# SOLUTIONS & ANSWERS FOR JEE MAINS-2021
# 18th March Shift 2

## [PHYSICS, CHEMISTRY & MATHEMATICS]

## PART – A – PHYSICS

### SECTION A

**Q.1** If the angular velocity of earth's spin is increased such that the bodies at the equator start floating, the duration of the day would be approximately :

[Take $g = 10\ ms^{-2}$, the radius of earth, $R = 6400 \times 10^3$ m, Take $\pi = 3.14$]

**Options**

1. 84 minutes
2. 60 minutes
3. 1200 minutes
4. does not change

**Ans:** **1**

**Sol:** Let $\omega \rightarrow$ angular velocity of earth

$R \rightarrow$ radius of earth

For objects to float

$mg = m\omega^2 R$

$\Rightarrow g = \omega^2 R$

$\omega = \sqrt{\frac{g}{R}}$

Let T → duration of a ay

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{g}{R}}} = 2\pi\sqrt{\frac{R}{g}}$

$= 2\pi\sqrt{\frac{6400 \times 10^3}{10}}$

$\Rightarrow \frac{T}{60} = 83.77\,\text{min}$

$\approx 84\,\text{min}$

**Q.2** A proton and an α-particle, having kinetic energies $K_p$ and $K_\alpha$ respectively, enter into a magnetic field at right angles.

The ratio of the radii of trajectory of proton to that of α-particle is 2 : 1. The ratio of $K_p : K_\alpha$ is :

**Options**

1. 8 : 1
2. 1 : 4
3. 1 : 8
4. 4 : 1

**Ans:** **4**

**Sol:** $r = \frac{mv}{Bq} = \frac{p}{Bq}$

$\Rightarrow r \propto \frac{p}{q}$

$\frac{r_p}{r_\alpha} = \frac{p_p}{p_\alpha} \times \frac{q_\alpha}{q_p}$

Given $\frac{r_p}{r_\alpha} = 2:1$

$\therefore \frac{p_p\ q_\alpha}{p_\alpha\ q_p} = \frac{2}{1}$

$\frac{p_p}{p_\alpha} = 2\frac{q_p}{q_\alpha} = 2 \times \frac{q_p}{2q_p} = 1$

$KE = \frac{p^2}{2m} \propto \frac{p^2}{m}$

$\therefore \frac{k_p}{k_\alpha} = \frac{p_p^2}{p_\alpha^2} \times \frac{m_\alpha}{m_p} = \frac{p_p^2}{p_\alpha^2} \times \frac{4m_p}{m_p}$

= 1 × 4 = 4 : 1

**Q.3** The angular momentum of a planet of mass M moving around the sun in an elliptical orbit is $\vec{L}$. The magnitude of the areal velocity of the planet is :

**Options**

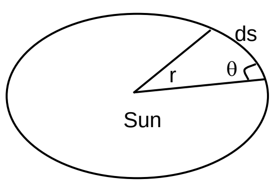
1. $\frac{L}{2M}$
2. $\frac{L}{M}$
3. $\frac{4L}{M}$
4. $\frac{2L}{M}$

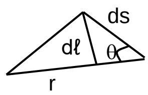
**Ans:** **1**

**Sol:** For a small displacement ds of the planet,

Area, dA = $\frac{1}{2}$ × base × height

= $\frac{1}{2} \times r \times d\ell$




$\sin\theta = \frac{d\ell}{ds}$

$\Rightarrow d\ell = ds\ \sin\theta$

$\therefore dA = \frac{1}{2} \times r \times ds \sin\theta$

Areal velocity = $\frac{dA}{dt} = \frac{1}{2} \times r \sin\theta \times \frac{ds}{dt} = \frac{1}{2} \times r \sin\theta \times V$

$= \frac{1}{2} \times \frac{m \times vr \sin\theta}{m} = \frac{L}{2m}$

**Q.4** Consider a uniform wire of mass M and length L. It is bent into a semicircle. Its moment of inertia about a line perpendicular to the plane of the wire passing through the centre is :

**Options**

1. $\frac{2}{5}\frac{ML^2}{\pi^2}$

2. $\frac{1}{4}\frac{ML^2}{\pi^2}$

3. $\frac{1}{2}\frac{ML^2}{\pi^2}$

4. $\frac{ML^2}{\pi^2}$

**Ans:** **4**

**Sol:** When the wire is bent into a semicircle,
Length of the wire → Perimeter of the semicircle.

$L = \pi r$

$\Rightarrow r = \frac{L}{\pi}$

$I = Mr^2 = M \times \left(\frac{L}{\pi}\right)^2$

$= \frac{ML^2}{\pi^2}$

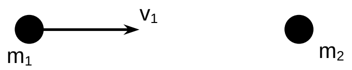
**Q.5** An object of mass $m_1$ collides with another object of mass $m_2$, which is at rest. After the collision the objects move with equal speeds in opposite direction. The ratio of the masses $m_2 : m_1$ is :

**Options**

1. 1 : 2
2. 3 : 1
3. 2 : 1
4. 1 : 1

**Ans:** **2**

**Sol:** **Before collision**




**After collision**

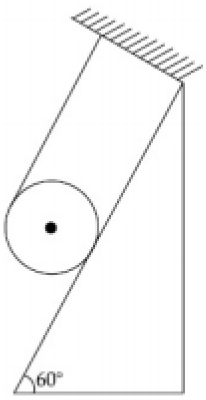

Applying law of conservation of momentum

$$m_1v_1 + m_2 \times 0 = -m_1v_2 + m_2v_2$$
$$m_1v_1 = -m_1v_2 + m_2v_2$$
$$v_1 = \frac{-m_1v_2 + m_2v_2}{m_1} = -v_2 + \frac{m_2}{m_1}v_2$$
$$v_1 + v_2 = \frac{m_2}{m_1}v_2$$
$$\frac{v_1 + v_2}{v_2} = \frac{m_2}{m_1}$$
$$e = \frac{2v_2}{v_1} = 1$$
$$\Rightarrow v_2 = \frac{v_1}{2}$$

Substituting in the expression of $v_1$,

$$v_1 = -\frac{v_1}{2} + \frac{m_2}{m_1} \times \frac{v_1}{2}$$
$$1 = -\frac{1}{2} + \frac{1}{2}\frac{m_2}{m_1}$$
$$\frac{1}{2}\frac{m_2}{m_1} = 1 + \frac{1}{2} = \frac{3}{2}$$
$$\frac{m_2}{m_1} = 3:1$$

**Q.6** A solid cylinder of mass m is wrapped with an inextensible light string and, is placed on a rough inclined plane as shown in the figure. The frictional force acting between the cylinder and the inclined plane is :




[The coefficient of static friction, $\mu_s$, is 0.4]

**Options**

1. $\frac{mg}{5}$
2. $0$
3. $\frac{7}{2}mg$
4. $5\ mg$

**Ans: 1**

**Sol:** If the solid cylinder is in equilibrium

T + f = mg sin 60 ------(1)

TR – fR = 0 -------------(2)

Solving (1) and (2)

$T = f_{req} = \frac{mg\sin\theta}{2}$

But limiting friction $< f_{req}$

$\mu mg\cos 60 = \frac{mg\sin 60}{2}$

$\therefore$ the solid cylinder will not remain in equilibrium

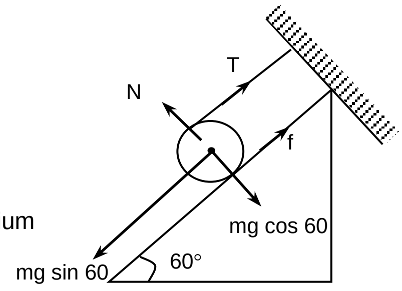
$\therefore$ friction is kinetic

$f_k = \mu_k N$

$= \mu_k mg\cos 60$

$= \mu_k mg \times \frac{1}{2}$

$= \frac{0.4mg}{2} = \frac{mg}{5}$




**Q.7** In a series LCR circuit, the inductive reactance ($X_L$) is 10 Ω and the capacitive reactance ($X_C$) is 4 Ω. The resistance (R) in the circuit is 6 Ω.

The power factor of the circuit is :

**Options**

1. $\frac{1}{2}$
2. $\frac{\sqrt{3}}{2}$
3. $\frac{1}{2\sqrt{2}}$
4. $\frac{1}{\sqrt{2}}$

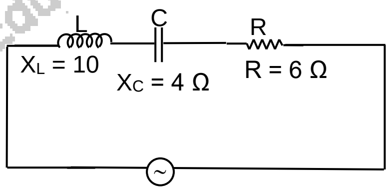
**Ans: 4**

**Sol**: $\cos\phi = \frac{R}{Z}$

Where $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{6^2 + (10-4)^2}$

$= \sqrt{6^2 + 6^2} = 6\sqrt{2}$

$\therefore \cos\phi = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}}$






**Q.8** The speed of electrons in a scanning electron microscope is $1 \times 10^7$ ms$^{-1}$. If the protons having the same speed are used instead of electrons, then the resolving power of scanning proton microscope will be changed by a factor of :

**Options**

1. 1837
2. $\frac{1}{\sqrt{1837}}$
3. $\frac{1}{1837}$
4. $\sqrt{1837}$

**Ans:** **1**

**Sol:** $\lambda = \frac{h}{p} = \frac{h}{mv}$

Resolving power $\propto \frac{1}{\lambda}$

$RP \propto \frac{mv}{h} \Rightarrow RP \propto m$

$\Rightarrow 1837$

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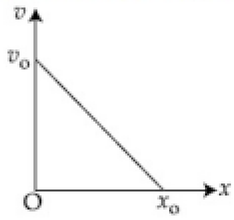
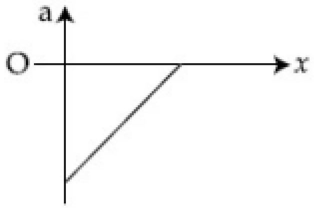
**Q.9** The velocity - displacement graph of a particle is shown in the figure.




The acceleration - displacement graph of the same particle is represented by :

**Options**

1.
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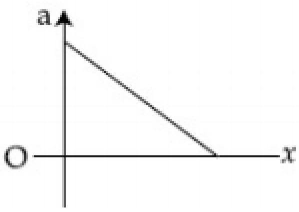


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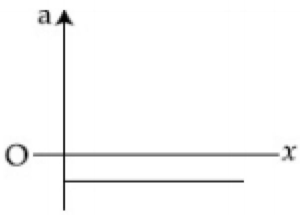


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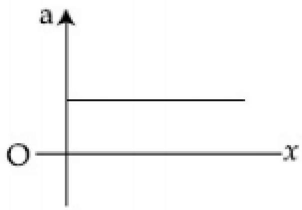



**Ans:** **1**

**Sol:** $v = -\left(\frac{v_0}{x_0}\right)x + v_0$

$$a = v\frac{dv}{x}$$

$$= \left[-\left(\frac{v_0}{x_0}\right)x + v_0\right]\left[-\frac{v_0}{x_0}\right]$$

$$a = \left[\frac{v_0}{x_0}\right]^2 x - \frac{{v_0}^2}{x_0}$$

**Q.10** The correct relation between $\alpha$ (ratio of collector current to emitter current) and $\beta$ (ratio of collector current to base current) of a transistor is :

**Options**

1. $\beta = \frac{\alpha}{1+\alpha}$
2. $\alpha = \frac{\beta}{1+\beta}$
3. $\beta = \frac{1}{1-\alpha}$
4. $\alpha = \frac{\beta}{1-\alpha}$

**Ans: 2**

**Sol:** $\alpha = \frac{I_C}{I_E}$ and $\beta = \frac{I_C}{I_B}$

$$I_E = I_B + I_C$$

$$\alpha = \frac{I_C}{I_B + I_C} = \frac{1}{\frac{I_B}{I_C} + \frac{I_C}{I_C}} = \frac{1}{\frac{I_B}{I_C} + 1}$$

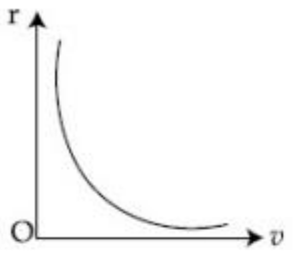
$$= \frac{1}{\frac{1}{\beta} + 1} = \frac{\beta}{1+\beta}$$

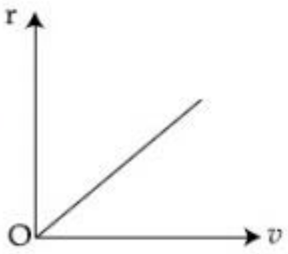
**Q.11**

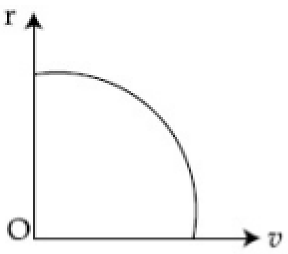
A particle of mass m moves in a circular orbit under the central potential field, $U(r) = -\frac{C}{r}$, where C is a positive constant.

