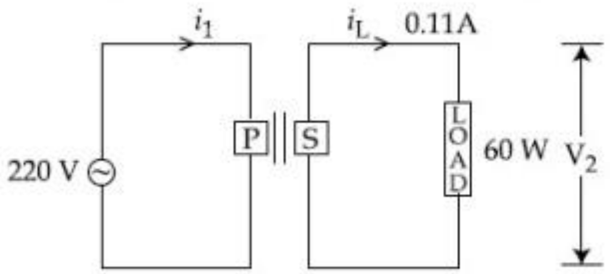
SOLUTIONS & ANSWERS FOR JEE MAINS-2021
16th March Shift 2

[PHYSICS, CHEMISTRY & MATHEMATICS]

PART – A – PHYSICS

SECTION A

Q.1 For the given circuit, comment on the type of transformer used.



Options
1. Auxilliary transformer
2. Step - up transformer
3. Auto transformer
4. Step down transformer

Ans: 2

Sol: $V_s = \frac{P}{I} = \frac{60}{0.11} = 545.45$

$V_p = 220 \Rightarrow V_s > V_p \Rightarrow$ step up transformer

Q.2 What will be the nature of flow of water from a circular tap, when its flow rate increased from 0.18 L/min to 0.48 L/min ? The radius of the tap and viscosity of water are 0.5 cm and 10^{-3} Pa s, respectively.
(Density of water : 10^3 kg/m^3)

Options
1. Steady flow to unsteady flow
2. Unsteady to steady flow
3. Remains steady flow
4. Remains turbulent flow

Ans: 2

Sol: $R_e = \frac{\rho V D}{\eta}$

If R_e < 1000 $\Rightarrow$ steady flow
1000 < Re < 2000 $\Rightarrow$ flow becomes unsteady
R_e > 2000 $\Rightarrow$ flow is turbulent

R_e initial = $\frac{10^3 \times 0.18 \times 10^{-3}}{\pi \times (0.5 \times 10^{-2})^2 \times 60} \times \frac{1 \times 10^{-2}}{10^{-3}}$ = 382.16

R_e final = $\frac{10^3 \times 0.48 \times 10^{-3}}{\pi \times (0.5 \times 10^{-2})^2 \times 60} \times \frac{1 \times 10^{-2}}{10^{-3}}$ = 1019.09

Q.3 A resistor develops 500 J of thermal energy in 20 s when a current of 1.5A is passed through it. If the current is increased from 1.5 A to 3 A, what will be the energy developed in 20 s.

Options
1. 1500 J
2. 2000 J
3. 1000 J
4. 500 J

Ans: 3

Sol: Energy, $Q = I^2Rt$

$\frac{Q_1}{Q_2} = \frac{I_1^2}{I_2^2}$ as R, t is the same for both case

$\frac{500}{Q_2} = \left(\frac{1.5}{3}\right)^2$

$Q_2 = 500 \times 4 = 2000$ J

Q.4 Find out the surface charge density at the intersection of point $x = 3$ m plane and x-axis, in the region of uniform line charge of 8 nC/m lying along the z-axis in free space.

Options
1. 47.88 C/m
2. 4.0 nC m^{-2}
3. 0.07 nC m^{-2}
4. 0.424 nC m^{-2}

Ans: 4

Sol: $\frac{2K\lambda}{r} = \frac{\sigma}{\varepsilon_0} (x = 3\,m)$

$\sigma = 0.424 \times 10^{-9} \frac{C}{m^2}$

Q.5 Calculate the value of mean free path (λ) for oxygen molecules at temperature 27°C and pressure 1.01×10^5 Pa. Assume the molecular diameter 0.3 nm and the gas is ideal. ($k = 1.38 \times 10^{-23}\ JK^{-1}$)

Options
1. 102 nm
2. 32 nm
3. 58 nm
4. 86 nm

Ans: 1

Sol: $\lambda = \frac{RT}{\sqrt{2}\pi d^2 N_A P}$

$\Rightarrow \lambda = 102\text{ nm}$

Q.6 The refractive index of a converging lens is 1.4. What will be the focal length of this lens if it is placed in a medium of same refractive index ? Assume the radii of curvature of the faces of lens are R_1 and R_2 respectively.

Options
1. Infinite
2. 1
3. Zero
4. $\frac{R_1 R_2}{R_1 - R_2}$

Ans: 1

Sol: $\frac{1}{f} = \left(\frac{\mu_\ell}{\mu_S} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

If $\mu_\ell = \mu_S \Rightarrow \frac{1}{f} = 0$

$\Rightarrow f = \infty$

Q.7 A charge Q is moving $\vec{dl}$ distance in the magnetic field $\vec{B}$. Find the value of work done by $\vec{B}$.

Options
1. -1
2. Zero
3. Infinite
4. 1

Ans: 2

Sol: The force on a point charge by magnetic field is perpendicular to $\vec{V}$

$\left[\because \vec{F} = q\vec{V} \times \vec{B}\right]$

$\therefore$ work done by magnetic force on the point charge is zero

Q.8 Amplitude of a mass-spring system, which is executing simple harmonic motion decreases with time. If mass = 500g, Decay constant = 20 g/s then how much time is required for the amplitude of the system to drop to half of its initial value ?

(ln 2 = 0.693)

Options
1. 34.65 s
2. 17.32 s
3. 15.01 s
4. 0.034 s

Ans: 1

Sol: $A = A_0 e^{-\gamma t} = A_0\, e^{-\frac{bt}{2m}}$

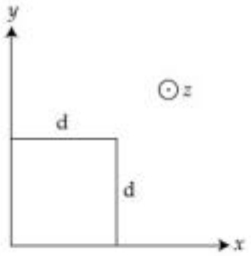
$$\frac{A_0}{2} = A_0\, e^{-\frac{bt}{2m}}$$

$$\frac{bt}{2m} = \ell n\ 2$$

$$t = \frac{2m}{b}\ell n 2 = \frac{2 \times 500 \times 0.693}{20}$$

$$\Rightarrow t = 34.65\ \text{s}$$

Q.9 The magnetic field in a region is given by $\vec{B} = B_o\left(\frac{x}{a}\right)\hat{k}$. A square loop of side d is placed with its edges along the x and y axes. The loop is moved with a constant velocity $\vec{v} = v_o\,\hat{i}$. The emf induced in the loop is :



Options

1. $\dfrac{B_o v_o d^2}{2a}$
2. $\dfrac{B_o v_o^2 d}{2a}$
3. $\dfrac{B_o v_o d^2}{a}$
4. $\dfrac{B_o v_o d}{2a}$

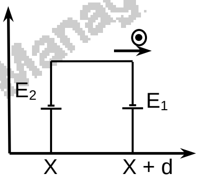
Ans: **3**

Sol: $E_1 = \dfrac{B_0(X+d)}{a} V_0 d$

$E_2 = \dfrac{B_0(X)}{a} V_0 d$

$E_{net} = E_1 - E_2$

$= \dfrac{B_0 V_0 d^2}{a}$



Q.10 In order to determine the Young's Modulus of a wire of radius 0.2 cm (measured using a scale of least count=0.001 cm) and length 1m (measured using a scale of least count=1 mm), a weight of mass 1 kg (measured using a scale of least count=1 g) was hanged to get the elongation of 0.5 cm (measured using a scale of least count 0.001 cm). What will be the fractional error in the value of Young's Modulus determined by this experiment ?

Options
1. 0.9%
2. 1.4 %
3. 0.14%
4. 9%

Ans: 2

Sol: $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{FL}{A\ell} = \frac{mg\,L}{\pi R^2 \ell}$

$$\frac{\Delta Y}{Y} = \frac{\Delta m}{m} + \frac{\Delta L}{L} + 2\frac{\Delta R}{R} + \frac{\Delta \ell}{\ell}$$

$$\frac{\Delta Y}{Y} \times 100 = 100\left[\frac{1}{1000} + \frac{1}{1000} + 2\left(\frac{0.001}{0.2}\right) + \frac{0.001}{0.5}\right]$$

$$= \frac{1}{10} + \frac{1}{10} + 1 + \frac{1}{5} = \frac{14}{10} = 1.4\%$$

Q.11 The half-life of Au^{198} is 2.7 days. The activity of 1.50 mg of Au^{198} if its atomic weight is 198 g mol^{-1} is, ($N_A = 6\times10^{23}$/mol).

Options
1. 240 Ci
2. 357 Ci
3. 252 Ci
4. 535 Ci

Ans: 2

Sol: $A = \lambda N$

$N = n N_A$ $\left(t_{1/2} = \frac{\ell n2}{\lambda}\right)$

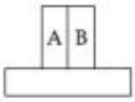
$$N = \left(\frac{1.5\times10^{-3}}{198}\right)N_A$$

$$A = \left(\frac{\ell n2}{t_{1/2}}\right)N$$

A = 365 Bq (1 curie = 3.7×10^{10} Bq)

Official key by NTA (2)

Q.12 A bimetallic strip consists of metals A and B. It is mounted rigidly as shown. The metal A has higher coefficient of expansion compared to that of metal B. When the bimetallic strip is placed in a cold bath, it will :



Options
1. Neither bend nor shrink
2. Bend towards the right
3. Not bend but shrink
4. Bend towards the left

Ans: 4

Sol: $\alpha_A > \alpha_B$

Length of both strips will decrease $\Delta L_A > \Delta L_B$

Q.13 Red light differs from blue light as they have :

Options
1. Same frequencies and same wavelengths
2. Different frequencies and different wavelengths
3. Different frequencies and same wavelengths
4. Same frequencies and different wavelengths

Ans: 2

Sol: Red light and blue light have different wavelength and different frequency.

Q.14 Calculate the time interval between 33% decay and 67% decay if half-life of a substance is 20 minutes.

