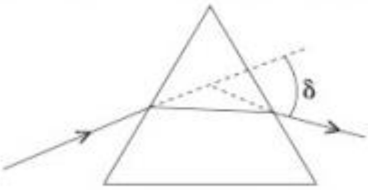
SOLUTIONS & ANSWERS FOR JEE MAINS-2021
16^{th} March Shift 1

[PHYSICS, CHEMISTRY & MATHEMATICS]

PART – A – PHYSICS

SECTION A

Q.1 The angle of deviation through a prism is minimum when



(A) Incident ray and emergent ray are symmetric to the prism

(B) The refracted ray inside the prism becomes parallel to its base

(C) Angle of incidence is equal to that of the angle of emergence

(D) When angle of emergence is double the angle of incidence

Choose the correct answer from the options given below :

Options
1. Statements (A), (B) and (C) are true
2. Only statement (D) is true
3. Statements (B) and (C) are true
4. Only statements (A) and (B) are true

Ans: 1

Sol: Basic concept

Q.2 A 25 m long antenna is mounted on an antenna tower. The height of the antenna tower is 75 m. The wavelength (in meter) of the signal transmitted by this antenna would be :

Options
1. 400
2. 200
3. 300
4. 100

Ans: 4

Sol: Length of antenna = $\frac{\lambda}{4}$

Given, length = 25 m

$\therefore \frac{\lambda}{4} = 25\,m$

$\lambda = 25 \times 4 = 100$ m

Q.3 One main scale division of a vernier callipers is 'a' cm and n^{th} division of the vernier scale coincide with $(n-1)^{th}$ division of the main scale. The least count of the callipers in mm is :

Options

1. $\dfrac{10\,a}{n}$
2. $\dfrac{10\,na}{(n-1)}$
3. $\dfrac{10\,a}{(n-1)}$
4. $\left(\dfrac{n-1}{10\,n}\right)a$

Ans: 1

Sol: Let 1 VSD = a' cm

Given 1 MSD = a cm

$(n-1)\,a = n\,a' \Rightarrow a' = \dfrac{(n-1)}{n}a$

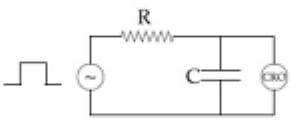
Least count = 1 MSD – VSD

$= (a - a')\text{ cm} = a - \dfrac{(n-1)}{n}a$

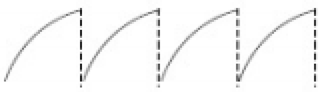
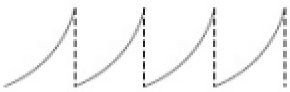
$= \dfrac{na - na + a}{n} = \dfrac{a}{n}\text{cm}$

$= \dfrac{a}{n} \times 10\text{ mm} = \dfrac{10a}{n}\text{mm}$

Q.4 An RC circuit as shown in the figure is driven by a AC source generating a square wave. The output wave pattern monitored by CRO would look close to :

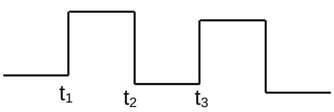


Options

1. 
2. 
3. 
4. 

Ans: 3

Sol:



$t_1 - t_2 \rightarrow$ charging
$t_2 - t_3 \rightarrow$ discharging

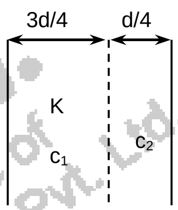
Q.5 For changing the capacitance of a given parallel plate capacitor, a dielectric material of dielectric constant K is used, which has the same area as the plates of the capacitor. The thickness of the dielectric slab is $\frac{3}{4}d$, where 'd' is the separation between the plates of parallel plate capacitor. The new capacitance (C′) in terms of original capacitance (C_0) is given by the following relation :

Options

1. $C' = \frac{4}{3+K}C_0$
2. $C' = \frac{4K}{K+3}C_0$
3. $C' = \frac{4+K}{3}C_0$
4. $C' = \frac{3+K}{4K}C_0$