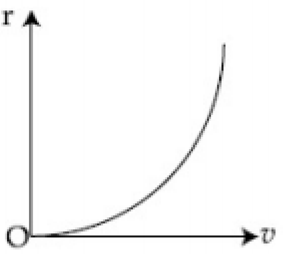
The correct radius – velocity graph of the particle's motion is :

**Options**

1. 


2. 


3. 


4. 




**Ans: 1**

**Sol:** $U = \frac{-C}{r}$

$F = \frac{-du}{dr} = \frac{-C}{r^2}$

$|F| = \frac{mv^2}{r}$

$\frac{C}{r^2} = \frac{mv^2}{r}$

$\Rightarrow v^2 \propto \frac{1}{r}$

**Q.12** Consider a sample of oxygen behaving like an ideal gas. At 300 K, the ratio of root mean square (rms) velocity to the average velocity of gas molecule would be :
(Molecular weight of oxygen is 32 g/mol; $R = 8.3\ J\ K^{-1}\ mol^{-1}$)

**Options**

1. $\sqrt{\frac{3\pi}{8}}$

2. $\sqrt{\frac{8}{3}}$

3. $\sqrt{\frac{8\pi}{3}}$

4. $\sqrt{\frac{3}{3}}$

**Ans: 1**

**Sol:** $v_{rms} = \sqrt{\frac{3RT}{M}}$

$v_{arg} = \sqrt{\frac{8RT}{\pi M}}$

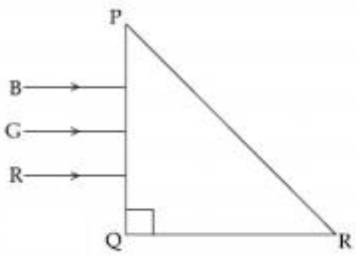
$\frac{v_{rms}}{v_{avg}} = \sqrt{\frac{3\pi}{8}}$

**13.**

**Ans: 4**

**Sol:** Magnetic field outside a toroid is zero and they form closed loops inside the toroid itself. Hence, it is correct that magnetic field lines are completely confined within a toroid. For a perfect diamagnetic material $\chi = 1$

**Q.14** Three rays of light, namely red (R), green (G) and blue (B) are incident on the face PQ of a right angled prism PQR as shown in the figure.



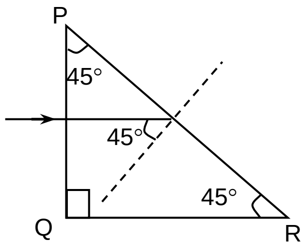

The refractive indices of the material of the prism for red, green and blue wavelength are 1.27, 1.42 and 1.49 respectively. The colour of the ray(s) emerging out of the face PR is :

**Options**
1. red
2. green
3. blue and green
4. blue

**Ans:** 1

**Sol:** The wavelength corresponding to which the incident angle is less than the critical angle will pass through the face PR

Critical angle $\theta_c = \sin^{-1}\left(\frac{1}{\mu}\right)$




If $\theta_c \geq 45°$, the light ray will pass.

$(\theta_c)_{red} = \sin^{-1}\left(\frac{1}{1.27}\right) = 51.94° > 45°$

$\therefore$ Red will pass

$(\theta_c)_{green} = \sin^{-1}\left(\frac{1}{1.42}\right) = 44.76° < 45°$

Hence, green will not pass

$(\theta_c)_{blue} = \sin^{-1}\left(\frac{1}{1.49}\right) = 42.15° < 45°$

Hence, blue will not pass

**Q.15** The decay of a proton to neutron is :

**Options**
1. not possible as proton mass is less than the neutron mass
2. always possible as it is associated only with $\beta^+$ decay
3. not possible but neutron to proton conversion is possible
4. possible only inside the nucleus

**Ans:** 4

**Sol:** **Option (4)**

**Q.16** The time taken for the magnetic energy to reach 25% of its maximum value, when a solenoid of resistance R, inductance L is connected to a battery, is :

**Options**
1. infinite
2. $\frac{L}{R}\ln 10$
3. $\frac{L}{R}\ln 5$
4. $\frac{L}{R}\ln 2$

**Ans:** 4

**Sol:** Magnetic energy = $\frac{1}{2}Li^2$

Given magnetic energy increases to 25% of maximum value

$$\frac{1}{2}Li^2 = \frac{25}{100} \times \frac{1}{2}L\ i_0^2$$

$$= \frac{1}{4} \times \frac{1}{2}L\,i_0^2$$

$$i^2 = \frac{i_0^2}{4}$$

$$\Rightarrow i = \frac{i_0}{2}$$

For charging $i = i_0\left(1 - R^{\frac{-Rt}{L}}\right)$

$$\Rightarrow t = \frac{L}{R}\ell n2$$

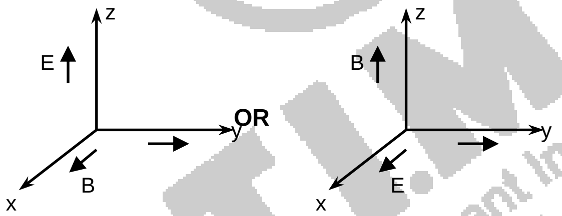
**Q.17** A plane electromagnetic wave propagating along *y*-direction can have the following pair of electric field $(\vec{E})$ and magnetic field $(\vec{B})$ components.

**Options**

1. $E_x, B_y$ or $E_y, B_x$
2. $E_y, B_y$ or $E_z, B_z$
3. $E_y, B_x$ or $E_x, B_y$
4. $E_x, B_z$ or $E_z, B_x$

**Ans: 4**

**Sol:**




**Q.18** The function of time representing a simple harmonic motion with a period of $\frac{\pi}{\omega}$ is :

**Options**

1. $\sin(\omega t) + \cos(\omega t)$
2. $3\cos\left(\frac{\pi}{4} - 2\omega t\right)$
3. $\cos(\omega t) + \cos(2\omega t) + \cos(3\omega t)$
4. $\sin^2(\omega t)$

**Ans: 2**

**Sol:** Time period, $T = \frac{2\pi}{\omega_0}$

Given time period = $\frac{\pi}{\omega}$

$\frac{\pi}{\omega} = \frac{2\pi}{\omega_0}$

$\Rightarrow \omega_0 = 2\omega \Rightarrow$ option (2) have angular frequency $2\omega$

Now, consider option (4)

$\sin^2 \omega t \rightarrow \frac{1}{2} \times 2\sin^2 \omega t = \frac{1}{2} \times 2\left(\frac{1-\cos 2\omega t}{2}\right)$

$= \frac{1}{2}(1-\cos 2\omega t) \Rightarrow$ angular frequency = $2\omega$

Both (2) and (4) has angular frequency $2\omega$, but (2) gives the direct answer.

**Q.19** An ideal gas in a cylinder is separated by a piston in such a way that the entropy of one part is $S_1$ and that of the other part is $S_2$. Given that $S_1 > S_2$. If the piston is removed then the total entropy of the system will be :

**Options**

1. $\frac{S_1}{S_2}$
2. $S_1 - S_2$
3. $S_1 + S_2$
4. $S_1 \times S_2$

**Ans:** **3**

**Sol:** Entropy is an extensive property. So it can be additive.

$\therefore$ entropy of the system after removing the partition = $S_1 + S_2$

**Q.20** For an adiabatic expansion of an ideal gas, the fractional change in its pressure is equal to (where $\gamma$ is the ratio of specific heats) :

**Options**

1. $-\gamma \frac{dV}{V}$
2. $-\frac{1}{\gamma} \frac{dV}{V}$
3. $\frac{dV}{V}$
4. $-\gamma \frac{V}{dV}$

**Ans:** **1**

**Sol:** For an adiabatic process,

$pv^\gamma$ = constant

On differentiation

$P\gamma dV + V^\gamma dP = 0$

$P\gamma dV = -V^\gamma dP$

$\frac{dP}{P} = -\frac{\gamma dV}{V}$

## Section B

**Q.1** A galaxy is moving away from the earth at a speed of 286 $kms^{-1}$. The shift in the wavelength of a redline at 630 nm is $x \times 10^{-10}$ m.

The value of $x$, to the nearest integer, is __________.

[Take the value of speed of light c, as $3 \times 10^8$ $ms^{-1}$]

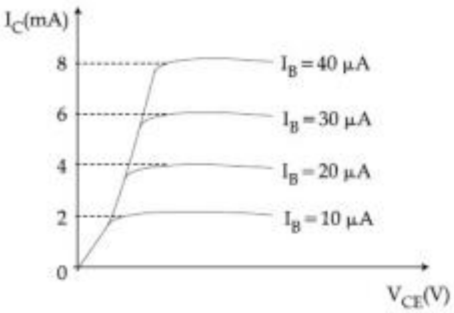
**Ans: 6.00**

**Sol:** $\frac{\Delta\lambda}{\lambda} C = V$

$$\Delta\lambda = \frac{V}{C}\lambda$$

$$= \frac{286}{3 \times 10^5} \times 630 \times 10^{-9} = 6 \times 10^{-10}$$

**Q.2** The typical output characteristics curve for a transistor working in the common-emitter configuration is shown in the figure.




The estimated current gain from the figure is _________.

**Ans: 200.00**

**Sol:** $\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{2 \times 10^{-3}}{10 \times 10^{-6}}$

$$\beta = \frac{1}{5} \times 10^3$$

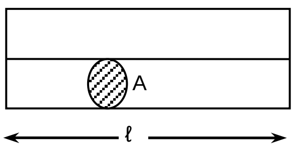
$$\beta = 2 \times 10^2 = 200$$

**Q.3** Two wires of same length and thickness having specific resistances 6 Ω cm and 3 Ω cm respectively are connected in parallel. The effective resistivity is ρ Ω cm. The value of ρ, to the nearest integer, is __________.

**Ans: 4.00**

**Sol:** In parallel

$$R_{eff} = \frac{R_1 R_2}{R_1 + R_2}$$

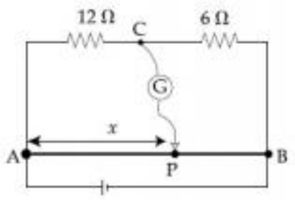



$$\frac{\rho\ell}{2A} = \frac{\rho_1 \frac{\ell}{A} \times \rho_2 \frac{\ell}{A}}{\rho_1 \frac{\ell}{A} + \rho_2 \frac{\ell}{A}}$$

$$\frac{\rho}{2} = \frac{6 \times 3}{6 + 3} = 2$$

$$\rho = 4$$

**Q.4** Consider a 72 cm long wire AB as shown in the figure. The galvanometer jockey is placed at P on AB at a distance $x$ cm from A. The galvanometer shows zero deflection.