Options
1. 13 minutes
2. 40 minutes
3. 20 minutes
4. 60 minutes

Ans: 3

Sol: $N_1 = N_0 e^{-\lambda t_1}$

$\frac{N_1}{N_0} = e^{-\lambda t_1}$

$0.67 = e^{-\lambda t_1}$

$$\ell n(0.67) = -\lambda t_1$$

$$N_2 = N_0\, e^{-\lambda t_2}$$

$$\frac{N_2}{N_0} = e^{-\lambda t_2}$$

$$0.33 = e^{-\lambda t_2}$$

$$\ell n(0.33) = -\lambda t_2$$

$$\ell n(0.67) - \ell n(0.33) = \lambda t_1 - \lambda t_2$$

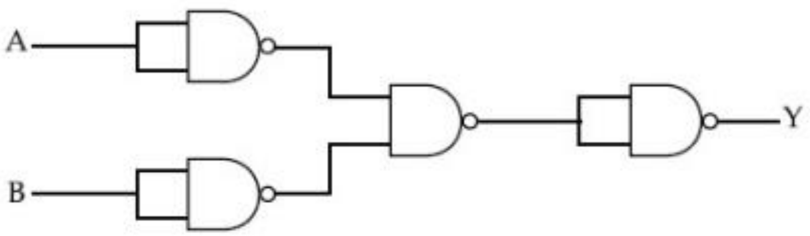
$$\lambda(t_1 - t_2) = \ell n\left(\frac{0.67}{0.33}\right)$$

$$\lambda(t_1 - t_2) = \ell n 2$$

$$t_1 - t_2 = \frac{\ell n 2}{\lambda} = t_{1/2}$$

$\therefore$ half life = $= t_{1/2}$ = 20 minute

Q.15 The following logic gate is equivalent to :



Options

1. NOR Gate
2. NAND Gate
3. AND Gate
4. OR Gate

Ans: **1**

Sol: Truth table for the given logic gate is

A	B	Out put (Y)
0	0	1
0	1	1
1	0	1
1	1	0

Q.16 A mosquito is moving with a velocity $\vec{v} = 0.5\, t^2 \hat{i} + 3t \hat{j} + 9\hat{k}$ m/s and accelerating in uniform conditions. What will be the direction of mosquito after 2 s ?

Options

1. $\tan^{-1}\left(\frac{5}{2}\right)$ from x-axis
2. $\tan^{-1}\left(\frac{2}{3}\right)$ from x-axis
3. $\tan^{-1}\left(\frac{5}{2}\right)$ from y-axis
4. $\tan^{-1}\left(\frac{2}{3}\right)$ from y-axis

Ans: **2**

Sol: $\vec{V} = 0.5t^2\hat{i} + 3t\,\hat{j} + 9\hat{k}$

$\vec{V}_{at\ t=2} = 2\hat{i} + 6\hat{j} + 9\hat{k}$

Angle made by direction of motion of mosquito will be

$\cos^{-1}\left(\frac{2}{11}\right)$ from x – axis = $\tan^{-1}\frac{\sqrt{117}}{2}$

$\cos^{-1}\left(\frac{6}{11}\right)$ from y – axis = $\tan^{-1}\frac{\sqrt{85}}{2}$

$\cos^{-1}\left(\frac{9}{11}\right)$ from z – axis = $\tan^{-1}\frac{\sqrt{40}}{9}$

No option matching

Official key by NTA (2)

Q.17 The de-Broglie wavelength associated with an electron and a proton were calculated by accelerating them through same potential of 100 V. What should nearly be the ratio of their wavelengths ? ($m_p = 1.00727u$ $m_e = 0.00055u$)

Options

1. $(1860)^2 : 1$
2. $41.4 : 1$
3. $43 : 1$
4. $1860 : 1$

Ans: **3**

Sol: $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}$

$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{m_2}{m_1}}$

$\frac{\lambda_e}{\lambda p} = \sqrt{\frac{m_p}{m_e}} = \sqrt{1831.4} = 42.79$

Q.18 Statement I : A cyclist is moving on an unbanked road with a speed of 7 kmh^{-1} and takes a sharp circular turn along a path of radius of 2m without reducing the speed. The static friction coefficient is 0.2. The cyclist will not slip and pass the curve. $(g=9.8\ m/s^2)$

Statement II : If the road is banked at an angle of 45°, cyclist can cross the curve of 2m radius with the speed of 18.5 kmh^{-1} without slipping.

In the light of the above statements, choose the correct answer from the options given below.

Options
1. Statement I is correct and statement II is incorrect
2. Both statement I and statement II are false
3. Statement I is incorrect and statement II is correct
4. Both statement I and statement II are true

Ans: **4**

Sol: Statement I $\Rightarrow V_{max} = \sqrt{\mu Rg} = \sqrt{0.2\times 2\times 9.8}$

V_{max} = 1.97 m/s

7 km/h = 1.944 m/s

Speed is lower than V_{max}, hence it can take safe turn

Statement II $\Rightarrow V_{max} = \sqrt{Rg\left(\frac{\tan\theta+\mu}{1-\mu\tan\theta}\right)}$

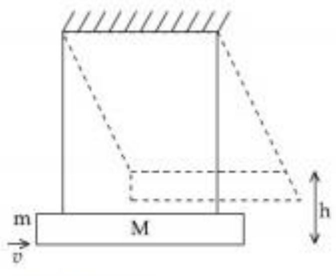
$$= \sqrt{2\times 9.8\times\left[\frac{1+0.2}{1-0.2}\right]} = 5.42\ m/s$$

18.5 km/ h = 5.14 m/s

Speed is lower than V_{max}, hence it can take safe turn.

Q.19 A large block of wood of mass M = 5.99 kg is hanging from two long massless cords. A bullet of mass m = 10 g is fired into the block and gets embedded in it. The (block + bullet) then swing upwards, their centre of mass rising a vertical distance h = 9.8 cm before the (block + bullet) pendulum comes momentarily to rest at the end of its arc. The speed of the bullet just before collision is :

(take $g = 9.8\ ms^{-2}$)



Options
1. 831.4 m/s
2. 811.4 m/s
3. 841.4 m/s
4. 821.4 m/s

Ans: **1**

Sol: According to energy conservation

$(M + m)\, g\, h = \frac{1}{2}(M+m)V_1^2$

Where $V_1 \rightarrow$ velocity after collision

$\therefore V_1 = \sqrt{2gh}$

Applying momentum conservation, (just before and just after collision)

$Mv = (M + m)\, V_1$

$$v = \left(\frac{M+m}{m}\right)V_1 = \frac{6}{10\times10^{-3}}\times\sqrt{2\times9.8\times9.8\times10^{-2}}$$

≈ 831.55 m/s

Q.20 Two identical antennas mounted on identical towers are separated from each other by a distance of 45 km. What should nearly be the minimum height of receiving antenna to receive the signals in line of sight?

(Assume radius of earth is 6400 km)

Options
1. 39.55 m
2. 158.2 m
3. 79.1 m
4. 19.77 m

Ans: 1

Sol: $D = 2\sqrt{2Rh}$

$$h = \frac{D^2}{8R} = \frac{45^2}{8\times6400}\text{ km} \cong 39.55\text{ m}$$

SECTION B

Q.1 The energy dissipated by a resistor is 10 mJ in 1 s when an electric current of 2 mA flows through it. The resistance is ________Ω. (Round off to the Nearest Integer)

Ans: 2500.00

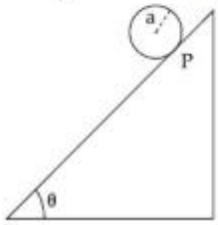
Sol: $Q = i^2 RT$

$$R = \frac{Q}{i^2 t} = \frac{10\times10^{-3}}{4\times10^{-6}\times1} = 2500\,\Omega$$

Q.2 A solid disc of radius 'a' and mass 'm' rolls down without slipping on an inclined plane making an angle θ with the horizontal. The acceleration of the disc will be $\frac{2}{b}g\sin\theta$ where b is ________. (Round off to the Nearest Integer)

(g = acceleration due to gravity

θ = angle as shown in figure)



Ans: 3.00

Sol: $a = \dfrac{g\sin\theta}{1+\dfrac{I}{mR^2}} = \dfrac{g\sin\theta}{1+\dfrac{1}{2}} = \dfrac{2}{3}g\sin\theta$

$\therefore b = 3$

Q.3 If one wants to remove all the mass of the earth to infinity in order to break it up completely. The amount of energy that needs to be supplied will be $\frac{x}{5}\frac{GM^2}{R}$ where x is __________

(Round off to the Nearest Integer)

(M is the mass of earth, R is the radius of earth, G is the gravitational constant)

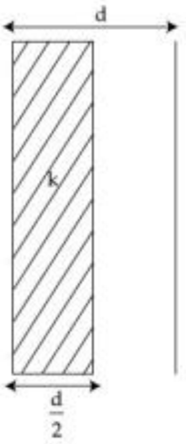
Ans: **3.00**

Sol: Energy given = $U_f - U_i$

$$= 0 - \left(-\frac{3}{5}\frac{GM^2}{R}\right) = \frac{3}{5}\frac{GM^2}{R}$$

X = 3

Q.4 In a parallel plate capacitor set up, the plate area of capacitor is 2 m^2 and the plates are separated by 1 m. If the space between the plates are filled with a dielectric material of thickness 0.5 m and area 2 m^2 (see fig) the capacitance of the set-up will be __________ ε_0. (Dielectric constant of the material = 3.2) (Round off to the Nearest Integer)



Ans: **3.04**

Sol: $C = \dfrac{\varepsilon_0 A}{\dfrac{d}{2K}+\dfrac{d}{2}} = \dfrac{2\varepsilon_0 A}{\dfrac{d}{K}+d}$

$= \dfrac{2\times 2\varepsilon_0}{\dfrac{1}{3.2}+1} = \dfrac{4\times 3.2}{4.2}\varepsilon_0 = 3.04\ \varepsilon_0$

Q.5 A body of mass 2 kg moves under a force of $(2\hat{i} + 3\hat{j} + 5\hat{k})$N. It starts from rest and was at the origin initially. After 4 s, its new coordinates are (8, b, 20). The value of b is __________.