Ans: 2

Sol: $c_0 \frac{\varepsilon_0 A}{d}$



c_1 and c_2 is in series. ∴ effective capacitance is given by

$$\frac{1}{c'} = \frac{1}{c_1} + \frac{1}{c_2} = \frac{3d/4}{\varepsilon_0 KA} + \frac{d/4}{\varepsilon_0 A}$$

$$= \frac{d}{4\varepsilon_0 A}\left[\frac{3+K}{K}\right]$$

$$c' = \frac{4\varepsilon_0 A K}{d[3+K]} = \frac{4K}{K+3}c_0$$

Q.6 A block of 200 g mass moves with a uniform speed in a horizontal circular groove, with vertical side walls of radius 20 cm. If the block takes 40 s to complete one round, the normal force by the side walls of the groove is :

Options

1. 9.859×10^{-2} N
2. 6.28×10^{-3} N
3. 9.859×10^{-4} N
4. 0.0314 N

Ans: 3

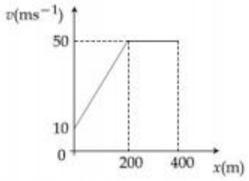
Sol: $N = m\omega^2 R = m \times \left(\frac{2\pi}{T}\right)^2 \times R = m\left[\frac{4\pi^2}{T^2}\right]R$

Given, m = 0.2 kg, T = 40 s, R = 0.2 m

Substituting the given values, normal force,

$$N = 0.2\left[\frac{4\pi^2}{40^2}\right] \times 0.2 = 9.859 \times 10^{-4} N$$

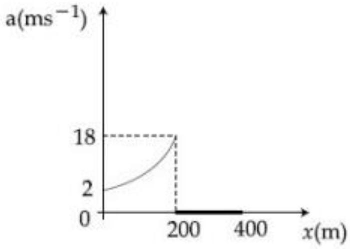
Q.7 The velocity-displacement graph describing the motion of a bicycle is shown in the figure.



The acceleration-displacement graph of the bicycle's motion is best described by :

Options

1.



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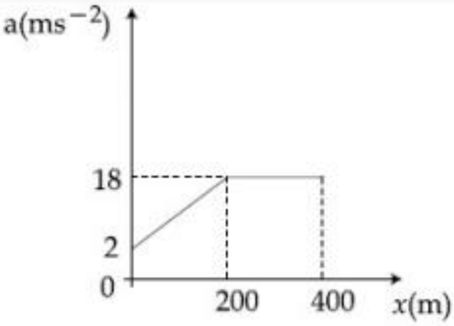

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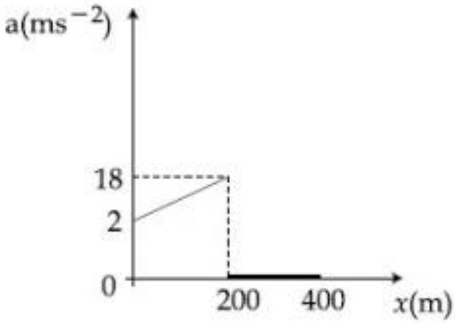
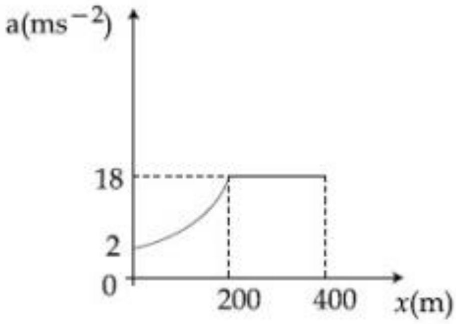


4.