The value of $x$, to the nearest integer, is ________.

**Ans: 48.00**

**Sol:** In balanced condition

$$\frac{12}{6} = \frac{x}{72 - x} \Rightarrow x = 48\text{cm}$$

**Q.5** An infinite number of point charges, each carrying 1 μC charge, are placed along the $y$-axis at $y = 1$ m, 2 m, 4 m, 8 m ............... .

The total force on a 1 C point charge, placed at the origin, is $x \times 10^3$ N.

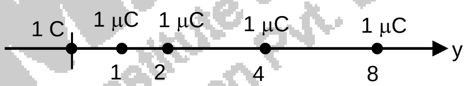
The value of $x$, to the nearest integer, is ________.

[Take $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9\ Nm^2/C^2$]

**Ans: 12.00**

**Sol:**
$$F = k(1C)(1\mu C)\left[1 + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{8^2} + .........\right]$$

$$= 9 \times 10^3\left[\frac{1}{1 - \frac{1}{4}}\right] = 12 \times 10^3 N$$




**Q.6** The radius of a sphere is measured to be $(7.50 \pm 0.85)$ cm. Suppose the percentage error in its volume is $x$.

The value of $x$, to the nearest $x$, is ________.

**Ans: 34.00**

**Sol:** We have $V = \frac{4}{3}\pi r^3$

Taking log and differentiating

$$\frac{dv}{v} = 3\frac{dr}{r}$$

$$= \frac{3 \times 0.85}{7.5} \times 100\% = 34\%$$

**Q.7** A TV transmission tower antenna is at a height of 20 m. Suppose that the receiving antenna is at.

(i) ground level

(ii) a height of 5 m.

The increase in antenna range in case (ii) relative to case (i) is n%.

The value of n, to the nearest integer, is ________.

**Ans: 50.00**

**Sol**: Range = $\sqrt{2Rh}$

Range (i) = $\sqrt{2Rh}$

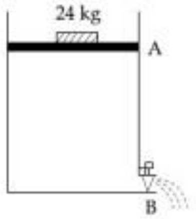
Range (ii) = $\sqrt{2Rh}$ + $\sqrt{2Rh'}$

$$n\% = \frac{\sqrt{2Rh'}}{\sqrt{2Rh}} \times 100\% = \frac{\sqrt{5}}{\sqrt{20}} \times 100 = 50\%$$

**Q.8** Consider a water tank as shown in the figure. It's cross-sectional area is 0.4 $m^2$. The tank has an opening B near the bottom whose cross-section area is 1 $cm^2$. A load of 24 kg is applied on the water at the top when the height of the water level is 40 cm above the bottom, the velocity of water coming out the opening B is $v$ $ms^{-1}$.

The value of $v$, to the nearest integer, is __________.

[Take value of g to be 10 $ms^{-2}$]




**Ans: 3.00**

**Sol:** Using Bernoulli's equation

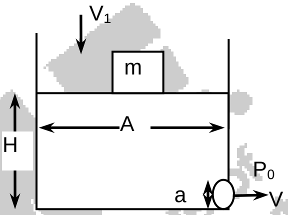
$$\left(P_0 + \frac{mg}{A}\right) + \rho gH + \frac{1}{2}\rho V_1^2 = P_0 + 0 + \frac{1}{2}\rho V^2$$

Neglecting $V_1$

$$\Rightarrow V = \sqrt{2gH + \frac{2mg}{A\rho}}$$

$$V = \sqrt{8 + 1.2} = 3.033 \text{ m/s}$$

i.e., $V \approx 3$ m/s




**Q.9** A ball of mass 4 kg, moving with a velocity of 10 $ms^{-1}$, collides with a spring of length 8 m and force constant 100 $Nm^{-1}$. The length of the compressed spring is $x$ m. The value of $x$, to the nearest integer, is _________.

**Ans: 6.00**

**Sol:** We have, according to work energy theorem

$$\frac{1}{2}mV^2 = \frac{1}{2}k\,y^2$$

$$y = \sqrt{\frac{m}{k}}V$$

$$y = \sqrt{\frac{4}{100}} \times 10$$

y = 2 m

i.e., final length of spring = 8 – 2 = 6 m

**Q.10** The projectile motion of a particle of mass 5 g is shown in the figure.


The initial velocity of the particle is $5\sqrt{2}\ ms^{-1}$ and the air resistance is assumed to be negligible. The magnitude of the change in momentum between the points A and B is $x \times 10^{-2}\ kgms^{-1}$. The value of $x$, to the nearest integer, is ________.

**Ans: 5.00**

**Sol:** $|\vec{u}| = |\vec{v}|$

$\vec{u} = u\cos 45 \uparrow + u\sin 45\hat{j}$

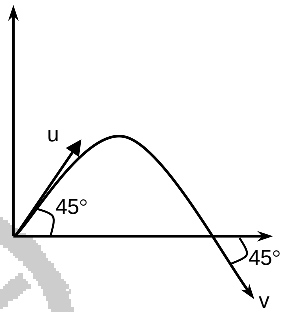
$\vec{v} = v\cos 45 \uparrow - v\sin 45\hat{j}$

$|\Delta\vec{P}| = |m(\vec{v} - \vec{u})|$

$\Delta P = 2mu\sin 45°$

$= 2 \times 5 \times 10^{-3} \times 5\sqrt{2} \times \frac{1}{\sqrt{2}}$

$= 50 \times 10^{-3} = 5 \times 10^{-2}$




# PART – B – CHEMISTRY

## SECTION A

**Q.1** In the reaction of hypobromite with amide, the carbonyl carbon is lost as :

**Options**
1. $CO_3^{2-}$
2. $CO_2$
3. $CO$
4. $HCO_3^-$

**Ans: 1**

**Sol:** $R-CO-NH_2 \xrightarrow{KOH/Br_2} RNH_2 + K_2CO_3 + 2KBr + 2H_2O$

Carbonyl carbon is lost as carbonate ion $(CO_3^{2-})$

**Q.2** In basic medium, $H_2O_2$ exhibits which of the following reactions ?

(A) $Mn^{2+} \rightarrow Mn^{4+}$

(B) $I_2 \rightarrow I^-$

(C) $PbS \rightarrow PbSO_4$

Choose the **most appropriate** answer from the options given below :

**Options**
1. (A), (B) only
2. (A) only
3. (B) only
4. (A), (C) only

**Ans:** 1

**Sol:** $PbS + 4H_2O \rightarrow PbSO_4 + 4H_2O$

The reaction is takes place in the acidic medium

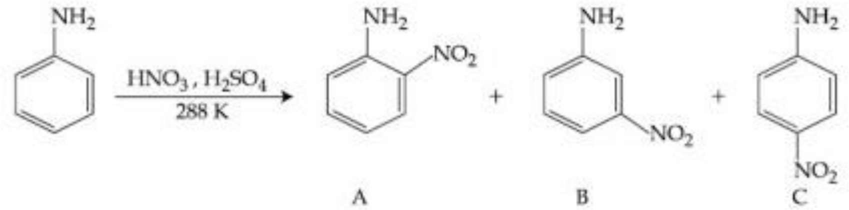
**Q.3** Deficiency of vitamin K causes :

**Options**
1. Increase in blood clotting time
2. Increase in fragility of RBC's
3. Decrease in blood clotting time
4. Cheilosis

**Ans:** 1

**Sol:** Deficiency of vitamin K increase in blood clotting lime

**Q.4**

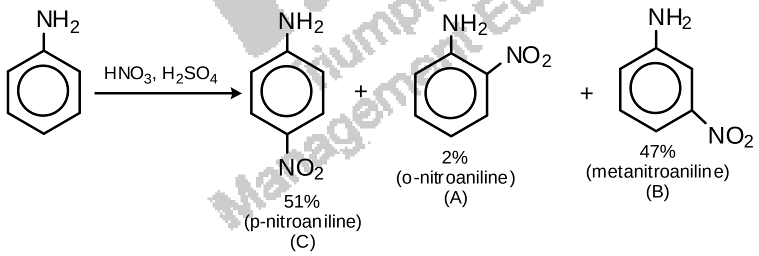



Consider the given reaction, percentage yield of :

**Options**
1. C > A > B
2. B > C > A
3. A > C > B
4. C > B > A

**Ans:** 4

**Sol:**



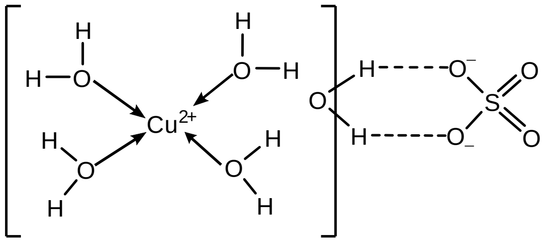

C > B > A

**Q.5** The secondary valency and the number of hydrogen bonded water molecule(s) in $CuSO_4 \cdot 5H_2O$, respectively, are :

**Options**
1. 5 and 1
2. 6 and 5
3. 4 and 1
4. 6 and 4

**Ans:** 3

**Sol:**




Secondary valency 4 and number of hydrogen bonded water molecule-1

Q.6 Match List - I with List - II :

| | List - I (Class of Chemicals) | | List - II (Example) |
|---|---|---|---|
| (a) | Antifertility drug | (i) | Meprobamate |
| (b) | Antibiotic | (ii) | Alitame |
| (c) | Tranquilizer | (iii) | Norethindrone |
| (d) | Artificial Sweetener | (iv) | Salvarsan |

Choose the most appropriate match :

Options
1. (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)
2. (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)
3. (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)
4. (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

**Ans:** 1

**Sol:**
(a) Antifertility Drug – Norethindrone
(b) Antibiotic – Salvarasan
(c) Tranquilizer – Meprobamate
(d) Artificial sweetener – Alitame



Q.7 Match List - I with List - II :

| | List - I | | List - II |
|---|---|---|---|
| (a) | Be | (i) | treatment of cancer |
| (b) | Mg | (ii) | extraction of metals |
| (c) | Ca | (iii) | incendiary bombs and signals |
| (d) | Ra | (iv) | windows of X-ray tubes |
| | | (v) | bearings for motor engines. |

Choose the most appropriate answer from the option given below :

Options
1. (a)-(iii), (b)-(iv), (c)-(ii), (d)-(v)
2. (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)
3. (a)-(iii), (b)-(iv), (c)-(v), (d)-(ii)
4. (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

**Ans:** 4

**Sol:** (a) Be – Windows of X-ray tubes
(b) Mg – Incendiary bombs and signals
(c) Ca – Extraction of metals
(d) Ra – Treatment of cancer

**Q.8** Given below are two statements :

**Statement I :** $C_2H_5OH$ and AgCN both can generate nucleophile.

**Statement II :** KCN and AgCN both will generate nitrile nucleophile with all reaction conditions.

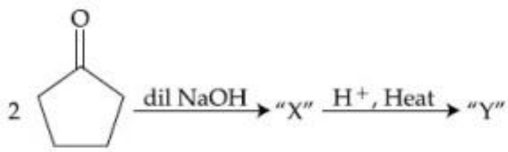
Choose the **most appropriate** option :

**Options**
1. Both statement I and statement II are true.
2. Statement I is false but statement II is true.
3. Statement I is true but statement II is false.
4. Both statement I and statement II are false.