(Round off to the Nearest Integer)

Ans: **12.00**

Sol: $\vec{a} = \frac{\vec{F}}{m} = \frac{2\hat{i}+3\hat{j}+5\hat{k}}{2}$

$= \hat{i}+1.5\hat{j}+2.5\hat{k}$

$\vec{\tau} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$= 0 + \frac{1}{2}\left(\hat{i}+1.5\hat{j}+2.5\hat{k}\right)(16)$

$= 8\hat{i}+12\hat{j}+20\hat{k}$

b = 12

Q.6 A swimmer can swim with velocity of 12 km/h in still water. Water flowing in a river has velocity 6 km/h. The direction with respect to the direction of flow of river water he should swim in order to reach the point on the other bank just opposite to his starting point is _________°. (Round off to the Nearest Integer)
(Find the angle in degrees)

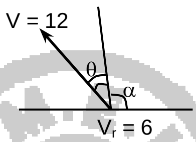
Ans: **120.00**

Sol: $12 \sin\theta = V_r$

$\sin\theta = \frac{1}{2}$

$\theta = 30°$

$\alpha = 120°$



Q.7 A closed organ pipe of length L and an open organ pipe contain gases of densities ρ_1 and ρ_2 respectively. The compressibility of gases are equal in both the pipes. Both the pipes are vibrating in their first overtone with same frequency. The length of the open pipe is $\frac{x}{3} L \sqrt{\frac{\rho_1}{\rho_2}}$ where x is _________. (Round off to the Nearest Integer)

Ans: **4.00**

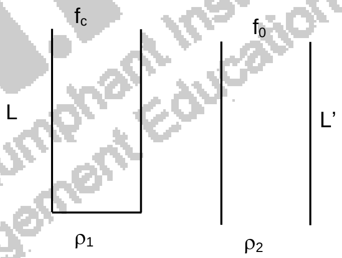
Sol: $f_c = f_0$

$\frac{3V_c}{4L} = \frac{2V_0}{2L'}$

$\frac{3V_c}{4L} = \frac{V_0}{L'}$

$L' = \frac{4L}{3}\frac{V_0}{V_c} = \frac{4L}{3}\sqrt{\frac{B\rho_1}{\rho_2 B}} = \frac{4L}{3}\sqrt{\frac{\rho_1}{\rho_2}}$

i.e., x = 4



Q.8 A deviation of 2° is produced in the yellow ray when prism of crown and flint glass are achromatically combined. Taking dispersive powers of crown and flint glass as 0.02 and 0.03 respectively and refractive index for yellow light for these glasses are 1.5 and 1.6 respectively. The refracting angles for crown glass prism will be _________° (in degree). (Round off to the Nearest Integer)

Ans: **12.00**

Sol: $\omega_1 = 0.02,\ \mu_1 = 1.5$

$\omega_2 = 0.03,\ \mu_2 = 1.6$

Achromatic combination

$\therefore \theta_{net} = 0$

$\theta_1 - \theta_2 = 0$

$\theta_1 = \theta_2$

$\omega_1\delta_1 = \omega_2\delta_2$

$\delta_{net} = \delta_1 - \delta_2 = 2°$

$\delta_1 - \frac{\omega_1\delta_1}{\omega_2} = 2°$

$\delta_1\left(1 - \frac{\omega_1}{\omega_2}\right) = 2°$

$\delta_1\left(1 - \frac{2}{3}\right) = 2°$

$\delta_1 = 6°$

$\delta_1 = (\mu_1 - 1)A_1$

$6° = (1.5 - 1)A_1$

$A_1 = 12°$

Q.9 A force $\vec{F} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ is applied on an intersection point of $x = 2$ plane and x-axis. The magnitude of torque of this force about a point (2, 3, 4) is _________. (Round off to the Nearest Integer)

Ans: 20.00

Sol: $\bar{\tau} = \bar{r} \times \bar{F}$

$\bar{r} = (2\hat{i}) - (2\hat{i} + 3\hat{j} + 4\hat{k}) = -3\hat{j} - 4\hat{k}$

and $\bar{F} = 4\hat{i} + 3\hat{j} + 4\hat{k}$

$$\bar{\tau} = \bar{r} \times \bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -3 & -4 \\ 4 & 3 & 4 \end{vmatrix}$$

$= \hat{i}(-12 + 12) - \hat{j}(0 + 16) + \hat{k}(0 + 12)$

$= -16\hat{i} + 12\hat{k}$

$\therefore |\bar{\tau}| = \sqrt{16^2 + 12^2} = 20$

Q.10 For an ideal heat engine, the temperature of the source is 127°C. In order to have 60% efficiency the temperature of the sink should be _________ °C. (Round off to the Nearest Integer)

Ans: -113.00

Sol: $n = 0.60 = 1 - \frac{T_L}{T_H}$

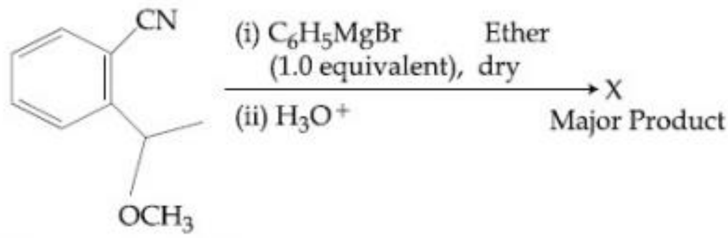
$\frac{T_L}{T_H} = 0.4 \Rightarrow T_L = 0.4 \times 400$

$= 160 \text{ K} = -113°\text{C}$

PART – B – CHEMISTRY

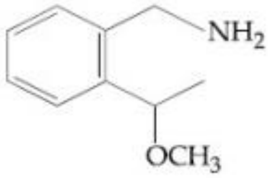
SECTION A

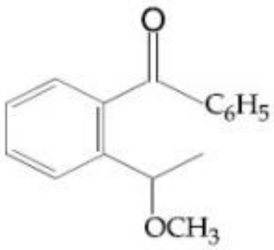
Q.1

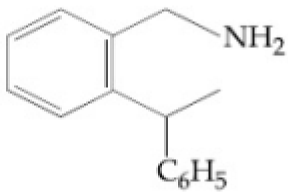


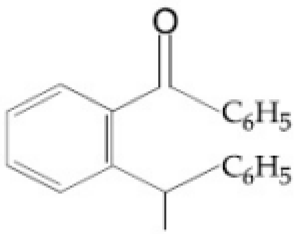
The structure of X is :

Options

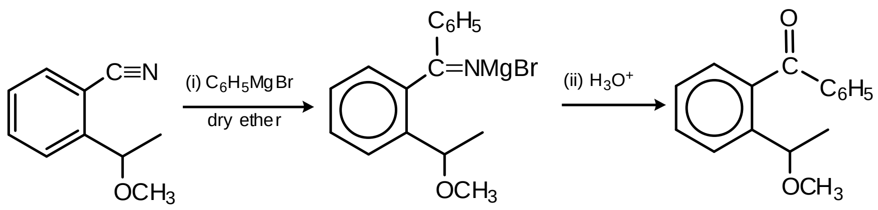
1. 

2. 

3. 

4. 

Ans: 2

Sol: 

Q.2 Fex_2 and Fey_3 are known when x and y are :

Options

1. x = Cl, Br, I and y = F, Cl, Br, I
2. x = F, Cl, Br, I and y = F, Cl, Br
3. x = F, Cl, Br and y = F, Cl, Br, I
4. x = F, Cl, Br, I and y = F, Cl, Br, I

Ans: 2

Sol: Fe forms halides of general formula
$FeX_3 \Rightarrow$ where X = F, Cl, Br
$FeX_2 \Rightarrow$ where X = F, Cl, Br, I

Q.3 The correct statements about H_2O_2 are :

(A) used in the treatment of effluents.

(B) used as both oxidising and reducing agents.

(C) the two hydroxyl groups lie in the same plane.

(D) miscible with water.

Choose the correct answer from the options given below :

Options

1. (A), (B) and (D) only
2. (B), (C) and (D) only
3. (A), (C) and (D) only
4. (A), (B), (C) and (D)

Ans: 1

Sol: The two hydroxyl group in H_2O_2 lie in different planes

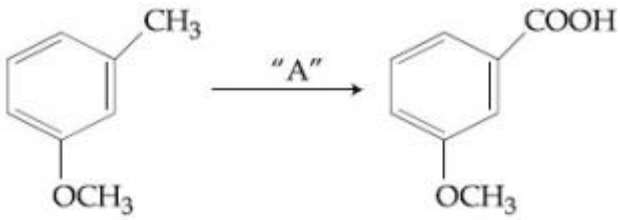
Q.4 Which of the following is least basic ?

Options
1. $(C_2H_5)_3\ddot{N}$
2. $(C_2H_5)_2\ddot{N}H$
3. $(CH_3CO)\ddot{N}HC_2H_5$
4. $(CH_3CO)_2\ddot{N}H$

Ans: 4

Sol: In $(CH_3CO)\ddot{N}H$, due to the presence of two electron in withdrawing CH_3CO– group, the availability of lone pair of electrons on nitrogen decreases. Hence it becomes less basic.

Q.5

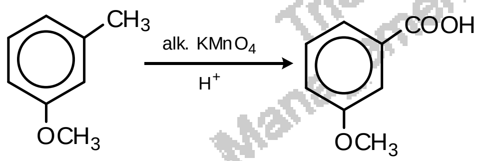


In the above reaction, the reagent "A" is :

Options
1. $NaBH_4$, H_3O^+
2. Alkaline $KMnO_4$, H^+
3. $LiAlH_4$
4. HCl, Zn – Hg

Ans: 2

Sol:



Q.6 Identify the elements X and Y using the ionisation energy values given below :

Ionization energy (kJ/mol)

	1^{st}	2^{nd}
X	495	4563
Y	731	1450

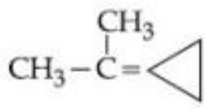
Options
1. X = F ; Y = Mg
2. X = Mg ; Y = Na
3. X = Na ; Y = Mg
4. X = Mg ; Y = F

Ans: 3

Sol: Since the second ionisation enthalpy of X is almost 10 times higher than the first it is evident that it is an alkali metal. Alkali metal after losing one electron will acquire a stable noble gas configuration and hence second IE will be very high. Since the first IE of Y is only little greater than X, if will be an alkaline earth metal.

Q.7 An unsaturated hydrocarbon X on ozonolysis gives A. Compound A when warmed with ammonical silver nitrate forms a bright silver mirror along the sides of the test tube. The unsaturated hydrocarbon X is :

Options

1. $CH_3-C\equiv C-CH_3$
2. $HC\equiv C-CH_2-CH_3$
3. 
4. $CH_3-C(CH_3)=C(CH_3)-CH_3$

Ans: 2

Sol: $HC\equiv C-CH_2-CH_3 \xrightarrow[\text{(ii) } H_2O]{\text{(i) } O_3} CH_3-CH_2-\underset{\underset{O}{\parallel}}{C}-CHO$

Since the compound contains an aldehydic group it will reduce ammoniacal silver nitrate solution to metallic silver.