Ans: 3

Sol: For $0 \le x \le 200$

$v = mx + c = \frac{1}{5}x + 10$

$a = v\frac{dv}{dx} = \left(\frac{x}{5} + 10\right)\left(\frac{1}{5}\right) = \frac{x}{25} + 2 \Rightarrow$ straight line till x = 200

For x > 200, v = constant $\Rightarrow$ a = 0

Q.8 In thermodynamics, heat and work are :

Options
1. Extensive thermodynamic state variables
2. Path functions
3. Point functions
4. Intensive thermodynamic state variables

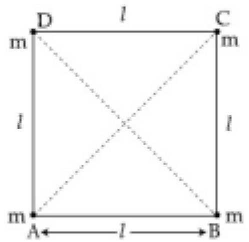
Ans: 2

Sol: According to first law of thermodynamics

$\Delta Q = \Delta U + \Delta W$

Since work done by the gas depends on the type of process (path) and Δu depends just on initial and final states, ΔQ also depends on process (path). $\therefore$ heat and work are treated as path functions in thermodynamics.

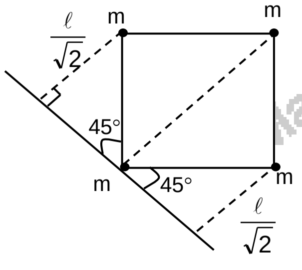
Q.9 Four equal masses, m each are placed at the corners of a square of length (l) as shown in the figure. The moment of inertia of the system about an axis passing through A and parallel to DB would be :



Options
1. $3\ ml^2$
2. ml^2
3. $2\ ml^2$
4. $\sqrt{3}\ ml^2$

Ans: 1

Sol:



Moment of inertia = $m(0)^2 + m\left(\frac{\ell}{\sqrt{2}}\right)^2 + m\left(\ell\sqrt{2}\right)^2 + m\left(\frac{\ell}{\sqrt{2}}\right)^2 = 3m\ell^2$

Q.10 For an electromagnetic wave travelling in free space, the relation between average energy densities due to electric (U_e) and magnetic (U_m) fields is :

Options
1. $U_e > U_m$
2. $U_e = U_m$
3. $U_e \neq U_m$
4. $U_e < U_m$

Ans: 2

Sol: Average energy density due to electric and magnetic fields is same.

Q.11 The pressure acting on a submarine is 3×10^5 Pa at a certain depth. If the depth is doubled, the percentage increase in the pressure acting on the submarine would be :
(Assume that atmospheric pressure is 1×10^5 Pa density of water is 10^3 kg m^{-3}, g = 10 ms^{-2})

Options
1. $\frac{3}{200}\%$
2. $\frac{5}{200}\%$
3. $\frac{200}{5}\%$
4. $\frac{200}{3}\%$

Ans: 4

Sol: $P_1 = \rho gh + P_0 = 3 \times 10^5 P_a$

$\therefore \rho gh = 2 \times 10^5$ Pa

$P_2 = 2\rho gh + P_0 = 4 \times 10^5 + 10^5 = 5 \times 10^5 \ P_a$

$\therefore \%$ increase in pressure $= \frac{P_2 - P_1}{P_1} \times 100 = \frac{5 \times 10^5 - 3 \times 10^5}{3 \times 10^5} \times 100 = \frac{200}{3}\%$

Q.12 A bar magnet of length 14 cm is placed in the magnetic meridian with its north pole pointing towards the geographic north pole. A neutral point is obtained at a distance of 18 cm from the center of the magnet. If $B_H = 0.4$ G, the magnetic moment of the magnet is (1 G = 10^{-4}T)

Options
1. 2.880×10^3 J T^{-1}
2. 2.880×10^2 J T^{-1}
3. 2.880 J T^{-1}
4. 28.80 J T^{-1}

Ans: 3

Sol: $\frac{2\mu_0}{4\pi}\frac{m}{r^2}\times\frac{7}{r} = 0.4\times10^{-4}$

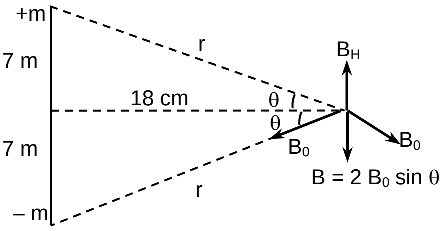
$\Rightarrow 2\times10^{-7}\times\frac{m\times7}{(7^2+18^2)^{3/2}}\times10^4 = 0.4\times10^{-4}$