**Ans:** **3**

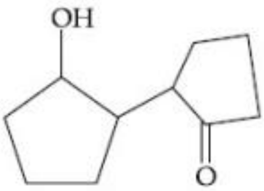
**Sol:** Option (3)

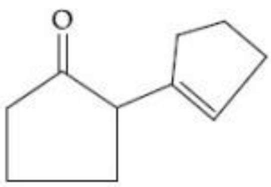
**Q.9**

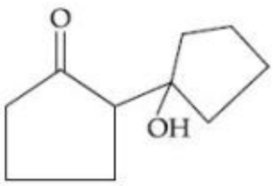


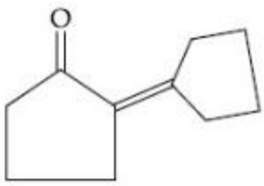

Consider the above reaction, the product 'X' and 'Y' respectively are :

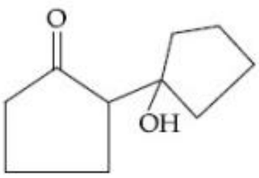
**Options**

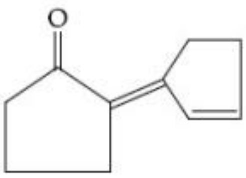
1. 

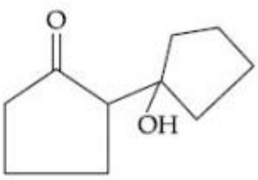
, 

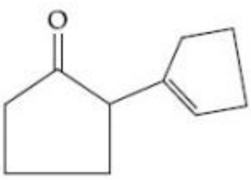

2. 

, 

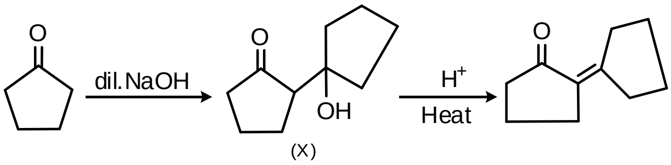

3. 

, 


4. 

, 


**Ans:** **2**

**Sol:** 


**Q.10** The oxide that shows magnetic property is :

**Options**
1. MgO
2. $SiO_2$
3. $Mn_3O_4$
4. $Na_2O$

**Ans:** 3

**Sol:** In $Mn_3O_4$, Mn is in +4 oxidation state, presence of unpaired electron make the compound paramagnetic

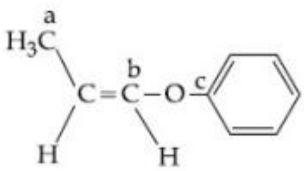
**Q.11** The oxidation states of nitrogen in NO, $NO_2$, $N_2O$ and $NO_3^-$ are in the order of :

**Options**
1. $NO > NO_2 > N_2O > NO_3^-$
2. $N_2O > NO_2 > NO > NO_3^-$
3. $NO_2 > NO_3^- > NO > N_2O$
4. $NO_3^- > NO_2 > NO > N_2O$

**Ans:** 4

**Sol:** $\overset{+5}{NO_3^-} > \overset{+4}{NO_2} > \overset{+2}{NO} > \overset{+1}{N_2O}$

**Q.12** In the following molecule,




Hybridisation of Carbon a, b and c respectively are :

**Options**
1. $sp^3, sp^2, sp^2$
2. $sp^3, sp^2, sp$
3. $sp^3, sp, sp^2$
4. $sp^3, sp, sp$

**Ans:** 1

**Sol:** a – $sp^3$
b – $sp^2$
c – $sp^2$

---

**Q.13** Match List - I with List - II :

| | List - I | | List - II |
|---|---|---|---|
| (a) | Mercury | (i) | Vapour phase refining |
| (b) | Copper | (ii) | Distillation Refining |
| (c) | Silicon | (iii) | Electrolytic Refining |
| (d) | Nickel | (iv) | Zone Refining |

Choose the **most appropriate** answer from the option given below :

Options
1. (a)-(i), (b)-(iv), (c)-(ii), (d)-(iii)
2. (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)
3. (a)-(ii), (b)-(iii), (c)-(i), (d)-(iv)
4. (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

**Ans:** **4**

**Sol:** (a) Mercury – Distillation
(b) Copper – Electrolytic refining
(c) Silicon – Zone refining
(d) Nickel – Vapour phase refining

**Q.14** Given below are two statements :

**Statement I :** Non-biodegradable wastes are generated by the thermal power plants.

**Statement II :** Bio-degradable detergents leads to eutrophication.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

Options
1. Statement I is false but statement II is true.
2. Both statement I and statement II are false.
3. Both statement I and statement II are true.
4. Statement I is true but statement II is false.

**Ans:** **3**

**Sol:** Both statement I and II are correct

**Q.15** The first ionization energy of magnesium is smaller as compared to that of elements X and Y, but higher than that of Z. The elements X, Y and Z, respectively, are :

Options
1. argon, lithium and sodium
2. neon, sodium and chlorine
3. chlorine, lithium and sodium
4. argon, chlorine and sodium

**Ans:** **4**

**Sol:** Argon (X) and chlorine (Y) has high IE than Mg but sodium (Z) lower IE than Mg

**Q.16** The charges on the colloidal CdS sol and $TiO_2$ sol are, respectively :

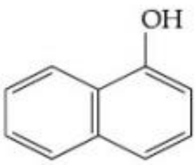
**Options**
1. negative and negative
2. positive and negative
3. positive and positive
4. negative and positive

**Ans:** 4

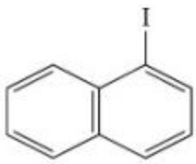
**Sol:** CdS is negatively charged sol
$TiO_2$ is a positively charged sol

**Q.17** Main Products formed during a reaction of 1-methoxy naphthalene with hydroiodic acid are :

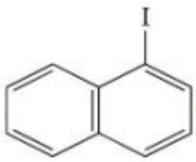
**Options**

1. 

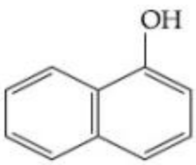
and $CH_3I$

2. 

and $CH_3OH$

3. 

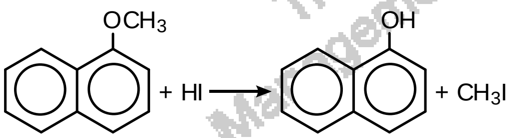
and $CH_3I$

4. 

and $CH_3OH$

**Ans:** 1

**Sol:**




**Q.18** Given below are two statements :

**Statement I :** Bohr's theory accounts for the stability and line spectrum of $Li^+$ ion.

**Statement II :** Bohr's theory was unable to explain the splitting of spectral lines in the presence of a magnetic field.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

**Options**
1. Both statement I and statement II are false.
2. Both statement I and statement II are true.
3. Statement I is false but statement II is true.
4. Statement I is true but statement II is false.

**Ans:** 3

**Sol:** Bohr's theory not explain the stability of multi electronic species it only explain the stability of hydrogen like (unielectronic) species

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**Q.19** An organic compound "A" on treatment with benzene sulphonyl chloride gives compound B. B is soluble in dil. NaOH solution. Compound A is :

**Options**
1. $C_6H_5-CH_2NHCH_3$
2. $C_6H_5-NHCH_2CH_3$
3. $C_6H_5-N-(CH_3)_2$
4. $C_6H_5-\underset{\displaystyle CH_3}{\underset{|}{CH}}-NH_2$

**Ans:** 4

**Sol:** Benzene sulphonyl chloride (Hinsberg reagent) give product with primary amine and it is soluble in NaOH

**Q.20** A hard substance melts at high temperature and is an insulator in both solid and in molten state. This solid is most likely to be a/an :

**Options**
1. Molecular solid
2. Ionic solid
3. Metallic solid
4. Covalent solid

**Ans:** 4

**Sol:** Covalent solids melt at high temperature, and it is an insulator in both solid and in molten state e.g., silicon dioxide

## SECTION B

**Q.1** The molar conductivities at infinite dilution of barium chloride, sulphuric acid and hydrochloric acid are 280, 860 and 426 S cm$^2$ mol$^{-1}$ respectively. The molar conductivity at infinite dilution of barium sulphate is _________ S cm$^2$ mol$^{-1}$. (Round off to the Nearest Integer).

**Ans:** 288

**Sol:** $\lambda^\circ_{m(BaSO_4)} = \lambda^\circ_{m(BaCl_2)} + \lambda^\circ_{m(H_2SO_4)} - 2\lambda^\circ_{m(HCl)} = 280 + 860 - 2(426) = 288$

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**Q.2** The solubility of $CdSO_4$ in water is $8.0 \times 10^{-4}$ mol L$^{-1}$. Its solubility in 0.01 M $H_2SO_4$ solution is _________ $\times 10^{-6}$ mol L$^{-1}$. (Round off to the Nearest Integer).
(Assume that solubility is much less than 0.01 M)

**Ans:** 64

**Sol:** $CdSO_4 \rightleftharpoons \underset{S}{Cd^{2+}} + \underset{S}{SO_4^{2-}}$

$K_{sp} = S^2 = (8 \times 10^{-4})^2 = 64 \times 10^{-8}$

$\underset{0.01\ M}{H_2SO_4} \rightleftharpoons \underset{2 \times 0.1\ M}{2H^+} + \underset{(S + 0.01) \approx (0.01)}{SO_4^{2-}}$

$CdSO_4 \rightleftharpoons Cd^{2+} + SO_4^{2-}$

S (S + 0.01) ≈ (0.01)

$64 \times 10^{-8} = S \times 0.01$

$S = 64 \times 10^{-6}$

**Q.3** The number of species below that have two lone pairs of electrons in their central atom is ________. (Round off to the Nearest Integer).

$SF_4$, $BF_4^-$, $ClF_3$, $AsF_3$, $PCl_5$, $BrF_5$, $XeF_4$, $SF_6$

Given 2.00

**Ans:** 2

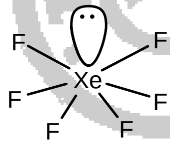
**Sol:**

| | | |
|---|---|---|
| $SF_4$ | 1 lone pair | 4 bond pair |
| $BF_4^-$ | 4 bond pair only | |
| $ClF_3$ | 2 lone pair | 3 bond pair |
| $AsF_3$ | 1 lone pair | 3 bond pair |
| $PCl_5$ | 5 bond pair only | |
| $XeF_4$ | 2 lone pair | 4 bond pair |
| $SF_6$ | 6 bond pair only | |

**Q.4** A xenon compound 'A' upon partial hydrolysis gives $XeO_2F_2$. The number of lone pair of electrons present in compound A is ________. (Round off to the Nearest Integer)

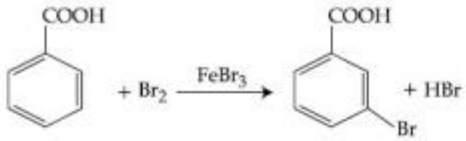
**Ans:** 19

**Sol:** $XeF_6$ -




The total number of lone pair = 1 lone from Xe +3 lone pair from each F (6 × 3 = 18)
= 19 lone pair

**Q.5**



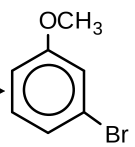

Consider the above reaction where 6.1 g of Benzoic acid is used to get 7.8 g of m-bromo benzoic acid. The percentage yield of the product is ________.

(Round off to the Nearest Integer).

[Given : Atomic masses : C : 12.0 u, H : 1.0 u, O : 16.0 u, Br : 80.0 u]

**Ans:** 78

**Sol:** $C_6H_5\text{—}COOH + Br_2 \xrightarrow{FeBr_3}$




+ HBr

122 g (1 mole) ⟶ 201 g (1 mole)

6.1 g ⟶ x

$$x = \frac{201 \times 6.1}{122} = 10.5 \approx 10$$

$$\therefore \% \text{ yield of product} = \frac{7.8}{10} \times 100 = 78$$

**Q.6** A solute A dimerizes in water. The boiling point of a 2 molal solution of A is 100.52°C. The percentage association of A is ________. (Round off to the Nearest Integer).