Q.8 Which of the following reduction reaction **CANNOT** be carried out with coke ?

Options

1. $Cu_2O \rightarrow Cu$
2. $ZnO \rightarrow Zn$
3. $Fe_2O_3 \rightarrow Fe$
4. $Al_2O_3 \rightarrow Al$

Ans: 4

Sol: Al_2O_3 cannot be reduced by coke because it is a highly stable oxide and aluminium has a strong affinity for oxygen. Hence Al_2O_3 has to be reduced to Al electrolytically.

Q.9 The green house gas/es is (are) :

(A) Carbon dioxide

(B) Oxygen

(C) Water vapour

(D) Methane

Choose the **most appropriate** answer from the options given below :

Options
1. (A) and (C) only
2. (A) only
3. (A) and (B) only
4. (A), (C) and (D) only

Ans: 4

Sol: Greenhouse gases are CO_2, CH_4, CFC, ozone, N_2O and water vapour.

Q.10 The exact volumes of 1 M NaOH solution required to neutralise 50 mL of 1 M H_3PO_3 solution and 100 mL of 2 M H_3PO_2 solution, respectively, are :

Options
1. 100 mL and 50 mL
2. 50 mL and 50 mL
3. 100 mL and 200 mL
4. 100 mL and 100 mL

Ans: 3

Sol: At the point of neutralization, $V_1N_1 = V_2N_2$

For NaOH × H_3PO_3, V × 1 = 50 × 2

V = 100 mL

For NaOH × H_3PO_2, V × 1 = 100 × 2

V = 200 mL

Q.11 The **INCORRECT** statement regarding the structure of C_{60} is :

Options
1. Each carbon atom forms three sigma bonds.
2. The six-membered rings are fused to both six and five-membered rings.
3. It contains 12 six-membered rings and 24 five-membered rings.
4. The five-membered rings are fused only to six-membered rings.

Ans: 3

Sol: C_{60} fullerene contains 20 six-membered rings and 12-five membered ring.

Q.12 The **INCORRECT** statements below regarding colloidal solutions is :

Options 1. An ordinary filter paper can stop the flow of colloidal particles.

2. The flocculating power of Al^{3+} is more than that of Na^{+}.

3. A colloidal solution shows colligative properties.

4. A colloidal solution shows Brownian motion of colloidal particles.

Ans: 1

Sol: Ordinary filter paper cannot be used for removing the dispersed phase because size of pores of filter paper is bigger than the size of colloidal particles which can easily pass through the pores of the ordinary filter paper.

Q.13 The secondary structure of protein is stabilised by :

Options 1. glycosidic bond

2. van der Waals forces

3. Hydrogen bonding

4. Peptide bond

Ans: 3

Sol: The secondary structure of proteins is stabilized by hydrogen bonding.

Q.14 Match List-I with List-II :

	List-I Test/Reagents/Observation(s)		List-II Species detected
(a)	Lassaigne's Test	(i)	Carbon
(b)	Cu(II) oxide	(ii)	Sulphur
(c)	Silver nitrate	(iii)	N, S, P, and halogen
(d)	The sodium fusion extract gives black precipitate with acetic acid and lead acetate	(iv)	Halogen Specifically

The correct match is :

Options 1. (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

2. (a)-(i), (b)-(iv), (c)-(iii), (d)-(ii)

3. (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)

4. (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Ans: 4

Sol: (a) Lassaignes test – N, S, P and halogens
(b) Cu (II) oxide – Carbon
(c) Silver nitrate – Halogen specifically
(d) The sodium fusion extract gives black precipitate with acetic acid and lead acetate solution – Sulphur

Q.15 Arrange the following metal complex/compounds in the increasing order of spin only magnetic moment. Presume all the three, high spin system.

(Atomic numbers Ce = 58, Gd = 64 and Eu = 63.)

(a) $(NH_4)_2[Ce(NO_3)_6]$ (b) $Gd(NO_3)_3$ and (c) $Eu(NO_3)_3$

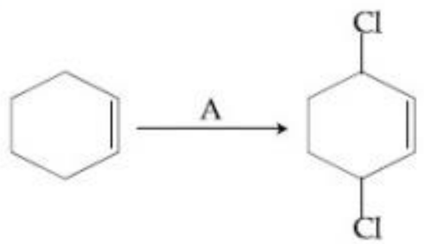
Answer is :

Options
1. (c) < (a) < (b)
2. (b) < (a) < (c)
3. (a) < (c) < (b)
4. (a) < (b) < (c)

Ans: 3

Sol: In $(NH_4)_2[Ce(NO_3)_6] \Rightarrow Ce^{+4}$
$Ce^{+4} \Rightarrow [Xe]\ 4f^0\ 5d^0\ 6s^0 \Rightarrow n = 0$
In $Gd(NO_3)_3 \Rightarrow Gd^{3+}$
$Gd^{3+} \Rightarrow [Xe]\ 4f^7 \Rightarrow n = 7$
In $Eu(NO_3)_3 \Rightarrow Eu^{3+}$
$Eu^{3+} \Rightarrow [Xe]\ 4f^6 \Rightarrow n = 6$
∴ The increasing order of spin only magnetic moment is
$Ce^{+4} < Eu^{+3} < Gd^{+3}$

Q.16

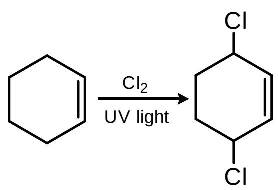


Identify the reagent(s) 'A' and condition(s) for the reaction

Options
1. A = HCl, $ZnCl_2$
2. A = HCl ; Anhydrous $AlCl_3$
3. A = Cl_2 ; dark, Anhydrous $AlCl_3$
4. A = Cl_2 ; UV light

Ans: 4

Sol:



The reaction proceeds through free radical intermediate formation.

Q.17 Statement I : Sodium hydride can be used as an oxidising agent.

Statement II : The lone pair of electrons on nitrogen in pyridine makes it basic.

Choose the CORRECT answer from the options given below :

Options
1. Both statement I and statement II are true
2. Statement I is true but statement II is false
3. Both statement I and statement II are false
4. Statement I is false but statement II is true

Ans: 4

Sol: NaH is used as a reducing agent.

Q.18 The characteristics of elements X, Y and Z with atomic numbers, respectively, 33, 53 and 83 are :

Options
1. X and Y are metalloids and Z is a metal.
2. X is a metalloid, Y is a non-metal and Z is a metal.
3. X, Y and Z are metals.
4. X and Z are non-metals and Y is a metalloid.

Ans: 2

Sol: X (Z = 33) → Arsenic (As)
Y (Z = 53) → Iodine (I)
Z (Z = 83) → Bismuth (Bi)
X → Metalloid Y → Non-metal Z → Metal

Q.19 Ammonolysis of Alkyl halides followed by the treatment with NaOH solution can be used to prepare primary, secondary and tertiary amines. The purpose of NaOH in the reaction is :

Options
1. to remove basic impurities
2. to increase the reactivity of alkyl halide
3. to activate NH_3 used in the reaction
4. to remove acidic impurities

Ans: 4

Sol: During ammonolysis of alkyl halide, the acid liberated during the reaction combines with the amine and forms amine salt. To liberate free amine from the amine salt, a base is needed.

Q.20 Which of the following polymer is used in the manufacture of wood laminates ?

Options
1. Phenol and formaldehyde resin
2. *cis*-poly isoprene
3. Urea formaldehyde resin
4. Melamine formaldehyde resin

Ans: 3

Sol: Urea formaldehyde resin is used in the manufacture of wood laminates.

SECTION B

Q.1 A and B decompose via first order kinetics with half-lives 54.0 min and 18.0 min respectively. Starting from an equimolar non reactive mixture of A and B, the time taken for the concentration of A to become 16 times that of B is ________ min. (Round off to the Nearest Integer).

Ans: 108

Sol: $A \xrightarrow{54} \frac{A}{2} \xrightarrow{54} \frac{A}{4}$

$B \xrightarrow{18} \frac{B}{2} \xrightarrow{18} \frac{B}{4} \xrightarrow{18} \frac{B}{8} \xrightarrow{18} \frac{B}{16} \xrightarrow{18} \frac{B}{32} \xrightarrow{18} \frac{B}{64}$

Initial concentrations of A & B are same, since $\frac{B}{64}$ is 16 times $\frac{A}{4}$ and so the time taken is 108 minutes

Q.2 $[Ti(H_2O)_6]^{3+}$ absorbs light of wavelength 498 nm during a d – d transition. The octahedral splitting energy for the above complex is __________ $\times 10^{-19}$ J. (Round off to the Nearest Integer). $h = 6.626 \times 10^{-34}$ Js; $c = 3 \times 10^8$ ms^{-1}

Ans: 4

Sol: $E = h\upsilon = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{498 \times 10^{-9}} = 0.0399 \times 10^{-17} = 3.99 \times 10^{-19}$ J

Q.3 Ga (atomic mass 70 u) crystallizes in a hexagonal close packed structure. The total number of voids in 0.581 g of Ga is __________ $\times 10^{21}$. (Round off to the Nearest Integer).
[Given : $N_A = 6.023 \times 10^{23}$]

Ans: 15

Sol: Total number of voids = 3n (where n ⇒ particles in unit cell)

$n = \frac{0.581}{70} \times 6.022 \times 10^{23} = 0.0499 \times 10^{23}$

∴ Total voids = $0.1499 \times 10^{23} = 14.99 \times 10^{21}$

Q.4 At 363 K, the vapour pressure of A is 21 kPa and that of B is 18 kPa. One mole of A and 2 moles of B are mixed. Assuming that this solution is ideal, the vapour pressure of the mixture is __________ kPa. (Round off to the Nearest Integer).

Ans: 19

Sol: $P_T = P_A^{\circ} x_A + P_B^{\circ} x_B$

$= 21 \times \frac{1}{3} + 18 \times \frac{2}{3} = 7 + 12 = 19\, pK_a$

Q.5 The number of orbitals with $n = 5$, $m_l = +2$ is ________. (Round off to the Nearest Integer).

Ans: 3

Sol: n = 5

$\ell = 0$	m = 0
1	m = +1, 0, –1
2	m = $\boxed{+2}$, +1, 0, –1, –2
3	m = +3, $\boxed{+2}$, +1, 0, –1, –2, –3
4	m = +4, +3, $\boxed{+2}$, +1, 0, –1, –2, –3, –4

Q.6 A 5.0 m mol dm^{-3} aqueous solution of KCl has a conductance of 0.55 mS when measured in a cell of cell constant 1.3 cm^{-1}. The molar conductivity of this solution is ________ $mSm^2\ mol^{-1}$. (Round off to the Nearest Integer).