$m = \frac{4\times10^{-2}\times(373)^{3/2}}{14}$

M = m × 14 cm = m × $\frac{14}{100}$ m

$= \frac{0.04\times(373)^{3/2}}{14}\times\frac{14}{100}$ = 4 × 10^{-4} × 7203.82

= 2.88 J/T



Q.13 Time period of a simple pendulum is T inside a lift when the lift is stationary. If the lift moves upwards with an acceleration g/2, the time period of pendulum will be :

Options

1. $\sqrt{\frac{3}{2}}\,T$
2. $\sqrt{3}T$
3. $\sqrt{\frac{2}{3}}\,T$
4. $\frac{T}{\sqrt{3}}$

Ans: 3

Sol: When lift is stationary, $T = 2\pi\sqrt{\frac{L}{g}}$.

When lift is moving up wards, pseudo force acts downwards.

$g_{eff} = g + \frac{g}{2} = \frac{3g}{2}$

$\therefore$ New time period, $T' = 2\pi\sqrt{\frac{L}{g_{eff}}} = 2\pi\sqrt{\frac{2L}{3g}} = \sqrt{\frac{2}{3}}T$

Q.14 A conducting wire of length 'l', area of cross-section A and electric resistivity ρ is connected between the terminals of a battery. A potential difference V is developed between its ends, causing an electric current.

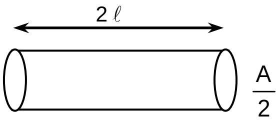
If the length of the wire of the same material is doubled and the area of cross-section is halved, the resultant current would be :

Options

1. $\frac{1}{4}\,\frac{VA}{\rho l}$
2. $\frac{3}{4}\,\frac{VA}{\rho l}$
3. $4\frac{VA}{\rho l}$
4. $\frac{1}{4}\,\frac{\rho l}{VA}$

Ans: 1

Sol: $R = \dfrac{\rho(2\ell)}{A/2} = \dfrac{4\rho\ell}{A}$



$I = \dfrac{V}{R} = \dfrac{V}{\frac{4\rho\ell}{A}} = \dfrac{VA}{4\rho\ell}$

Q.15 The maximum and minimum distances of a comet from the Sun are 1.6×10^{12} m and 8.0×10^{10} m respectively. If the speed of the comet at the nearest point is $6\times10^4\ ms^{-1}$, the speed at the farthest point is :

Options
1. 4.5×10^3 m/s
2. 6.0×10^3 m/s
3. 1.5×10^3 m/s
4. 3.0×10^3 m/s

Ans: 4

Sol: According to law of conservation of angular momentum,
$mv_1r_1 = mv_2r_2$

$v_1 = \dfrac{mv_2r_2}{mr_1} = \dfrac{48\times10^{14}}{1.6\times10^{12}} = 3000\,m/s = 3\times10^3$ m/s

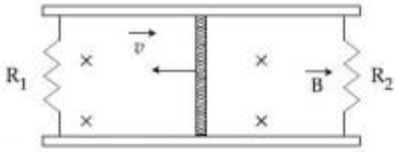
Q.16 The volume V of an enclosure contains a mixture of three gases, 16 g of oxygen, 28 g of nitrogen and 44 g of carbon dioxide at absolute temperature T. Consider R as universal gas constant. The pressure of the mixture of gases is :

Options
1. $\dfrac{4RT}{V}$
2. $\dfrac{88RT}{V}$
3. $\dfrac{3RT}{V}$
4. $\dfrac{5}{2}\dfrac{RT}{V}$

Ans: 4

Sol: $PV = (n_1 + n_2 + n_3)\,RT = \left[\dfrac{16}{32}+\dfrac{28}{28}+\dfrac{44}{44}\right]RT = \dfrac{5}{2}\dfrac{RT}{V}$

Q.17 A conducting bar of length L is free to slide on two parallel conducting rails as shown in the figure



Two resistors R_1 and R_2 are connected across the ends of the rails. There is a uniform magnetic field $\vec{B}$ pointing into the page. An external agent pulls the bar to the left at a constant speed v.