[Use : $K_b$ for water = 0.52 K kg mol$^{-1}$

Boiling point of water = 100°C]

**Ans: 100**

**Sol:** $\Delta T_b = i \times K_b \times m$

$0.52 = i \times 52 \times 2$

$i = \frac{1}{2}$

$\alpha = \frac{i-1}{1-\frac{1}{n}}$

$\alpha = \frac{\frac{1}{2}-1}{1-\frac{1}{2}} \Rightarrow 1$

α = 1 % of dissociation = 100

**Q.7** In Tollen's test for aldehyde, the overall number of electron(s) transferred to the Tollen's reagent formula $[Ag(NH_3)_2]^+$ per aldehyde group to form silver mirror is ________. (Round off to the Nearest Integer).

**Ans: 2**

**Sol:** $RCHO + 2[Ag(NH_3)_2]^+ + 3OH^- \rightarrow RCOO^- + 2Ag\downarrow + 4NH_3 + 2H_2O$

The overall number of electrons is transferred per aldehyde group = 2

**Q.8** 10.0 mL of $Na_2CO_3$ solution is titrated against 0.2 M HCl solution. The following titre values were obtained in 5 readings :

4.8 mL, 4.9 mL, 5.0 mL, 5.0 mL and 5.0 mL.

Based on these readings, and convention of titrimetric estimation the concentration of $Na_2CO_3$ solution is ________ mM.

(Round off to the Nearest Integer).

**Ans: 50**

**Sol:** $N_{(Na_2CO_3)} \times C_{(Na_2CO_3)} = N_{(HCl)} \times V_{(HCl)}$

$N_{(Na_2CO_3)} \times 10 = -0.2 \times 5$

Normality of $Na_2CO_3$ = $\frac{1}{10} = 0.1N = 0.05M$

Concentration of $Na_2CO_3$ solution = 50 milli moles

**Q.9** A reaction has a half life of 1 min. The time required for 99.9% completion of the reaction is ________ min. (Round off to the Nearest Integer).

[Use : ln 2 = 0.69; ln 10 = 2.3]

**Ans: 10**

**Sol:** For first order reaction $t_{99.9\%} = 10t_{1/2}$

$t_{99.9\%} = 10 \times 1 = 10$ min

**Q.10** The gas phase reaction

$$2A(g) \rightleftharpoons A_2(g)$$

at 400 K has $\Delta G^\circ = +25.2$ kJ $mol^{-1}$.

The equilibrium constant $K_C$ for this reaction is ________ $\times 10^{-2}$. (Round off to the Nearest Integer).

[Use : $R = 8.3$ J $mol^{-1}$ $K^{-1}$, ln 10 = 2.3

$\log_{10} 2 = 0.30$, 1 atm = 1 bar]

[antilog $(-0.3) = 0.501$]

**Ans: 166**

**Sol:** $\Delta G^\circ = -2.303$ RT log $K_p$

$25.2 = -2.3 \times 8.3 \times 10^{-3} \times 400 \times \log K_p$

$$\log K_p = \frac{-25.2}{2.3 \times 8.3 \times 10^{-3} \times 400} = -3.31$$

$K_p = 5 \times 10^{-4}$

$K_p = K_c (RT)^{\Delta n}$

$$K_c = \frac{K_p}{(RT)^{-1}} = \frac{5 \times 10^{-1}}{(8.3 \times 400)^{-1}} \Rightarrow 16600 \times 10^{-4} \Rightarrow 166 \times 10^{-2}$$

# PART – C – MATHEMATICS

## SECTION A

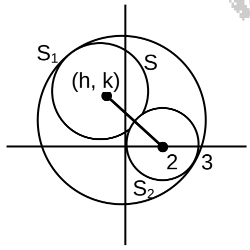
**Q.1** Let $S_1 : x^2 + y^2 = 9$ and $S_2 : (x-2)^2 + y^2 = 1$. Then the locus of center of a variable circle S which touches $S_1$ internally and $S_2$ externally always passes through the points :

**Options**

1. $\left(0, \pm \sqrt{3}\right)$
2. $(1, \pm 2)$
3. $\left(\frac{1}{2}, \pm \frac{\sqrt{5}}{2}\right)$
4. $\left(2, \pm \frac{3}{2}\right)$

**Ans: 4**

**Sol:**




Centre of $S_1 : C_1(0,0)$, radius = 3

Centre of $S_2 : C_2(2,0)$, radius = 1

Centre of $S : C_3(h,k)$, radius = r

$$C_2C_3 = \sqrt{(2-h)^2 + k^2} = r + 1 \_\_\_\_\_\_\_ (1)$$

$C_1C_3 = \sqrt{h^2 + k^2} = 3 - r$ ________(2)

$\sqrt{(2-h)^2 + k^2} = 3 - \sqrt{h^2 + k^2} + 1$

$\Rightarrow \sqrt{(2-h)^2 + k^2} = 4 - \sqrt{h^2 + k^2}$

$\Rightarrow \sqrt{h^2 + k^2} + \sqrt{(2-h)^2 + k^2} = 4$

Clearly $\left(2, \pm\frac{3}{2}\right)$ satisfies the above equation

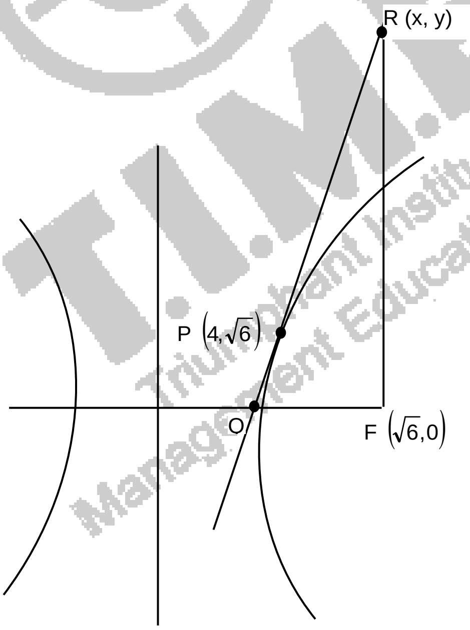
**Q.2** Consider a hyperbola $H : x^2 - 2y^2 = 4$. Let the tangent at a point $P\ (4, \sqrt{6})$ meet the $x$-axis at Q and latus rectum at $R\ (x_1, y_1)$, $x_1 > 0$. If F is a focus of H which is nearer to the point P, then the area of $\Delta QFR$ is equal to .

**Options**

1. $\frac{7}{\sqrt{6}} - 2$
2. $\sqrt{6} - 1$
3. $4\sqrt{6}$
4. $4\sqrt{6} - 1$

**Ans: 1**

**Sol:**




$\frac{x^2}{4} - \frac{y^2}{2} = 1 \Rightarrow a^2 = 4, b^2 = 2$

$c = \sqrt{a^2 + b^2} = \sqrt{6}$

Tangent at $(4, \sqrt{6}$ ) is

$\frac{xx_1}{a^2} - \frac{y.y_1}{b^2} = 1$

$\Rightarrow x(4) - 2y.\sqrt{6} = 4$

$4x - 2\sqrt{6}y = 4$

$\Rightarrow 2x - \sqrt{6}y = 2$

$Q(1,0), R\left(\sqrt{6}, \dfrac{2\sqrt{6}-1}{\sqrt{6}}\right)$

$\text{Area} = \dfrac{1}{2}bh = \dfrac{1}{2}\left(\sqrt{6}-1\right)\times 2\dfrac{\left(\sqrt{6}-1\right)}{\sqrt{6}} = \dfrac{6-2\sqrt{6}+1}{\sqrt{6}} = \dfrac{7}{\sqrt{6}} - 2$

---

**Q.3** If P and Q are two statements, then which of the following compound statement is a tautology ?

**Options**
1. $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow P$
2. $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow (P \wedge Q)$
3. $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow Q$
4. $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow \sim P$

**Ans:** 4

**Sol:** consider (4)

| p | q | $p \Rightarrow q$ | $(p \Rightarrow q)$^~q | ~p | Que |
|---|---|---|---|---|---|
| T | T | T | F | F | T |
| T | F | F | F | F | T |
| F | T | T | F | T | T |
| F | F | T | T | T | T |

**Q.4** Let in a Binomial distribution, consisting of 5 independent trials, probabilities of exactly 1 and 2 successes be 0.4096 and 0.2048 respectively. Then the probability of getting exactly 3 successes is equal to :

**Options**
1. $\dfrac{32}{625}$
2. $\dfrac{128}{625}$
3. $\dfrac{80}{243}$
4. $\dfrac{40}{243}$

**Ans:** 1

**Sol:** n=5

$P(x=1) = {}^5C_1 pq^4 = 0.4096$ ______(1)

$P(x=2) = {}^5C_2\, p^2q^3 = 0.2048$ _______(2)

$P(x=3) = {}^5C_3\, p^3q^2$

$\dfrac{(2)}{(1)} \Rightarrow \dfrac{(5.4/1.2)}{5}.\dfrac{p}{q} = \dfrac{1}{2}$

$\Rightarrow 2\dfrac{p}{q} = \dfrac{1}{2} \Rightarrow \dfrac{p}{q} = \dfrac{1}{4}$

$p(x=3) = {}^5C_3.p^2q^2 = p(x=2)\times\dfrac{p}{q} = 0.2048\times\dfrac{1}{4} = \dfrac{32}{625}$

**Q.5** Let the system of linear equations

$4x + \lambda y + 2z = 0$

$2x - y + z = 0$

$\mu x + 2y + 3z = 0,\ \lambda,\ \mu \epsilon \mathbf{R}.$

has a non-trivial solution. Then which of the following is true ?

**Options**
1. $\lambda = 2,\ \ \mu \epsilon \mathbf{R}$
2. $\mu = -6,\ \ \lambda \epsilon \mathbf{R}$
3. $\lambda = 3,\ \ \mu \epsilon \mathbf{R}$
4. $\mu = 6,\ \ \lambda \epsilon \mathbf{R}$

**Ans:** 4

**Sol:** $\begin{vmatrix} 4 & \lambda & 2 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$

$\Rightarrow \lambda\mu + 2\mu - 6\lambda - 12 = 0$

$\Rightarrow (\lambda + 2)(\mu - 6) = 0$

$\Rightarrow \mu = 6, \lambda \in R$

**Q.6** Let $f : \mathbf{R} \to \mathbf{R}$ be a function defined as

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin 2x}{2x}, & \text{if } x < 0 \\ b & , \text{if } x = 0 \\ \dfrac{\sqrt{x + bx^3} - \sqrt{x}}{b\, x^{5/2}} & , \text{if } x > 0 \end{cases}$$

If $f$ is continuous at $x = 0$, then the value of $a + b$ is equal to :

**Options**
1. $-\dfrac{3}{2}$
2. $-2$
3. $-3$
4. $-\dfrac{5}{2}$

**Ans:** 1

**Sol:** $\lim\limits_{x \to 0} - f(x) = \dfrac{(a+1)}{2} + 1$

$$\lim_{x \to 0} + f(x) = \lim_{x \to 0}\left[\frac{\sqrt{x + bx^3} - \sqrt{x}}{bx^{5/2}}\right] = \lim_{x \to 0} \frac{\sqrt{x}.\left[\sqrt{1 + bx^2} - 1\right]}{b.x^{1/2}.x^2} = \lim_{x \to 0}\left[\frac{\sqrt{1 + bx^2 - 1}}{bx^2}\right] = \lim_{x \to 0} \frac{\dfrac{1 \ \ (2bx)}{2\sqrt{1 + bx^2}}}{2bx} = \frac{1}{2}$$

f(0)=b

$\Rightarrow b = \dfrac{1}{2}$

$\dfrac{a+1}{2} + 1 = \dfrac{1}{2} \Rightarrow \dfrac{a+1}{2} = \dfrac{-1}{2} \Rightarrow a + 1 = -1 \Rightarrow a = -2$

$a + b = -2 + \dfrac{1}{2} = \dfrac{-3}{2}$

**Q.7** If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in R$, then the value of $27\sec^6\alpha + 8\operatorname{cosec}^6\alpha$ is equal to :