Ans: **14**

Sol: Molar conductivity, $\Lambda_m = \frac{\kappa}{1000 \times M}$

$M = 5 \times 10^{-3}$ mol L^{-1}

$\kappa = C \times G^* = 0.55 \times \frac{1.3}{10^{-2}} = 71.5\ MS\ m^{-1}$

$\therefore \Lambda_m = \frac{71.5}{10^3 \times 5 \times 10^{-3}} = 14.3\,MS\ m^2\ mol^{-1} \approx 14$

Q.7 Sulphurous acid (H_2SO_3) has $Ka_1 = 1.7 \times 10^{-2}$ and $Ka_2 = 6.4 \times 10^{-8}$. The pH of 0.588 M H_2SO_3 is ________. (Round off to the Nearest Integer).

Ans: **1**

Sol: $K_{a_1} = 1.7 \times 10^{-2}$ $\quad C = 0.588\,M$

$\therefore [H^+]_1 = \sqrt{K_{a_1} C} = \sqrt{1.7 \times 10^{-2} \times 0.588} = 0.99 \times 10^{-1} = 0.1$

$K_{a_2} = 6.4 \times 10^{-8}$ $\quad C = 0.588\,M$

$\therefore [H^+]_2 = \sqrt{K_{a_2} C} = \sqrt{6.4 \times 10^{-2} \times 0.588} = 1.94 \times 10^{-4}$

Since $[H^+]_2 << [H^+]_1$ we neglect $[H^+]_2$

$\therefore$ pH = $-\log 10^{-1} = 1$

Q.8 When 35 mL of 0.15 M lead nitrate solution is mixed with 20 mL of 0.12 M chromic sulphate solution, ________ $\times 10^{-5}$ moles of lead sulphate precipitate out. (Round off to the Nearest Integer).

Ans: **525**

Sol: $3Pb(NO_3)_2 + Cr_2(SO_4)_2 \rightarrow 3PbSO_4 + 2Cr(NO_3)_3$

35×0.15 $\Rightarrow$ 5.25 m.moles (LR) $\quad$ 20×0.12 = 2.4 m.moles $\quad$ $\Rightarrow$ 5.25 m. moles

$\therefore$ No. of moles of $PbSO_4 \Rightarrow 5.25 \times 10^{-3}$ moles
$= 525 \times 10^{-5}$ moles

Q.9 At 25°C, 50 g of iron reacts with HCl to form $FeCl_2$. The evolved hydrogen gas expands against a constant pressure of 1 bar. The work done by the gas during this expansion is ________ J.

(Round off to the Nearest Integer).

[Given : $R = 8.314\ J\ mol^{-1}\ K^{-1}$. Assume, hydrogen is an ideal gas]

[Atomic mass of Fe is 55.85 u]

Ans: **2218**

Sol: $Fe + 2HCl \rightarrow FeCl_2 + H_2$

1 mol Fe $\Rightarrow$ 1 mol H_2

$\frac{50}{55.85} = 0.895$ mole Fe $\Rightarrow 0.895$ mol H_2

$w = -P\Delta V = -\Delta n_g RT$

$= -0.895 \times 8.314 \times 298 = -2218.05$ J

Q.10 In Duma's method of estimation of nitrogen, 0.1840 g of an organic compound gave 30 mL of nitrogen collected at 287 K and 758 mm of Hg pressure. The percentage composition of nitrogen in the compound is _________. (Round off to the Nearest Integer).

[Given : Aqueous tension at 287 K = 14 mm of Hg]

Ans: **19**

Sol: $\frac{P_1V_1}{T_1} = \frac{P_0V_0}{T_0}$

$V_0 = \frac{P_1V_1}{T_1} \times \frac{T_0}{P_0} = \frac{744 \times 30}{287} \times \frac{273}{760} = 27.93$ mL

$\%$ of $N_2 = \frac{V_0}{8W} = \frac{27.93}{8 \times 0.1840} = 18.9$

PART – C – MATHEMATICS

SECTION A

Q.1 Let C_1 be the curve obtained by the solution of differential equation $2xy\frac{dy}{dx} = y^2 - x^2, x > 0$.

Let the curve C_2 be the solution of $\frac{2xy}{x^2 - y^2} = \frac{dy}{dx}$. If both the curves pass through (1, 1), then

the area enclosed by the curves C_1 and C_2 is equal to :

Options

1. $\frac{\pi}{4} + 1$
2. $\pi + 1$
3. $\pi - 1$
4. $\frac{\pi}{2} - 1$

Ans: 4

Sol: $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

put $y = vx$

$\Rightarrow v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2x.vx} = \frac{v^2 - 1}{2v}$

$x\frac{dv}{dx} = \frac{v^2 - 1}{2v} - v = \frac{-v^2 - 1}{2v}$

$\Rightarrow \frac{2v}{v^2 + 1}dv = \frac{dx}{x}$

$\Rightarrow \ (\log v^2 + 1) = \log cx$

$\Rightarrow \ \frac{y^2}{x^2} + 1 = cx$

Putting $x, y = 1$

$\Rightarrow c = 2$

$\Rightarrow x^2 + y^2 - 2x = 0$

Similarly

$\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}$

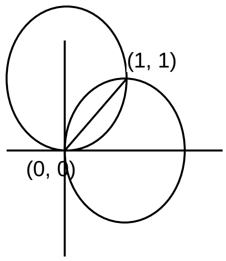
Put $y = vx$

$v + x\frac{dv}{dx} = \frac{2x\ vx}{x^2 - v^2x^2} = \frac{2v}{1 - v^2} =$

$x\frac{dv}{dx} = \frac{2v}{1 - v^2} - v = \frac{v + v^3}{1 - v^2}$

Solving we get

$x^2 + y^2 - 2y = 0$



Required area = $2\int_0^1 \left(\sqrt{2x - x^2} - x\right) dx = \frac{\pi}{2} - 1$

Q.2 Let C be the locus of the mirror image of a point on the parabola $y^2 = 4x$ with respect to the line $y = x$. Then the equation of tangent to C at P(2, 1) is :

Options
1. $2x + y = 5$
2. $x - y = 1$
3. $x + 2y = 4$
4. $x + 3y = 5$

Ans: 1

Sol: $y^2 = 4x$

Mirror image of $y^2 = 4x$ on $y = x$ is $x^2 = 4y$

Differentiating w.r t. x

$2x = 4\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{x}{2}$

At (2, 1) $\frac{dy}{dx} = 1$

Equation of tangent $y - 1 = 1\ (x - 2)$

$\Rightarrow y - 1 = x - 2$

$\Rightarrow x - y = 1$

Q.3

Let $\alpha \in \mathbb{R}$ be such that the function $f(x) = \begin{cases} \dfrac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}, & x \neq 0 \\ \alpha, & x = 0 \end{cases}$ is

continuous at $x=0$, where $\{x\}=x-[x]$, $[x]$ is the greatest integer less than or equal to x. Then :

Options

1. $\alpha = \dfrac{\pi}{\sqrt{2}}$
2. no such α exists
3. $\alpha = \dfrac{\pi}{4}$
4. $\alpha = 0$

Ans: 2

Sol: $\lim_{x \to 0} f(x) = f(0)$ (since function is continuous)

HL = $\lim_{x\to 0} \dfrac{\cos^{-1}(1-x^2)\sin^{-1}(1-x)}{x(1-x)(1+x)} = \lim_{x\to 0} \dfrac{\cos^{-1}(1-x^2)}{x} = \dfrac{\pi}{2}$

Putting $1 - x^2 = \cos\theta$

$$\frac{\pi}{2} \lim_{x\to 0^+} \frac{\theta}{\sqrt{1-\cos\theta}} = \frac{\pi}{2} \lim_{\theta\to 0+} \frac{\theta}{\sqrt{2\sin \theta/2}} = \frac{\pi}{\sqrt{2}}$$

LHL = $\lim_{x\to 0^-} \dfrac{\cos^{-1}\left(1-(1+x^2)\right)\sin^{-1}(-x)}{(1+x)-(1+x^3)} = \lim_{x\to 0^-} \dfrac{\frac{\pi}{2}(-\sin^{-1}x)}{(1+x)(2+x)(-x)}$

$= \dfrac{\pi}{4}$

$\therefore$ LHL $\neq$ RHL

$\Rightarrow$ Function is not continuous for any α

$\Rightarrow$ no such α exists

Q.4 Consider a rectangle ABCD having 5, 7, 6, 9 points in the interior of the line segments AB, CD, BC, DA respectively. Let α be the number of triangles having these points from different sides as vertices and β be the number of quadrilaterals having these points from different sides as vertices. Then $(\beta - \alpha)$ is equal to :

Options

1. 717
2. 795
3. 1890
4. 1173

Ans: 1

Sol: No: of triangles = α = 5.6.7 + 6.7.9 + 7.9.5 + 9.5.6

= 210 + 378 + 315 +270 = 1173

No: of quadrilaterals = β = 5.6.7.9 = 1890

$\beta - \alpha$ = 1890 – 1173 = 717

Q.5 The least value of $|z|$ where z is complex number which satisfies the inequality

$\exp\left(\frac{(|z|+3)\ (|z|-1)}{||z|+1|}\log_e 2\right) \geq \log_{\sqrt{2}} |5\sqrt{7}+9i|$, $i=\sqrt{-1}$, is equal to :

Options
1. 8
2. 2
3. $\sqrt{5}$
4. 3

Ans: 4

Sol: $exp\left[\frac{(|z|+3)(|z|-1)}{||z|+1|}\ln 2\right]x \geq \log_{\sqrt{2}}|5\sqrt{7}+9i|$

$\Rightarrow 2^{\frac{(|z|+3)(|z|-1)}{(|z|+1)}} \geq 2^3$

$\Rightarrow \frac{(|z|+3)(|z|-1)}{|z|+1} \geq 3$

$\Rightarrow (|z|+3)(|z|-1) \geq 3(|z|+1)$

$\Rightarrow |z|^2 + 2|z| - 3 \geq 3|z| + 3$

$\Rightarrow |z|^2 - |z| - 6 \geq 0$

$\Rightarrow (|z|-3)(|z|+2) \geq 0$

$\Rightarrow |z| - 3 \geq 0 \Rightarrow |z| \geq 3$

$\Rightarrow$ least value = 3

Q.6 Given that the inverse trigonometric functions take principal values only. Then, the number of real values of x which satisfy $\sin^{-1}\left(\frac{3x}{5}\right) + \sin^{-1}\left(\frac{4x}{5}\right) = \sin^{-1}x$ is equal to :