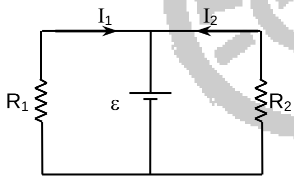
The correct statement about the directions of induced currents I_1 and I_2 flowing through R_1 and R_2 respectively is :

Options

1. Both I_1 and I_2 are in anticlockwise direction
2. I_1 is in anticlockwise direction and I_2 is in clockwise direction
3. I_1 is in clockwise direction and I_2 is in anticlockwise direction
4. Both I_1 and I_2 are in clockwise direction

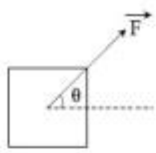
Ans: 3

Sol:



Q.18 A block of mass m slides along a floor while a force of magnitude F is applied to it at an angle θ as shown in figure. The coefficient of kinetic friction is μ_K. Then, the block's acceleration 'a' is given by :

(g is acceleration due to gravity)



Options

1. $\frac{F}{m}\cos\theta + \mu_K\left(g - \frac{F}{m}\sin\theta\right)$
2. $\frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$
3. $\frac{F}{m}\cos\theta - \mu_K\left(g + \frac{F}{m}\sin\theta\right)$
4. $-\frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$

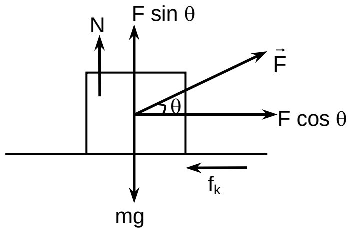
Ans: 2

Sol: $N = mg - f \sin\theta$

$F\cos\theta - \mu_k N = ma$

$F\cos\theta - \mu_k (mg - F\sin\theta) = ma$

$a = \frac{F}{m}\cos\theta - \mu_k\left(g - \frac{F}{m}\sin\theta\right)$



Q.19 The stopping potential in the context of photoelectric effect depends on the following property of incident electromagnetic radiation :

Options
1. Amplitude
2. Frequency
3. Phase
4. Intensity

Ans: 2

Sol: Stopping potential varies linearly with frequency of incident radiation.

Q.20 A plane electromagnetic wave of frequency 500 MHz is travelling in vacuum along y-direction. At a particular point in space and time, $\vec{B} = 8.0 \times 10^{-8}\ \hat{z}\,T$. The value of electric field at this point is :
(speed of light $= 3 \times 10^8\ ms^{-1}$)

$\hat{x}, \hat{y}, \hat{z}$ are unit vectors along x, y and z directions.

Options
1. $-24\ \hat{x}\ V/m$
2. $24\ \hat{x}\ V/m$
3. $-2.6\ \hat{y}\ V/m$
4. $2.6\ \hat{x}\ V/m$

Ans: 1

Sol: $f = 5 \times 10^8$ Hz

Electromagnetic wave is travelling to wards $+\hat{j}$

$\vec{B} = 8 \times 10^{-8}\hat{z}\,T$

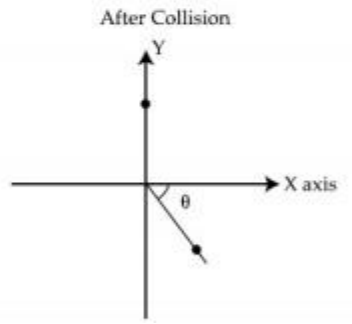
$\vec{E} = \vec{B} \times \vec{C} = \left(8 \times 10^{-8}\hat{z}\right) = \left(3 \times 10^{8}\hat{j}\right) = -24\hat{x}\ Vm^{-1}$

SECTION B

Q.1 A ball of mass 10 kg moving with a velocity $10\sqrt{3}\ \text{m s}^{-1}$ along X-axis, hits another ball of mass 20 kg which is at rest. After collision, the first ball comes to rest and the second one disintegrates into two equal pieces. One of the pieces starts moving along Y-axis at a speed of 10 m/s. The second piece starts moving at a speed of 20 m/s at an angle θ (degree) with respect to the X-axis.