**Options**
1. 250
2. 350
3. 400
4. 500

**Ans:** 1

**Sol:** Let $\sin^2\alpha = t \Rightarrow \cos^2\alpha = 1 - t$

$\sin^4\alpha = t^2, \cos^4\alpha = 1 - 2t + t^2$

$15t^2 + 10(1 - 2t + t^2) = 6$

$\Rightarrow 25t^2 - 20t + 4 = 0$

$\Rightarrow (5t-2)^2 \Rightarrow t = \frac{2}{5} \Rightarrow \sin^2 t = \frac{2}{5}; \cos^2 t = 3/5$

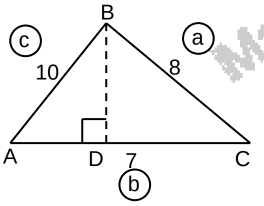
$27\sec^6\alpha + 8\operatorname{cosec}^6\alpha = \frac{27}{\cos^6\alpha} + \frac{8}{\sin^6\alpha} = \frac{27}{(3/5)^3} + \frac{8}{(2/5)^3} = 125 + 125 = 250$

**Q.8** In a triangle ABC, if $|\overrightarrow{BC}| = 8$, $|\overrightarrow{CA}| = 7$, $|\overrightarrow{AB}| = 10$, then the projection of the vector $\overrightarrow{AB}$ on $\overrightarrow{AC}$ is equal to :

**Options**
1. $\frac{115}{16}$
2. $\frac{25}{4}$
3. $\frac{127}{20}$
4. $\frac{85}{14}$

**Ans:** 4

**Sol:**




$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{49 + 100 - 64}{2(7)(10)} = \frac{17}{28}$

$AD = 10 \times \cos A = 10 \times \frac{17}{28} = \frac{85}{14}$

**Q.9**

Let $f : \mathbf{R} - \{3\} \to \mathbf{R} - \{1\}$ be defined by $f(x) = \frac{x-2}{x-3}$.

Let $g : \mathbf{R} \to \mathbf{R}$ be given as $g(x) = 2x - 3$. Then, the sum of all the values of $x$ for which

$f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$ is equal to.

**Options**

1. 3
2. 2
3. 5
4. 7

**Ans: 3**

**Sol:**

$$y = f(x) = \frac{x-2}{x-3}$$

$$\Rightarrow xy - 3y = x - 2 \Rightarrow xy - x = 3y - 2$$

$$x(y-1) = 3y - 2 \Rightarrow x = \frac{3y-2}{y-1}$$

$$f^{-1}(y) = \frac{3y-2}{y-1} \Rightarrow f^{-1}(x) = \frac{3x-2}{x-1}$$

$$\text{Let } y = g(x) = 2x - 3 \Rightarrow x = \frac{y+3}{2}$$

$$\Rightarrow g^{-1}(y) = \frac{y+3}{2} \Rightarrow g^{-1}(x) = \frac{x+3}{2}$$

$$f^{-1}(x) + g^{-1}(x) - \frac{3x-2}{x-1} + \frac{x+3}{2} = \frac{13}{2}$$

$$\frac{6x - 4 + x^2 + 2x - 3}{2(x-1)} = \frac{13}{2}$$

$$x^2 + 8x - 7 = 13x - 13$$

$$x^2 - 5x + 6 = 0$$

$$x = 2 \text{ or } 3$$

Sum=5

**Q.10**

Let a tangent be drawn to the ellipse $\frac{x^2}{27} + y^2 = 1$ at $(3\sqrt{3}\cos\theta, \sin\theta)$ where $\theta \in \left(0, \frac{\pi}{2}\right)$.

Then the value of $\theta$ such that the sum of intercepts on axes made by this tangent is minimum is equal to :

**Options**

1. $\frac{\pi}{3}$
2. $\frac{\pi}{4}$
3. $\frac{\pi}{6}$
4. $\frac{\pi}{8}$

**Ans: 3**

**Sol:** $\frac{x^2}{27}+y^2=1$

Tangent at $(3\sqrt{3}\cos\theta, \sin\theta)$ is

$\frac{x.3\sqrt{3}\cos\theta}{27}+y.\sin\theta=1$

$\frac{x.(\sqrt{3}\cos\theta)}{9}+y\sin\theta=1$

$\Rightarrow \frac{x}{(9/\sqrt{3}\cos\theta)}+\frac{y}{\text{cosec}\theta}=1$

Sum of intercepts $=3\sqrt{3}\sec\theta+\text{cosec}\theta$

Let y= $3\sqrt{3}\sec\theta+\text{cosec}\theta$

$\frac{dy}{d\theta}=3\sqrt{3}\sec\theta\tan\theta-\text{cosec}\theta+\cot\theta=0$

$\frac{3\sqrt{3}\sin\theta}{\cos^2\theta}=\frac{\cos\theta}{\sin^2\theta}\Rightarrow\frac{\sin^3\theta}{\cos^3\theta}=\frac{1}{3\sqrt{3}}$

$\tan^3\theta=\frac{1}{3\sqrt{3}}=\left(\frac{1}{\sqrt{3}}\right)^3\Rightarrow\theta=\pi/6$

**Q.11**

Let $y=y(x)$ be the solution of the differential equation $\frac{dy}{dx}=(y+1)\left((y+1)e^{x^2/2}-x\right)$, $0<x<2.1$, with $y(2)=0$. Then the value of $\frac{dy}{dx}$ at $x=1$ is equal to :

**Options**

1. $\frac{-e^{3/2}}{(e^2+1)^2}$

2. $\frac{e^{5/2}}{(1+e^2)^2}$

3. $-\frac{2e^2}{(1+e^2)^2}$

4. $\frac{5e^{1/2}}{(e^2+1)^2}$

**Ans:** 1

**Sol:** $\frac{dy}{dx}=(y+1)\left[(y+1)e^{x^2/2}-x\right]$

$y+1=u$

$\frac{dy}{dx}=\frac{du}{dx}$

$\frac{du}{dx}=u\left[ue^{\frac{x^2}{2}}-x\right]$

$\frac{du}{dx}=-ux+u^2e^{\frac{x^2}{2}}$

$$\frac{du}{dx}+ux=u^2e^{\frac{x^2}{2}}$$

$$u^{-2}.\frac{du}{dx}+u^{-1}.x=e^{\left(\frac{x^2}{2}\right)}$$

Let $u^{-1}=V$

$$\Rightarrow -u^{-2}.\frac{du}{dx}=\frac{dv}{dx}$$

$$\therefore -\frac{dv}{dx}+Vx=e^{\frac{x^2}{2}}$$

$$\frac{dv}{dx}-Vx=-e^{\frac{x^2}{2}}$$

$$I.F=e^{\int -xdx}=e^{-\frac{x^2}{2}}$$

$$V.e^{-\frac{x^2}{2}}=-\int e^{\frac{x^2}{2}}.e^{-\frac{x^2}{2}}dx=-x+c$$

$$V=(-x+c)e^{\frac{x^2}{2}}$$

$$\frac{1}{y+1}=(-x+c)e^{\frac{x^2}{2}}$$

$$\frac{1}{0+1}=(-2+c)e^2\Rightarrow -2+c=e^{-2}$$

$$\Rightarrow c=e^{-2}+2$$

$$y+1=\frac{1}{(c-x)}.e^{-\frac{x^2}{2}}$$

$$\frac{dy}{dx}=\frac{1}{(c-x)}.\,e^{-\frac{x^2}{2}}\left(\frac{-1}{2}\right).2x+e^{-\frac{x^2}{2}}\left(\frac{-1}{(c-x)^2}\right)(-1)$$

$$=\frac{-1.e^{-1/2}}{c-1}+\frac{e^{-1/2}}{(c-1)^2}=\frac{e^{-1/2}}{c-1}\left[-1+\frac{1}{c-1}\right]=\frac{e^{-1/2}}{(c-1)}.\frac{(2-c)}{c-1}$$

$$=\frac{e^{-1/2}.\left(-e^{-2}\right)}{\left(e^{-2}+1\right)^2}=\frac{-e^{-5/2}}{\left(1+\frac{1}{e^2}\right)^2}=\frac{-e^{-5/2}.e^4}{(e^2+1)^2}=\frac{-e^{3/2}}{\left(e^2+1\right)^2}$$

**Q.12** Let $g(x)=\int_0^x f(t)dt$, where $f$ is continuous function in [0, 3] such that $\frac{1}{3}\le f(t)\le 1$ for all $t \in [0, 1]$ and $0\le f(t)\le \frac{1}{2}$ for all $t \in (1, 3]$. The largest possible interval in which $g(3)$ lies is :

**Options**

1. $\left[\frac{1}{3}, 2\right]$
2. $\left[-\frac{3}{2}, -1\right]$
3. $[1, 3]$
4. $\left[-1, -\frac{1}{2}\right]$

**Ans: 1**

**Sol:** $g(x)=\int_0^x f(t)dt$

$g(3)=\int_0^3 f(t)dt=\int_0^1 f(t)dt+\int_1^3 f(t)dt$

Maximum value of g(3)is $\int_0^1 1dt+\int_1^3 \frac{1}{2}dt$

$=[t]_0^1+\frac{1}{2}[t]_1^3=1+\frac{1}{2}(3-1)=2$

Similarly, minimum value is $\frac{1}{3}$

**Q.13** Let in a series of 2n observations, half of them are equal to a and remaining half are equal to $-a$. Also by adding a constant b in each of these observations, the mean and standard deviation of new set become 5 and 20, respectively. Then the value of $a^2+b^2$ is equal to :

**Options**
1. 250
2. 425
3. 925
4. 650

**Ans: 2**

**Sol:** $\underbrace{a,a,.........a.}_{n\,terms}\ -\underbrace{a,-a,-a,.......-a}_{n\,terms}$

$\bar{x}$ before adding 'b' is 0

$\bar{x}$ after adding b is b

$\therefore b=5$

$\sigma_{new}=\sigma_{old}$

$\sigma_{old}=\sqrt{\frac{\sum xi^2}{n}-\left(\frac{\sum xi}{n}\right)^2}=\sqrt{\frac{na^2+na^2-0^2}{2n}}$

$\sqrt{a^2-0}=a=20$

$a^2+b^2=425$

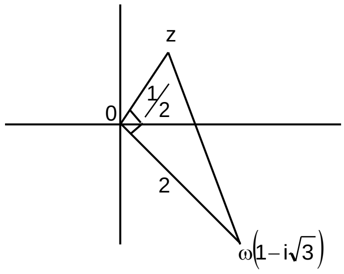


**Q.14** Let a complex number be $w=1-\sqrt{3}\,i$. Let another complex number z be such that $|zw|=1$ and $\arg(z)-\arg(w)=\frac{\pi}{2}$. Then the area of the triangle with vertices origin, z and w is equal to :

**Options**
1. $\frac{1}{2}$
2. $\frac{1}{4}$
3. 2
4. 4

**Ans: 1**

**Sol:**




$|z|.|w| = 1 \Rightarrow |z| = \frac{1}{\sqrt{1+3}} = \frac{1}{2}$

Area $\frac{1}{2} \times 2 \times \frac{1}{2} = \frac{1}{2}$

**Q.15** Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors perpendicular to each other and $|\vec{a}| = |\vec{b}|$. If $|\vec{a} \times \vec{b}| = |\vec{a}|$, then the angle between the vectors $\left(\vec{a} + \vec{b} + \left(\vec{a} \times \vec{b}\right)\right)$ and $\vec{a}$ is equal to :

**Options**

1. $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$
2. $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$
3. $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$
4. $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$



**Ans: 4**

**Sol:** $\left|\vec{a}\right| = \left|\vec{b}\right| = k$

$\left|\vec{a}\right|.\left|\vec{b}\right| \sin\theta = \left|\vec{a}\right| \Rightarrow \sin\theta = \frac{1}{k} = \sin 90$