Options
1. 2
2. 1
3. 0
4. 3

Ans: 3

Sol: $\sin^{-1}\frac{3x}{5} + \sin^{-1}\frac{4x}{5} = \sin^{-1}x$

Let A = $\sin^{-1}\frac{3x}{5}$ B = $\sin^{-1}\frac{4x}{5}$

A + B = $\sin^{-1}x$

$\sin(A+B) = x$

$\sin A \cos B + \cos A \sin B = x$

$\Rightarrow \frac{3x}{5}\sqrt{1-\frac{16x^2}{25}} + \frac{4x}{5}\sqrt{1-\frac{9x^2}{25}} = x$

$\Rightarrow x\left(3\sqrt{25-16x^2} + 4\sqrt{25-9x^2}\right) = 25x$

Solving we get, x=0, 1, -1

$\Rightarrow$ no: of real solution is 3

Q.7 Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 5\hat{k}$. If $\vec{r} \times \vec{a} = \vec{b} \times \vec{r}$, $\vec{r} \cdot (\alpha\hat{i} + 2\hat{j} + \hat{k}) = 3$

and $\vec{r} \cdot (2\hat{i} + 5\hat{j} - \alpha\hat{k}) = -1$, $\alpha \in R$, then the value of $\alpha + |\vec{r}|^2$ is equal to :

Options
1. 11
2. 15
3. 9
4. 13

Ans: **2**

Sol:

$\bar{r} \times \bar{a} - \bar{b} \times \bar{r} = 0$

$\Rightarrow \bar{r} \times a + \bar{r} \times b = 0$

$\Rightarrow \bar{r} \times (\bar{a} + \bar{b}) = 0$

$\Rightarrow \bar{r} = \lambda(\bar{a} + \bar{b}) = \lambda(3i - j + 2k)$

Now, $\bar{r}.(\alpha i + 2j + k) = 3$

$\Rightarrow \lambda(3\alpha - 2 + 2) = 3$

$\lambda\alpha = 1$

Also $\bar{r}.(2i + 5j - \alpha k) = -1$

$\Rightarrow \lambda(6 - 5 - 2\alpha) = -1$

$\Rightarrow \lambda - 2 = -1$

$\Rightarrow \lambda = 1 \Rightarrow \alpha = 1$

$\therefore \bar{r} = 3i - j + 2k$

$\therefore \alpha + |r|^2 = 1 + (9 + 1 + 4) = 15$

Q.8 Let A denote the event that a 6-digit integer formed by 0, 1, 2, 3, 4, 5, 6 without repetitions, be divisible by 3. Then probability of event A is equal to :

Options
1. $\frac{4}{9}$
2. $\frac{3}{7}$
3. $\frac{11}{27}$
4. $\frac{9}{56}$

Ans: **1**

Sol

Total cases $= 6 \times 6!$

<u>Favourable cases</u>

No: is divisible by 3 can be formed with

0 1 2 4 5 6 $= 5 \times 5!$

0 1 2 3 4 5 $= 5 \times 5!$!

1 2 3 4 5 6 $=$ 6!

Required Probability $= \frac{6\,!+5\times5\,!+5\times5\,!}{6\times6\,!} = \frac{16}{36} = \frac{4}{9}$

Q.9 Let $A = \{2, 3, 4, 5, \ldots, 30\}$ and '$\simeq$' be an equivalence relation on $A \times A$, defined by $(a, b) \simeq (c, d)$, if and only if $ad = bc$. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair $(4, 3)$ is equal to :

Options
1. 8
2. 5
3. 7
4. 6

Ans: 3

Sol

$(a,b) \cong (c,d) \Leftrightarrow ad = bc$

$(4,3) \cong (c,d) \Leftrightarrow 4d = 3c$

$\Rightarrow \frac{c}{d} = \frac{4}{3}$

$\Rightarrow$ The elements is the equivalence relation are

{(4,3), (8,6), (12,9) (16,12), (20,15), (24,18) (28,21)}

$\therefore$ No: of ordered pairs=7

Q.10 If the points of intersections of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the circle $x^2 + y^2 = 4b$, $b > 4$ lie on the curve $y^2 = 3x^2$, then b is equal to :

Options
1. 12
2. 5
3. 10
4. 6

Ans: 1

Sol

$\frac{x^2}{16} + \frac{y^2}{b^2} = 1 \qquad (1)$

$x^2 + y^2 = 4b \qquad (2)$

From (2) $y^2 = 4b - x^2$

Sub is (1) $\frac{x^2}{16} + \frac{4b - x^2}{b^2} = 1$

$\Rightarrow (b^2 - 16)x^2 = 16b^2 - 64b$

$\Rightarrow x^2 = \frac{16b}{b+4}$

$y^2 = 4b - \frac{16b}{b+4} = \frac{4b^2}{b+4}$

Now $y^2 = 3x^2$

$\Rightarrow \frac{4b^2}{b+4} = \frac{48b}{b+4} \Rightarrow b = 0, 12$

But $b > 0$

$\Rightarrow b = 12$

Q.11 If the foot of the perpendicular from point (4, 3, 8) on the line $L_1 : \frac{x-a}{l} = \frac{y-2}{3} = \frac{z-b}{4}$, $l \neq 0$ is (3, 5, 7), then the shortest distance between the line L_1 and line $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is equal to :

Options

1. $\frac{1}{\sqrt{3}}$
2. $\sqrt{\frac{2}{3}}$
3. $\frac{1}{2}$
4. $\frac{1}{\sqrt{6}}$

Ans: 4

Sol Direction ratios of line $\frac{x-a}{\ell} = \frac{y-2}{3} = \frac{z-b}{4}$ is ℓ, 3, 4

Let P (4, 3, 8), Q (3, 5, 7)

Direction ratios of PQ is (–1, 2, –1)

PQ $\perp$ L1 $\Rightarrow -\ell + 6 - 4 = 0$

$\Rightarrow \ell = 2$

Now, Q (3,5,7) lie on the line L_1

$\Rightarrow \frac{3-a}{2} = \frac{3}{3} = \frac{7-b}{4}$

$\Rightarrow a - 1, \; b = 3$

Now, shortest distance between $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

And $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$

$$b_1 \times b_2 = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -i + 2j - k$$

$a_2 - a_1 = i + 2j + 2k$

$$d = \left| \frac{(-i + 2j - k).(i + 2j + 2k)}{\sqrt{1+4+1}} \right| = \left| \frac{-1+4-2}{\sqrt{6}} \right| = \frac{1}{\sqrt{6}}$$

Q.12 Let $P(x) = x^2 + bx + c$ be a quadratic polynomial with real coefficients such that $\int_0^1 P(x)\,dx = 1$ and $P(x)$ leaves remainder 5 when it is divided by $(x-2)$. Then the value of $9(b+c)$ is equal to :

Options

1. 7
2. 9
3. 11
4. 15

Ans: 1

Sol

$P(x) = x^2 + bx + c$

$P(2) = 5$

$\Rightarrow 5 = 4 + 2b + c$

$\Rightarrow 2b + c = 1 \qquad (1)$

$\int_0^1 P(x)dx = 1$

$\Rightarrow \int_0^1 (x^2 + bx + c)dx = 1$

$\Rightarrow \left(\frac{x^3}{3} + \frac{bx^2}{2} + cx\right)_0^1 = 1 \Rightarrow 3b + 6c = 4$ ___(2)

Solving $b = \frac{2}{9}, \quad c = \frac{5}{9}$

$\therefore 9\ (b + c) = 9\left(\frac{2}{9} + \frac{5}{9}\right) = 7$

Q.13

The maximum value of $f(x) = \begin{vmatrix} \sin^2 x & 1 + \cos^2 x & \cos 2x \\ 1 + \sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}, x \in \mathbf{R}$ is :

Options

1. $\sqrt{7}$
2. 5
3. $\frac{3}{4}$
4. $\sqrt{5}$

Ans: 4

Sol

$$f(x) = \begin{vmatrix} \sin^2 x & 1 + \cos^2 x & \cos 2x \\ 1 + \sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}$$

$$C_1 \to C_1 + C_2 \Rightarrow \begin{vmatrix} 2 & 1 + \cos^2 x & \cos 2x \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

$$R_1 \Rightarrow R_1 - R_2 \Rightarrow \begin{vmatrix} 0 & 1 & 0 \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin^2 x \end{vmatrix}$$

$f(x) = \cos 2x - 2\sin 2x$

Max $f(x) = \sqrt{1^2 + (-2)^2} = \sqrt{5}$

Q.14 If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} + (\tan x)\, y = \sin x$, $0 \le x \le \frac{\pi}{3}$, with $y(0) = 0$, then $y\left(\frac{\pi}{4}\right)$ equal to :

Options

1. $\left(\frac{1}{2\sqrt{2}}\right) \log_e 2$
2. $\frac{1}{2} \log_e 2$
3. $\frac{1}{4} \log_e 2$
4. $\log_e 2$

Ans: 1

Sol $\frac{dy}{dx} + (\tan x)y = \sin x$

I.F $e^{\int \tan x dx} = e^{\log \sec x} = \sec x$

$y.\sec x = \int \tan x dx + C$

$\Rightarrow y \sec x = \log \sec x + C$

$y(0) = 0 \rightarrow 0 = C$

$\Rightarrow y \sec x = \log \sec x$

When $x = {}^{\pi}/_{4} ; y.\sqrt{2} = \log \sqrt{2}$

$\Rightarrow y = \frac{1}{2\sqrt{2}} \log 2$

Q.15 Let f be a real valued function, defined on $\mathbf{R} - \{-1, 1\}$ and given by

$$f(x) = 3 \log_e \left|\frac{x-1}{x+1}\right| - \frac{2}{x-1}.$$

Then in which of the following intervals, function $f(x)$ is increasing ?