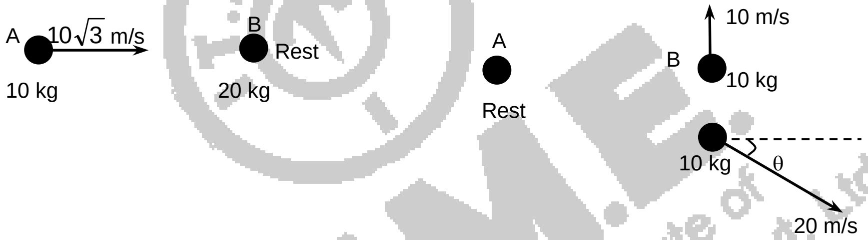
The configuration of pieces after collision is shown in the figure.

The value of θ to the nearest integer is _________.



Ans: 30.00

Sol: **Before collision** **After collision**



Applying low of conservation of momentum along x-axis $\vec{P}_i = \vec{P}_f$

$10 \times 10\sqrt{3} = 200\ \cos\theta$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30°$

Q.2 The first three spectral lines of H-atom in the Balmer series are given $\lambda_1, \lambda_2, \lambda_3$ considering the Bohr atomic model, the wave lengths of first and third spectral lines $\left(\frac{\lambda_1}{\lambda_3}\right)$ are related by a factor of approximately 'x' $\times 10^{-1}$.

The value of x, to the nearest integer, is _________.

Ans: 15.00

Sol: For first line

$$\frac{1}{\lambda_1} = RZ^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = RZ^2 \times \frac{5}{36} ------(1)$$

For third line,

$$\frac{1}{\lambda_3} = RZ^2\left(\frac{1}{2^2} - \frac{1}{5^2}\right) = RZ^2 \times \frac{21}{100} ------(2)$$

$$(2) \div (1) \Rightarrow \frac{\lambda_3}{\lambda_1} = \frac{21}{100} \times \frac{36}{5} = 1.512 = 15.12 \times 10^{-1} \approx 15$$

Q.3 Consider a 20 kg uniform circular disk of radius 0.2 m. It is pin supported at its center and is at rest initially. The disk is acted upon by a constant force F = 20 N through a massless string wrapped around its periphery as shown in the figure.

Suppose the disk makes n number of revolutions to attain an angular speed of 50 rad s^{-1}.

The value of n, to the nearest integer, is ________.

[Given : In one complete revolution, the disk rotates by 6.28 rad]

Ans: 20.00

Sol: $\tau = I\alpha \Rightarrow \alpha = \frac{\tau}{I} = \frac{FR}{\frac{mR^2}{2}} = \frac{2F}{mR}$

$\alpha = \frac{2 \times 200}{20 \times 0.2} = 10 \text{ rad/s}^2$

$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

$50^2 = 0^2 + 2 \times 10 \times \Delta\theta$

$\Delta\theta = \frac{2500}{20} = 125 \text{ rad}$

Number of revolution = $\frac{125}{2\pi} \approx 20$ revolutions

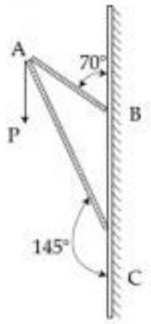
Q.4 The resistance $R = \frac{V}{I}$, where $V = (50 \pm 2)V$ and $I = (20 \pm 0.2)A$. The percentage error in R is 'x' %.

The value of 'x' to the nearest integer is ________.

Ans: 5.00

Sol: $\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100 = \frac{2}{50} \times 100 + \frac{0.2}{20} \times 100 = 4 + 1 = 5\%$

Q.5 Consider a frame that is made up of two thin massless rods AB and AC as shown in the figure. A vertical force $\vec{P}$ of magnitude 100 N is applied at point A of the frame.