$\Rightarrow k = 1$

$\therefore \left|\vec{a}\right| = \left|\vec{b}\right| = 1, \theta = 90^0$

Let $\phi$ be the angle between $\vec{a} + \vec{b} + \left(\vec{a} \times \vec{b}\right)$ and $\vec{a}$

$\vec{a}.\left[\vec{a} + \vec{b} + \left(\vec{a} \times \vec{b}\right)\right]$

$= \left|\vec{a}\right|^2 + \vec{a}.\vec{b} + \vec{a}.\left(\vec{a} \times \vec{b}\right) = 1$

$|a| = 1$

$$\left|\vec{a}+\vec{b}+(\vec{a}+\vec{b})\right| = \sqrt{\left|\vec{a}\right|^2+\left|\vec{b}\right|^2+\left|\vec{a}+\vec{b}\right|^2} = \sqrt{1+1+1} = \sqrt{3}$$

$$\left[\therefore \vec{a}\perp\vec{b}\perp\vec{a}\times\vec{b} \text{ and } \left|\vec{a}\right| = \left|\vec{b}\right| = \left(\vec{a}\times\vec{b}\right) = 1\right]$$

$$\cos\phi = \frac{\vec{a}.\left[\vec{a}+\vec{b}+\left(\vec{a}\times\vec{b}\right)\right]}{\left|\vec{a}\right|.\left|\vec{a}+\vec{b}+\left(\vec{a}\times\vec{b}\right)\right|} = \frac{1}{1\times\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\therefore \phi = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

**Q.16** Let $S_1$ be the sum of first 2n terms of an arithmetic progression. Let $S_2$ be the sum of first 4n terms of the same arithmetic progression. If $(S_2-S_1)$ is 1000, then the sum of the first 6n terms of the arithmetic progression is equal to :

**Options**
1. 3000
2. 1000
3. 5000
4. 7000

**Ans: 1**

**Sol:** $S_1 = S_{2n} = \frac{2n}{2}[2a+(2n-1)d]$

$S_2 = S_{4n} = \frac{4n}{2}[2a+(4n-1)d]$

$S_2 - S_1 = 2n[2a+(4n-1)d] - n[2a+(2n-1)d]$

$4na + 2n(4n-1)d - 2na - n(2n-1)d$

$= 2na + [8n^2 - 2n - 2n^2 + n]d = 1000$

$2na + (6n^2 - n)d = 1000$

$\Rightarrow n[2a+(6n-1)d] = 1000$

$S_{6n} = \frac{6n}{2}[2a+(6n-1)d] = 3\times1000 = 3000$

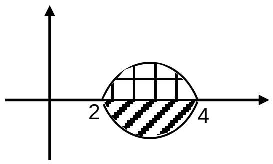


**Q.17** The area bounded by the curve $4y^2 = x^2(4-x)(x-2)$ is equal to :

**Options**
1. $\frac{\pi}{8}$
2. $\frac{3\pi}{8}$
3. $\frac{3\pi}{2}$
4. $\frac{\pi}{16}$

**Ans: 3**

**Sol:**




$$y^2 = \frac{1}{4}x^2(4-x)(x-2)$$
$$y = \frac{1}{2}x\sqrt{(4-x)(x-2)}$$

$$A = 2\int_2^4 \frac{1}{2}x\sqrt{(4-x)(x-2)}dx = \int_2^4 x\sqrt{(4-x)(x-2)}\ dx$$

Let $x = 3-t \Rightarrow 4-x = 4-(3-t) = 1+t$

$x-2 = 3-t-2 = 1-t$

If $x = 2, t = 1$ and if $x = 4, t = -1$

$$\Rightarrow \int_1^{-1}(3-t).\sqrt{(1+t)(1-t)}\ dt = \int_1^{-1}3\sqrt{1-t^2}\ dt - \int_1^{-1}t\sqrt{1-t^2}\ dt$$

$$3\times 2\int_0^1\sqrt{1-t^2}\ dt + 0 \text{ (taking modulus value)}$$

$$= 6\left[\frac{t}{2}\sqrt{1-1^2} + \frac{1}{2}\sin^{-1}t\right]_0^1 = 6\left[\left(0+\frac{\pi}{4}\right)-0\right] = \frac{3\pi}{2}$$

**Q.18** Define a relation R over a class of $n\times n$ real matrices A and B as "ARB iff there exists a non-singular matrix P such that $PAP^{-1} = B$". Then which of the following is true ?

**Options**
1. R is symmetric, transitive but not reflexive,
2. R is reflexive, symmetric but not transitive
3. R is reflexive, transitive but not symmetric
4. R is an equivalence relation

**Ans:** 4

**Sol:** $ARB \Rightarrow PAP^{-1} = B$

$ARA \Rightarrow PAP^{-1} = A$ which is true if P=I

$\therefore$ reflective

Let ARB

$\Rightarrow PAP^{-1} = B$

$BRA \Rightarrow QBQ^{-1} = A$ which is true when Q= $P^{-1}$

$\therefore ARB \Rightarrow BRA$

$\therefore$ symmetric

Let ARB and BRC

$\Rightarrow PAP^{-1} = B$ ______(1)

$QBQ^{-1} = C$ ______(2)

$\Rightarrow Q..(PAP^{-1}).Q^{-1} = C$

$\Rightarrow (QP).A(P^{-1}Q^{-1}) = C$

$\Rightarrow (QP).A.(QP)^{-1} = C$

$\Rightarrow ARC$

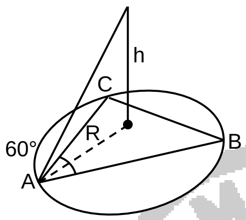
$\therefore$ Transitive

**Q.19** A pole stands vertically inside a triangular park ABC. Let the angle of elevation of the top of the pole from each corner of the park be $\frac{\pi}{3}$. If the radius of the circumcircle of $\Delta ABC$ is 2, then the height of the pole is equal to :

**Options**
1. $\frac{1}{\sqrt{3}}$
2. $\frac{2\sqrt{3}}{3}$
3. $\sqrt{3}$
4. $2\sqrt{3}$

**Ans: 4**

**Sol:**



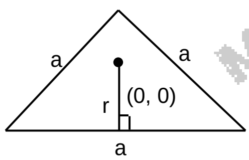

$$h = R\tan 60 = \sqrt{3}R = 2\sqrt{3}$$

**Q.20** Let the centroid of an equilateral triangle ABC be at the origin. Let one of the sides of the equilateral triangle be along the straight line $x+y=3$. If R and r be the radius of circumcircle and incircle respectively of $\Delta ABC$, then $(R+r)$ is equal to :

**Options**
1. $2\sqrt{2}$
2. $3\sqrt{2}$
3. $7\sqrt{2}$
4. $\frac{9}{\sqrt{2}}$

**Ans: 4**

**Sol:**




$$r = \left|\frac{0+0-3}{\sqrt{1+1}}\right| = \frac{3}{\sqrt{2}}$$

$R = 2r$ (in an equilateral triangle)

$\therefore R = 3\sqrt{2}$

$$R + r = 3\sqrt{2} + \frac{3}{\sqrt{2}} = \frac{9}{\sqrt{2}}$$

## SECTION B

**Q.1**

The term independent of $x$ in the expansion of $\left[\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right]^{10}$, $x \neq 1$, is equal to ________.

**Ans:** 210

**Sol:** $1^{st}$ term $=\frac{x+1}{x^{2/3}-x^{1/3}+1}$

Let $x^{1/3}=y$

$\Rightarrow 1^{st}$ term $=\frac{y^3+1}{y^2-y+1}=\frac{(y+1)(y^2-y+1)}{y^2-y+1}=y+1=x^{1/3}+1$

$2^{nd}$ term $=\frac{x-1}{\sqrt{x}(\sqrt{x}-1)}=\frac{(\sqrt{x}+1)(\sqrt{x}-1)}{\sqrt{x}(\sqrt{x}-1)}=1+\frac{1}{\sqrt{x}}$

The given expression $=\left[(x^{1/3}+1)-(1+x^{1/2})\right]^{10}=(x^{1/3}+-x^{-1/2})^{10}$

$Tr+1\,{}^{10}C_r\left(x^{1/3}\right)^{10-r}(-x^{-1/2})^r={}^{10}C_r(-1)^r.x^{\frac{10-r}{3}}.x^{-\frac{r}{2}}$

${}^{10}C_r(-1)^r.x^{\frac{20-5r}{6}}$

$\therefore 20-5r=0 \Rightarrow r=4$

$\therefore$ Term independent of $x$ is ${}^{10}C_4(-1)^4=\frac{10.9.8.7}{1.2.3.4}=210$

**Q.2**

Let I be an identity matrix of order $2\times2$ and $P=\begin{bmatrix}2 & -1\\ 5 & -3\end{bmatrix}$. Then the value of $n\in N$ for which $P^n=5I-8P$ is equal to ________.

**Ans: 6**

**Sol:** For $2\times2$ matrix,

$P^2-$ (sum of the diagonal elements ) $P+|I|=0$

$\Rightarrow P^2+P-I=0$

$\Rightarrow P^2=I-P$

$P^3=P(I-P)=P-P^2=P-(I-P)=2P-I$

$P^5=P^3.P^2=(2P-1)(I-P)=2P-2P^2-I+P=3P-2(I-P)-I=5P-3I$

$P^6=P(5P-3I)=5P^2-3P=5I-5P-3I=5I-8P$

$\therefore n=6$

**Q.3**

Let $f: R\to R$ satisfy the equation $f(x+y)=f(x).f(y)$ for all $x, y\in R$ and $f(x)\neq0$ for any $x\in R$. If the function $f$ is differentiable at $x=0$ and $f'(0)=3$, then $\lim_{h\to0}\frac{1}{h}(f(h)-1)$ is equal to ________.

**Ans: 3**

**Sol:** $f(x)=e^{kx}$

$f'(x)=k\,e^{kx}\Rightarrow f'(0)=ke^{\theta}=k=3$

$\therefore f(x) = e^{3x}$

$$\lim_{h\to 0}\left[\frac{f(h)-1}{h}\right] = \lim_{h\to 0}\frac{[f'(h)]}{1} = f'(0) = 3$$

**Q.4** If $\sum_{r=1}^{10} r!\,(r^3 + 6r^2 + 2r + 5) = \alpha\,(11!)$,

then the value of $\alpha$ is equal to ________.

**Ans: 160**

**Sol:** $r^3 + 6r^2 + 2r + 5 = (r+1)(r+2)(r+3) - 9r - 1$

$= (r+1)(r+2)(r+3) - 9(r+1) + 8$

$\therefore r!(r^3 + 6r^2 + 2r + 5) = r!\left[(r+1)(r+2)(r+3) - 9(r+1) + 8\right]$

$= (r+3)! - 9(r+1)! + 8r!$

$1^{st}$ : 4! -9(2!)+8(1!)

$2^{nd}$ :5!-9(3!)+8(2!)

$3^{rd}$ :6!-9(4!)+8(3!)

$4^{th}$ :7!-9(5!)+8(4!)

$5^{th}$ :8!-9(6!)+8(5!)

$6^{th}$ :9!-9(7!)+8(6!)

$7^{th}$ :10!-9(8!)+8(7!)

$8^{th}$ :11!-9(9!)+8(8!)

$9^{th}$ :12!-9(10!)+8(9!)

$10^{th}$ 13!-9(11!)+8(10!)

Let S denote 1!+2!+3!+….+13!

$\text{sum} = S - (1!+2!+3!) - 9\left[S - (1!+12!+13!)\right] + 8\left[S - (11!+12!+13!)\right]$

$= (S - 9S + 8S) - 1 - 2 - 6 + 9 + 9(12!) + 9(13! - 8(11!) - 8(12!) - 8(13!)$

$= 12! + 13! - 8(11!) = (12)(11!) + (13)(12)(11!) - 8(11!) = 200(11!)$

$\therefore \alpha = 200$

**Q.5** If $f(x)$ and $g(x)$ are two polynomials such that the polynomial $P(x) = f(x^3) + x\,g(x^3)$ is divisible by $x^2 + x + 1$, then $P(1)$ is equal to ________.

