=\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

n = multiple of 8
Number of two digit numbers is S = 11 (16, 24,.....96)

Q.7 Consider the following frequency distribution :

Class :	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Frequency :	α	110	54	30	β

If the sum of all frequencies is 584 and median is 45, then $|\alpha-\beta|$ is equal to _____.

Given **43**
Answer :

Ans: 164

Sol:

Class	Frequency	C.F
10-20	α	α
20-30	110	$\alpha + 110$
30-40	54	$\alpha + 164$
40-50	30	$\alpha + 194$
50-60	β	$\alpha + \beta + 194 = 584$

N= $\sum f = 584$
$\alpha+\beta = 390$

Median (m) $= l + \left[\frac{\left(\frac{N}{2}\right)-c}{f}\right] \times c$

$N=\frac{584}{2}=292$

$m = 45 = 40 + \left[\frac{292-(\alpha+164)}{30}\right] \times 10$

$45 = 40 + \left(\frac{128-\alpha}{3}\right)$

$5 = \frac{128-\alpha}{3}$

$15 = 128 - \alpha$
$\alpha = 113$
$\beta = 277$
$|\alpha-\beta| = |113-277| = 164$

Q.8 If α, β are roots of the equation $x^2 + 5(\sqrt{2})x + 10 = 0$, $\alpha > \beta$ and $P_n = \alpha^n - \beta^n$ for each positive integer n, then the value of $\left(\frac{P_{17}P_{20} + 5\sqrt{2}\,P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}\,P_{18}^2}\right)$ is equal to _____.

Given --
Answer :

Ans: 1.00

Sol: $x^2 + 5\sqrt{2}x + 10 = 0 \; ; \alpha > \beta$

$p_n = \alpha^n - \beta^n$

$p_{17} = \alpha^{17} - \beta^{17}, p_{18} = \alpha^{18} - \beta^{18}$

$\alpha^2 + 10 = 5\sqrt{2}\alpha$.....(1)

$\beta^2 + 10 = -5\sqrt{2}\beta$.....(2)

$$\Rightarrow \quad \frac{p_{17}\left((\alpha^{20} - \beta^{20}) + 5\sqrt{2}(\alpha^{19} - \beta^{19})\right)}{p_{18}\left((\alpha^{15} - \beta^{19}) + 5\sqrt{2}(\alpha^{18} - \beta^{18})\right)} = \frac{p_{17}\left(\alpha^{18}(\alpha^2 + 5\sqrt{2}\alpha) - \beta^{18}(5\sqrt{2}\beta + \beta^2)\right)}{p_{18}\left(\alpha^{17}(\alpha^2 + 5\sqrt{2}\alpha) - \beta^{17}(5\sqrt{2}\beta + \beta^2)\right)}$$

From (1) & (2), required answer is 1

Q.9 If the value of $\left(1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \text{..... upto } \infty\right)^{\log_{(0.25)}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \text{..... upto } \infty\right)}$

is l, then l^2 is equal to _____.

Given **0**
Answer :

Ans: 3.00

Sol: $\left(1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + ...\right)^{\log_{0.25}\left(\frac{1}{3}\left(\frac{1}{2}\right)\right)} = l$

$\left(1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + ...\right)^{log_{\frac{1}{4}}\frac{1}{2}} = a$

Let $1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + .. = b$

$(b-1) = \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + ...(1)$

$\frac{1}{3}(b-1) = \frac{2}{3^2} + \frac{6}{3^3} + ...(2)$

From (1) – (2), we get

$\frac{2}{3}(b-1) = \frac{2}{3} + \frac{4}{3^2} + \frac{4}{3^3} + ..$

$\frac{2}{3}(b-1) = \frac{2}{3} + \frac{4}{3^2}\left(\frac{1}{1-\frac{1}{3}}\right)$

b – 1 = 2 & b = 3

So , $3^{log_{\frac{1}{4}}\frac{1}{2}} = a$

$3^{\frac{1}{2}} = a$

$a^2=3$

Q.10 Let $\vec{p} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{q} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. If a vector $\vec{r} = \left(\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}\right)$ is perpendicular to each of the vectors $\left(\vec{p} + \vec{q}\right)$ and $\left(\vec{p} - \vec{q}\right)$, and $\left|\vec{r}\right| = \sqrt{3}$, then $|\alpha| + |\beta| + |\gamma|$ is equal to _____.

Given --
Answer :

Ans: 3.00

Sol: $(\vec{p}+\vec{q})\times(\vec{p}-\vec{q})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 5 & 2\\ 1 & 1 & 0\end{vmatrix}=-2\hat{i}+2\hat{j}-2\hat{k}$

$$\vec{r}=\pm\sqrt{3}\,\frac{(\vec{p}+\vec{q})\times(\vec{p}-\vec{q})}{\left|(\vec{p}+\vec{q})\times(\vec{p}-\vec{q})\right|}$$

$$=\pm(-\hat{i}+\hat{j}-\hat{k})$$

So $|\alpha|=1,|\beta|=1$, $|\gamma|=1$

$|\alpha|+|\beta|+|\gamma|=3$