Options

1. $(-\infty, \frac{1}{2}] - \{-1\}$
2. $(-\infty, -1) \cup \left([\frac{1}{2}, \infty) - \{1\}\right)$
3. $(-\infty, \infty) - \{-1, 1\}$
4. $(-1, \frac{1}{2}]$

Ans: 1

Sol $f(x) = 3\log_e\left|\frac{x-1}{x+1}\right| - \frac{2}{x-1} = 3\ln(x-1) - 3\ln(x+1) - \frac{2}{x-1}$

$f'(x) = \frac{3}{x-1} - \frac{3}{x+1} + \frac{2}{(x-1)^2} = \frac{8x-4}{(x-1)^2(x+1)} = \frac{8(x-2)(x+1)}{(x-1)^2(x+1)^2} \ge 0$

$\Rightarrow x \in (-\infty,-1) \cup \left[\frac{1}{2},1\right] \cup (1,\infty)$

$\Rightarrow x \in \left(-\infty,\frac{1}{2}\right] \cup \{-1\}$

Q.16 Let the lengths of intercepts on x-axis and y-axis made by the circle $x^2+y^2+ax+2ay+c=0$, $(a<0)$ be $2\sqrt{2}$ and $2\sqrt{5}$, respectively. Then the shortest distance from origin to a tangent to this circle which is perpendicular to the line $x+2y=0$, is equal to :

Options
1. $\sqrt{6}$
2. $\sqrt{10}$
3. $\sqrt{7}$
4. $\sqrt{11}$

Ans: 1

Sol: Given eq is $x^2+y^2+ax+2ay+c=0$

$2\sqrt{g^2-c} = 2\sqrt{\frac{a^2}{4}-c} = 2\sqrt{2}$

$\Rightarrow \frac{a^2}{4}-c=2 \Rightarrow a^2-4c=8$ ______ (1)

$2\sqrt{f^2-c} = 2\sqrt{a^2-c} = 2\sqrt{5}$

$\Rightarrow a^2-c=5$ __________ (2)

Solving $a=-2, \quad c=-1$

$\Rightarrow$ Circle is $x^2+y^2-2x-4y-1=0$

$\Rightarrow (x-1)^2+(y-2)^2=6$

Slope of $x+2y=0$ is $\frac{-1}{2}$

Slope of $\perp^r = 2$

Equation of tangent $y=mx \pm a\sqrt{1+m^2}$

$\Rightarrow y=2x \pm \sqrt{30} \Rightarrow 2x-y \pm \sqrt{30}=0$

Perpendicular distance from (0,0)= $\left|\frac{\pm\sqrt{30}}{\sqrt{4+1}}\right| = \sqrt{6}$

Q.17 Let A(−1, 1), B(3, 4) and C(2, 0) be given three points. A line $y=mx$, $m>0$, intersects lines AC and BC at point P and Q respectively. Let A_1 and A_2 be the areas of ΔABC and ΔPQC respectively, such that $A_1=3A_2$, then the value of m is equal to :

Options
1. 3
2. 1
3. 2
4. $\frac{4}{15}$

Ans: 2

Sol: Let $P(x, mx_1)$ and Q be (x_2, mx_2)

$$A_1 = \frac{1}{2}\begin{vmatrix} 3 & 4 & 1 \\ 2 & 0 & 1 \\ -1 & 1 & 1 \end{vmatrix} = \frac{13}{2}$$

$$A_2 = \frac{1}{2}\begin{vmatrix} x_1 & mx_1 & 1 \\ x_2 & mx_2 & 1 \\ 2 & 0 & 1 \end{vmatrix} = m|x_1 - x_2|$$

$$A_1 = 3A_2 \quad \Rightarrow \frac{13}{2} = m|x_1 - x_2| \_\_\_\_\_\_ (1)$$

Equation of AC =x + 3y= 2
Equation of BC.= 4x - y= -8

Solving x+3y=2 and y = mx $\quad x_1 = \frac{2}{1+3m}$

Solving 4x-y+8=0 and y= mx $\quad x_2 = \frac{8}{4-m}$

$$\Rightarrow |x_1 - x_2| = \frac{26m}{(3m+1)(4-m)} \_\_\_\_\_\_\_\_\_\_ (2)$$

From (1) and (2)
m = 1

Q.18 If (x, y, z) be an arbitrary point lying on a plane P which passes through the points (42, 0, 0), (0, 42, 0) and (0, 0, 42), then the value of the expression

$$3 + \frac{x-11}{(y-19)^2\,(z-12)^2} + \frac{y-19}{(x-11)^2\,(z-12)^2} + \frac{z-12}{(x-11)^2\,(y-19)^2} - \frac{x+y+z}{14(x-11)\,(y-19)\,(z-12)}$$

is equal to :

Options
1. 0
2. 39
3. -45
4. 3

Ans: **4**

Sol: Equation of the plane passing through (42,0,0), (0,42,0) and (0,0,42) is x+ y + z=42

$\Rightarrow x + y + z - 42 = 0$

$\Rightarrow$ (x – 11) + (y – 19) + (z – 12) = 0

Let $x - 11 = p \quad y - 19 = q \quad z - 12 = r \quad \Rightarrow p + q + r = 0$

Given expression is

$$3 + \frac{x-11}{(y-19)^2(z-12)^2} + \frac{y-19}{(x-11)^2(z-12)^2} + \frac{z-12}{(x-1)^2(y-19)^2} - \frac{x+y+z}{14(x-11)(y-19)(z-12)}$$

$$\Rightarrow 3 + \frac{p}{q^2r^2} + \frac{q}{p^2r^2} + \frac{r}{p^2q^2} - \frac{42}{14pq^2}$$

$$= 3 + \frac{p3 + q^3 + r^3 - 3pqr}{p^2q^2r^2}$$

$= 3 \quad$ since $p^3 + q^3 + r^3 = 3pqr$

Q.19 Let $f : S \to S$ where $S = (0, \infty)$ be a twice differentiable function such that $f(x+1) = xf(x)$. If $g : S \to \mathbb{R}$ be defined as $g(x) = \log_e f(x)$, then the value of $|g''(5) - g''(1)|$ is equal to :

Options

1. $\frac{205}{144}$
2. $\frac{187}{144}$
3. 1
4. $\frac{197}{144}$

Ans: **1**

Sol: $f(x+1) = x\ f(x)$

Taking log as both sides

$\ln\ f(x+1) = \ln x + \ln f(x)$

$\Rightarrow g(x+1) = \ln x + g(x)$

$\Rightarrow g(x+1) - g(x) = \ln x$

$\Rightarrow g''(x+1) - g''(x) = \frac{-1}{x^2}$

$g''(5) - g''(1) = \frac{-1}{1^2}\ \frac{-1}{2^2}\ \frac{-1}{3^2}\ \frac{-1}{4^2}$

$|g''(5) - g''(1)| = \frac{205}{144}$

Q.20 Consider the integral

$$I = \int_0^{10} \frac{[x]\, e^{[x]}}{e^{x-1}}\, dx,$$

where $[x]$ denotes the greatest integer less than or equal to x. Then the value of I is equal

to :

Options

1. $45\ (e-1)$
2. $45\ (e+1)$
3. $9\ (e+1)$
4. $9\ (e-1)$

Ans: **1**

Sol: $I = \int_0^{10} \frac{[x]\, e^{[x]}}{e^{x-1}}\, dx = \int_0^{10} |x|\ e^{[x]-x+1}$

$= \int_0^1 0dx + \int_1^2 1\, e^{2-x} + \int_2^3 2e^{3-x}dx + ... \int_9^{10} 9e^{10-x}dx$

$= -\sum_{n=0}^{9} n\, (e^{n+1-x})_n^{n+1}$

$$= -\sum_{n=0}^{9} n\left(e^0 - e^1\right)$$

$$= (e-1)\frac{9.10}{2} = 45\ (e-1)$$

SECTION B

Q.1 If the distance of the point (1, −2, 3) from the plane $x+2y-3z+10=0$ measured parallel to the line, $\frac{x-1}{3}=\frac{2-y}{m}=\frac{z+3}{1}$ is $\sqrt{\frac{7}{2}}$, then the value of $|m|$ is equal to ________.

Ans: 2

Sol: Direction cosines of line

$$\left(\frac{3}{\sqrt{m^2+10}}, \frac{-m}{\sqrt{m^2+10}}, \frac{1}{\sqrt{m^2+10}}\right)$$

A point on the plane is

$$\left(1+\frac{3n}{\sqrt{m^2+10}}, -2+\frac{-mn}{\sqrt{m^2+10}}, 3+\frac{n}{\sqrt{m^2+10}}\right)$$

Since it lie on the plane

$$1+\frac{3n}{\sqrt{m^2+10}}-4-\frac{2mn}{\sqrt{m^2+10}}, -9-\frac{-3n}{\sqrt{m^2+10}}+10=0$$

$$\Rightarrow n^2m^2 = m^2+10$$

$$\Rightarrow \frac{7}{2}m^2 = m^2+10 \Rightarrow \frac{5}{2}m^2 = 10 \Rightarrow m^2 = 4 \Rightarrow m = \pm 2$$

$$\therefore |m| = 2$$

Q.2 In ΔABC, the lengths of sides AC and AB are 12 cm and 5 cm, respectively. If the area of ΔABC is 30 cm^2 and R and r are respectively the radii of circumcircle and incircle of ΔABC, then the value of 2R + r (in cm) is equal to ________.

Ans: 15

Sol: Given

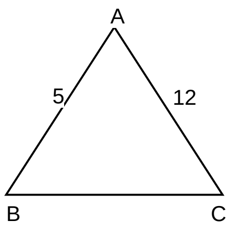
$$\frac{1}{2}5 \times 12 \sin A = 30$$

$$\Rightarrow \sin A = 1$$

$$\Rightarrow A = 90^0$$

$$\Rightarrow BC = 13 = 2R$$

$$r = \frac{\Delta}{s} = \frac{30}{\frac{5+12+13}{2}} = 2$$



$\therefore$ 2R + r = 13 + 2 = 15

Q.3 Let n be a positive integer. Let $A = \sum_{k=0}^{n}(-1)^k\, {}^nC_k \left[\left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \left(\frac{15}{16}\right)^k + \left(\frac{31}{32}\right)^k\right]$

If $63A = 1 - \frac{1}{2^{30}}$, then n is equal to ________.