**Ans: 0**

**Sol:** $f(x^3) + xg(x^3)$ is divided by $x^2 + x + 1$

$\Rightarrow f(x^3) + xg(x^3) = Q(x).(x^2 + x + 1)$

When Q(x) is some polynomial put x = $\omega$

$\Rightarrow f(\omega^3) + \omega g(\omega^3) = 0$

$\Rightarrow f(1) + \omega g(1) = 0$ ___(1)

Put x= $\omega^2$ $f(\omega^6) + \omega^2 g(\omega^6) = 0$

$\Rightarrow f(1) + \omega^2 g(1) = 0$ _____(2)

$(1) + (2) \Rightarrow 2f(1) + (\omega + \omega^2).g(1) = 0$

$\Rightarrow 2f(1) - g(1) = 0$

$\Rightarrow g(1) = 2f(1)$

$\therefore (1) \Rightarrow f(1) + \omega.2f(1) = 0$

$\Rightarrow f(1)[1 + 2\omega] = 0 \Rightarrow f(1) = 0$

$\Rightarrow g(1) = 0$

$\therefore P(1) = f(1) + 1.g(1) = 0 + 0 = 0$

**Q.6** Let $y = y(x)$ be the solution of the differential equation $xdy - ydx = \sqrt{(x^2 - y^2)}\,dx$, $x \geq 1$, with $y(1) = 0$. If the area bounded by the line $x = 1$, $x = e^{\pi}$, $y = 0$ and $y = y(x)$ is $\alpha e^{2\pi} + \beta$, then the value of $10(\alpha + \beta)$ is equal to ________.

**Ans:** 4

**Sol:**
$$xdy - ydx = \sqrt{x^2 - y^2}\,dx$$
$$x.\frac{dy}{dx} - y = \sqrt{x^2 - y^2}$$
$$x\frac{dy}{dx} = y + \sqrt{x^2 - y^2}$$
$$\frac{dy}{dx} = \frac{y}{x} + \sqrt{1 - \frac{y^2}{x^2}}$$
$$y = Vx \Rightarrow \frac{dy}{dx} = V + x\frac{dv}{dx}$$
$$V + x\frac{dv}{dx} = V + \sqrt{1 - v^2} \Rightarrow x\frac{dv}{dx} = \sqrt{1 - v^2}$$
$$\int \frac{dv}{\sqrt{1 - v^2}} - \int \frac{dx}{x} \Rightarrow \sin^{-1} V = \log x + C \Rightarrow \sin^{-1}\left(\frac{y}{x}\right) = \log x + C$$
If $x = 1, y = 0$
$$\Rightarrow \sin^{-1}\left(\frac{0}{1}\right) = \log 1 + C \Rightarrow C = 0$$
$$\sin^{-1}\left(\frac{y}{x}\right) = \log x$$
$$\frac{y}{x} = \sin(\log x) \Rightarrow y = x.\sin(\log x)$$
Area= $\int_1^{e^{\pi}} x.\sin(\log x)dx$
$$\int x.\sin(\log x)dx = \int \sin(\log x).xdx = \sin(\log x).\int xdx - \int \cos(\log x).\frac{1}{x}.\frac{x^2}{2}dx$$
$$= \frac{x^2}{2}\sin(\log x) - \frac{1}{2}\int \cos(\log x).\ xdx$$
$$= \frac{x^2}{2}\sin(\log x) - \frac{1}{2}\left[\cos(\log x).\int xdx - \int -\sin(\log x).\frac{1}{x}.\frac{x^2}{2}dx\right]$$
$$= \frac{x^2}{2}\sin(\log x) - \frac{1}{2}\left[\frac{x^2}{2}\cos(\log x) + \frac{1}{2}\int x\sin(\log x)dx\right]$$
$$\frac{x^2}{2}\sin(\log x) - \frac{x^2}{4}\cos(\log x) - \frac{1}{4}\int x.\sin(\log x)dx$$
$$I = \frac{x^2}{2}\sin(\log x) - \frac{x^2}{4}\cos(\log x) - \frac{I}{4}$$
$$\Rightarrow \frac{5I}{4} = \frac{x^2}{2}\sin(\log x) - \frac{x^2}{4}\cos(\log x)$$
$$I = \frac{4}{5}\left[\frac{x^2}{2}\sin(\log x) - \frac{x^2}{4}\cos(\log x)\right] + c$$
$$\therefore I = \frac{2}{5}x^2\sin(\log x) - \frac{x^2}{5}\cos(\log x) + c$$

$$\text{Area} = \frac{1}{5}\left[2x^2 \sin(\log x) - x^2 \cos(\log x)\right]^{e^{\pi}}$$

$$= \frac{1}{5}\left[\left\{2e^{2\pi} \sin\{\log(e^{\pi})\} - e^{2\pi} \cos\{\log(e^{\pi})\}\right\} - \{2(1)\sin(\log 1) - 1\cos(\log 1)\}\right]$$

$$\frac{1}{5}\left[\left(2e^{2\pi} . \sin\pi - e^{2\pi} \cos\pi\right) - (0-1)\right] = \frac{1}{5}\left[-e^{2\pi}(1) + 1\right] = \frac{1}{5}\left(e^{2\pi} + 1\right)$$

$$\alpha = \frac{1}{5}, \beta = \frac{1}{5}$$

$$10(\alpha + \beta) = 10\left(\frac{1}{15} + \frac{1}{5}\right) = 10 \times \frac{2}{5} = 4$$

**Q.7** Let ${}^nC_r$ denote the binomial coefficient of $x^r$ in the expansion of $(1+x)^n$.

If $\sum_{k=0}^{10} (2^2 + 3k)\ {}^nC_k = \alpha.3^{10} + \beta.2^{10}$, $\alpha, \beta \in R$, then $\alpha + \beta$ is equal to ________.

Given –

**Ans:** 19

**Sol:** Let us take the value of n=10 (to set the desired sum)

$$S = \sum_{k=0}^{10} 4.\ {}^{10}C_k + 3\sum_{k=0}^{10} k.\ {}^{10}C_k$$

$$= 4\left({}^{10}C_0 + {}^{10}C_1 + .... + {}^{10}C_{10}\right) + 3\left(0.{}^{10}C_0 + + 1.{}^{10}C_1 + .... + 10.{}^{10}C_{10}\right)$$

$$4.2^{10} + 3(0+10).2^{10-1}$$

$$\left[\because a_0\,{}^nC_0 + a_1.\,{}^nC_1 + a_2.\,{}^nC_2 + ... + a_n.\,{}^nC_n = (a_0 + a_n)2^{2-1} \text{ if } a_0, a_1, a_2, .....a_n, \text{ are in AP}\right]$$

$$4.2^{10} + 3(10).2^9 = (4+15)2^{10} = 19.\ 2^{10}$$

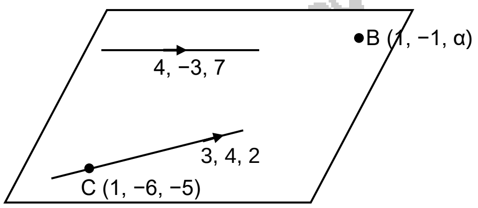
$$\alpha = 19,\ \beta = 0$$

$$\Rightarrow k + \beta| = 19$$

**Q.8** Let P be a plane containing the line $\frac{x-1}{3} = \frac{y+6}{4} = \frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4} = \frac{y-2}{-3} = \frac{z+5}{7}$. If the point $(1, -1, \alpha)$ lies on the plane P, then the value of $|5\alpha|$ is equal to ________.

**Ans:** 38

**Sol:**




$$\vec{AB} = 0\hat{i} + 5\hat{j} + (\alpha + 5)\hat{k}$$

$$\vec{b} = 3\hat{i} + 4\hat{j} + 2\hat{k}$$

$$\vec{c} = 4\hat{i} - 3\hat{j} + 7\hat{k}$$

These three vectors are coplanar

$$\Rightarrow \begin{vmatrix} 0 & 5 & \alpha+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix} = 0$$

$$\Rightarrow -5(21-8)+(\alpha+5)(-9-16)=0$$
$$-65-25\alpha-125=0$$
$$25\alpha=-190$$
$$5\alpha=\frac{-190}{250}\times 5=-38$$
$$|5\alpha|=38$$

**Q.9** Let P($x$) be a real polynomial of degree 3 which vanishes at $x=-3$. Let P($x$) have local minima at $x=1$, local maxima at $x=-1$ and $\int_{-1}^{1} P(x)dx=18$, then the sum of all the coefficients of the polynomial P($x$) is equal to ________.

**Ans: 8**

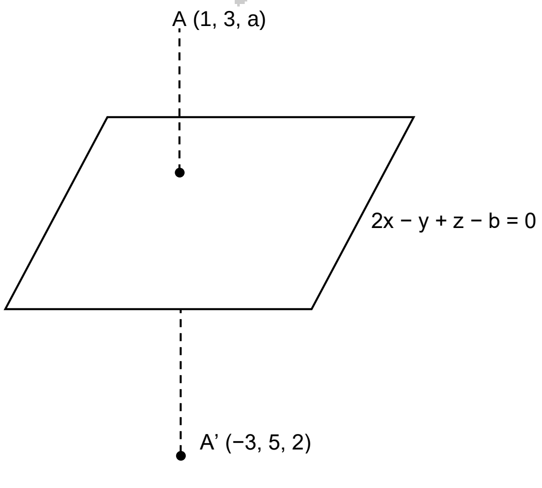
**Sol:** $P'(x)=\lambda(x+1)(x-1)=\lambda(x^2-1)$

$$p(x)=\lambda\left(\frac{x^3}{3}-x\right)+\mu$$
$$x=-3\Rightarrow p(x)=\lambda(-9+3)+\mu=0\Rightarrow -6\lambda+\mu=0\Rightarrow \mu=6\lambda$$
$$\therefore p(x)=\lambda\left(\frac{x^3-3x}{3}\right)+6\lambda=\lambda\left[\frac{x^3-3x}{3}+6\right]$$
$$p(x)=\frac{\lambda}{3}(x^3-3x+18)=A(x^3-3x+18)\text{ where } A=\frac{\lambda}{3}$$
$$\int_{-1}^{1}p(x)dx=A\int_{-1}^{1}(x^3-3x+18)dx=A\left(0+0+18\int_{-1}^{1}1dx\right.$$
$$18.A\times 2\int_{0}^{1}1.dx=36A[x]_0^1=36A=18\Rightarrow A=\frac{1}{2}$$
$$\therefore p(x)=\frac{1}{2}(x^3-3x+18)$$
$$\text{Sum of the coefficients}=\frac{1}{2}-\frac{3}{2}+9=8$$

**Q.10** Let the mirror image of the point (1, 3, a) with respect to the plane $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})-b=0$ be $(-3, 5, 2)$. Then, the value of $|a+b|$ is equal to ________.

**Ans: 1**

**Sol:**



A' is given by solving,

$$\frac{x-1}{2}=\frac{y-3}{-1}=\frac{z-a}{1}=-2\frac{(2-3+a-b)}{4+1+1}$$

$$\Rightarrow \frac{x-1}{2}=\frac{y-3}{-1}=\frac{z-a}{1}=\frac{b+1-a}{3}$$

Substituting (-3,5,2)

$$-2=-2=\frac{2-a}{1}=\frac{b+1-a}{3}$$

$$\Rightarrow 2-a=-2 \Rightarrow a=4$$

$$\Rightarrow \frac{b+1-4}{3}=-2 \Rightarrow b-3=-6 \Rightarrow b=-3$$

$$|a+b|=1$$