Ans: 6

Sol: $$\sum_{k.0}^{n} {}^{n}C_k\left[\left(\frac{-1}{2}\right)^k+\left(\frac{-3}{4}\right)^k+\left(\frac{-7}{8}\right)^k+\left(\frac{-15}{16}\right)^k+\left(\frac{-31}{32}\right)^k\right]$$

$$=\left(1-\frac{1}{2}\right)^n+\left(1-\frac{3}{4}\right)^n+\left(1-\frac{7}{8}\right)^n+\left(1-\frac{15}{16}\right)^n+\left(1-\frac{31}{32}\right)^n$$

$$\Rightarrow A=\frac{1}{2^n}+\frac{1}{4^n}+\frac{1}{8^n}+\frac{1}{16^n}+\frac{1}{32^n}$$

$$\Rightarrow A=\frac{1}{2^n}\left(\frac{1-\left(\frac{1}{2n}\right)^5}{1-\frac{1}{2n}}\right)=A=\left(\frac{1-\frac{1}{2^{5n}}}{2^n-1}\right)$$

Rearranging

$$(2^n-1)A=1-\frac{1}{2^{5n}}$$

Comparing with 63A = $1-\frac{1}{2^{30}}$

5n = 30

⇒ n = 6

Q.4 Let $\vec{c}$ be a vector perpendicular to the vectors $\vec{a}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$. If $\vec{c}\cdot\left(\hat{i}+\hat{j}+3\hat{k}\right)=8$ then the value of $\vec{c}\cdot\left(\vec{a}\times\vec{b}\right)$ is equal to ________.

Ans: 28

Sol: $$\bar{c}=\lambda\left(a\times\bar{b}\right)$$

$$a\times b=\begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix}=3i-2j+k$$

c.(i + j+ 3k) = 8 ⇒ 3λ – 2λ + 3λ = 8

$$\Rightarrow \lambda=2$$

$$\Rightarrow c=2\left(a\times\bar{b}\right)$$

So, ⇒c. (a × b) = 2|a × b|^2 = 2 (9 + 4 + 1) = 28

Q.5 Let

$S_n(x)=\log_{a^{1/2}}x+\log_{a^{1/3}}x+\log_{a^{1/6}}x+\log_{a^{1/11}}x+\log_{a^{1/18}}x+\log_{a^{1/27}}x+\ldots$ up to n-terms,

where a > 1. If $S_{24}(x)=1093$ and $S_{12}(2x)=265$, then value of a is equal to ________.

Ans: 16

Sol: $S_n(x)$ can be written as

$S_n(x)=(2+3+6+---+\ n\ terms)\log_a x$

General term = p^2-2p+3

$$S_n(x)=\sum_{p=1}^{n}(p^2-2p+3)\log_a x$$

$$S_{24}(x)=\log_a x\sum_{p=1}^{24}(p^2-2p+3)$$

$$1093=4372\ \log_a x$$

$$\log_a x=\frac{1}{4}\qquad \Rightarrow x=a^{1/4}\ ----------(1)$$

$$S_{12}(2x) = \log_a(2x)\sum_{p=1}^{12}(p^2 - 2p + 3)$$

$$265 = 530\log_a(2x)$$

$$\log_a(2x) = \frac{1}{2} \Rightarrow 2x = a^{\frac{1}{2}} - - - - (2)$$

From (1) and (2) a = 16

Q.6 Consider the statistics of two sets of observations as follows :

	Size	Mean	Variance
Observation I	10	2	2
Observation II	n	3	1

If the variance of the combined set of these two observations is $\frac{17}{9}$, then the value of n is equal to ________.

Ans: 5

Sol: $\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2} + \frac{n_1 n_2}{(n_1+n_2)^2}(\bar{x}_1 - \bar{x}_2)^2$

$\Rightarrow \frac{17}{9} = \frac{10\times 2 + n}{n+10} + \frac{10n}{(n+10)^2}(3-2)^2$

$\frac{17}{9} = \frac{(n+20)(n+10) + 10n}{(n+10)^2}$

Solving, we get $n = \frac{-5}{2}, 5$

$\Rightarrow$ n = 5

Q.7 For real numbers α, β, γ and δ, if

$$\int \frac{(x^2-1) + \tan^{-1}\left(\frac{x^2+1}{x}\right)}{(x^4+3x^2+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx$$

$$= \alpha\log_e\left(\tan^{-1}\left(\frac{x^2+1}{x}\right)\right) + \beta\tan^{-1}\left(\frac{\gamma(x^2-1)}{x}\right) + \delta\tan^{-1}\left(\frac{x^2+1}{x}\right) + C$$

where C is an arbitrary constant, then the value of $10(\alpha + \beta\gamma + \delta)$ is equal to ________.

Ans: 6

Sol: Given integral is

$$\int \frac{x^2-1}{(x^4+3x^2+1)\tan^{-1}\left(x+\frac{1}{x}\right)}dx + \int \frac{dx}{x^4+3x^2+1}$$

$$= \int \frac{\left(1-\frac{1}{x^2}\right)}{\left(\left(x+\frac{1}{x}\right)^2+1\right)\tan^{-1}\left(x+\frac{1}{x}\right)}dx + \frac{1}{2}\int\frac{(x^2+1)-(x^2-1)}{x^4+3x^2+1}dx$$

Put $\tan^{-1}\left(x+\frac{1}{x}\right) = t$

$$\Rightarrow \int \frac{dt}{t} + \frac{1}{2}\int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x+\frac{1}{x}\right)^2+5}dx - \frac{1}{2}\int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)^2+1}$$

Put $x - \frac{1}{x} = u$ and $x + \frac{1}{x} = v$

$$\Rightarrow \log\ t + \frac{1}{2}\int \frac{du}{u^2+5} - \frac{1}{2}\int \frac{dv}{v^2+1}$$

$$= \log \tan^{-1}\left(x+\frac{1}{x}\right) + \frac{1}{2\sqrt{5}}\tan^{-41}\left(\frac{x-\frac{1}{x}}{\sqrt{5}}\right) - \frac{1}{2}\tan^{-1}\left(x+\frac{1}{x}\right) + c$$

Comparing, we get

$$\alpha = 1 \quad \beta = \frac{1}{2\sqrt{5}} \qquad \gamma = \frac{1}{\sqrt{5}} \qquad \delta = \frac{-1}{2}$$

$$\Rightarrow 10(\alpha + \beta\gamma + \delta) = 10\left(1 + \frac{1}{10} - \frac{1}{2}\right) = 6$$

Q.8 Let $\frac{1}{16}$, a and b be in G.P. and $\frac{1}{a}$, $\frac{1}{b}$, 6 be in A.P., where a, b > 0. Then 72(a + b) is equal to __________.

Ans: **14**

Sol: $\frac{1}{16}$, a, b are in G.P $\Rightarrow a^2 = \frac{b}{16} - - - (1)$

$\frac{1}{a}, \frac{1}{b}, 6$ are in A.P $\Rightarrow \frac{2}{b} = \frac{1+6a}{a} \Rightarrow b = \frac{2a}{1+6a} - - - (2)$

$$16a^2 = \frac{2a}{1+6a}$$

$\Rightarrow 8a^2 + 48a^3 - a = 0$

$\Rightarrow a = 0, \frac{-1}{4}, \frac{1}{12}$

Since a > 0; $a = \frac{1}{2}$

$\Rightarrow b = \frac{1}{9}$

$\therefore 72(a+b) = 72\left(\frac{1}{12} + \frac{1}{9}\right) = 14$

Q.9 Let $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} x + a, & x < 0 \\ |x - 1|, & x \geq 0 \end{cases} \text{ and } g(x) = \begin{cases} x + 1, & x < 0 \\ (x-1)^2 + b, & x \geq 0 \end{cases}$$

where a, b are non-negative real numbers. If $(gof)(x)$ is continuous for all $x \in \mathbb{R}$, then a + b is equal to __________.

Ans: **1**

Sol: $$g(f(x)) = \begin{cases} f(x) + 1 & f(x) < 0 \\ (f(x) - 1)^2 + b & f(x) \geq 0 \end{cases}$$

$$g(f(x)) = \begin{cases} x+a+1 & x+a<0,\ x<0 \\ |x-1|+1 & |x-1|<0,\ x\ge 0 \\ (x-a-1)^2+b & x+a\ge 0,\ x<0 \\ (|x-1|-1)^2+b & |x-1|\ge 0,\ x\ge 0 \end{cases}$$

$$= \begin{cases} x+a+1 & x\in[-\infty,-a) \\ (x-a-1)^2+b & x\in[-a,0) \\ (|x-1|-1)^2+b & x\in[0,\infty) \end{cases}$$

g(f(x)) is continuous at x = –a and x = 0

At $x = -a$

1 = b + 1

$\Rightarrow$ b = 0

At x = 0

$(a-1)^2 + b = b$

$\Rightarrow$ a = 1

$\Rightarrow$ a + b = 1 + 0 = 1

Q.10 Let $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ be two 2×1 matrices with real entries such that $A = XB$, where $X = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$, and $k \in R$. If $a_1^2 + a_2^2 = \frac{2}{3}(b_1^2 + b_2^2)$ and $(k^2+1)\, b_2^2 \ne -2\, b_1\, b_2$, then the value of k is __________.

Ans: 1

Sol: A = XB

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$\Rightarrow b_1 - b_2 = \sqrt{3}\, a_1$ ---- (1)

$b_1 + kb_2 = \sqrt{3}\, a_2$ --- (2)

Given $a_1^2 + a_2^2 = \frac{2}{3}\left(b_1^2 + b_2^2\right)$--- (3)

Squaring and adding (1) and (2)

$$b_1^2 + b_2^2 - 2b_1b_2 + b_1^2 + k^2b_2^2 + 2b_1b_2k = 3\left(a_1^2 + a_2^2\right)$$

$$\Rightarrow \frac{2}{3}b_1^2 + \frac{\left(1+k^2\right)}{3}b_2^2 + \frac{2}{3}b_1b_2(k-1) = a_1^2 + a_2^2$$

Equating (3) and (4), we get

$$\frac{2}{3}b_1^2 + \frac{\left(1+k^2\right)}{3}b_2^2 + \frac{2}{3}b_1b_2(k-1) = \frac{2}{3}\left(b_1^2 + b_2^2\right)$$

Comparing we get

$\frac{k^2+1}{3} = \frac{2}{3}$ and $\frac{2}{3}(k-1) = 0$

$k^2 \ne 1$ k = 1

$k = \pm 1$

$\therefore$ k = 1