Suppose the force is $\vec{P}$ resolved parallel to the arms AB and AC of the frame.

The magnitude of the resolved component along the arm AC is xN.

The value of x, to the nearest integer, is _________.

[Given : $\sin(35°) = 0.573$, $\cos(35°) = 0.819$

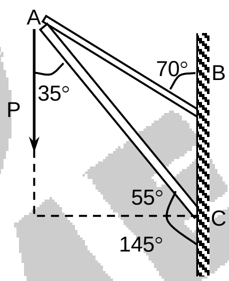
$\sin(110°) = 0.939$, $\cos(110°) = -0.342$]

Ans: 82.00

Sol: Component along

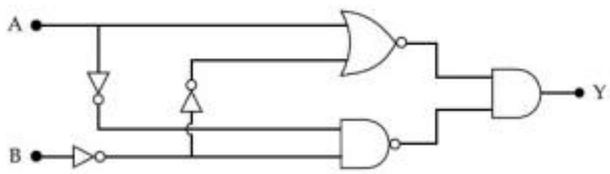
AC = 100 cos 35

= 100 × 0.819 = 81.9 N = 82 N



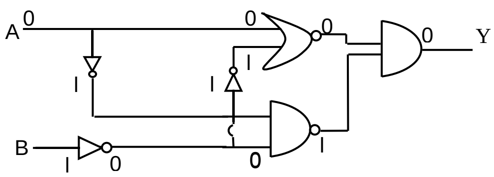
Q.6 In the logic circuit shown in the figure, if input A and B are 0 to 1 respectively, the output at Y would be 'x'.

The value of x is _________.

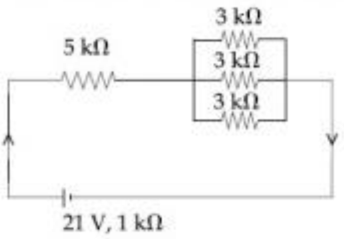


Ans: 0.00

Sol:



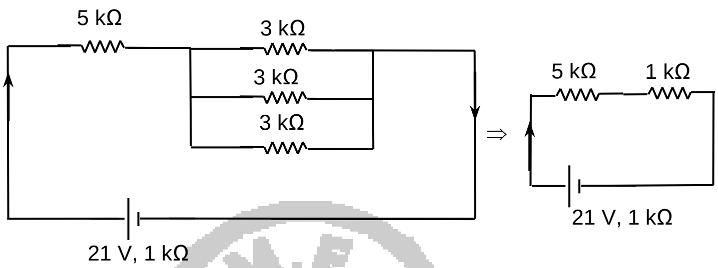
Q.7 In the figure given, the electric current flowing through the 5 kΩ resistor is 'x' mA.



The value of x to the nearest integer is ________.

Ans: 3.00

Sol:



$$I = \frac{21}{5+1+1} = 3\text{mA}$$

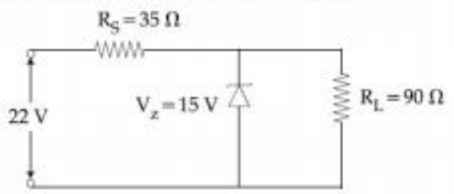
Q.8 A fringe width of 6 mm was produced for two slits separated by 1 mm apart. The screen is placed 10 m away. The wavelength of light used is 'x' nm.

The value of 'x' to the nearest integer is ________.

Ans: 600.00

Sol: $\beta = \frac{D}{d}\lambda \Rightarrow \lambda = \frac{\beta d}{D} = \frac{6\times10^{-3}\times10^{-3}}{10}$ = 6×10^{-7} = 600×10^{-9} m = 600 nm

Q.9 The value of power dissipated across the zener diode ($V_z = 15$ V) connected in the circuit as shown in the figure is $x \times 10^{-1}$ watt.



The value of x, to the nearest integer, is ________.

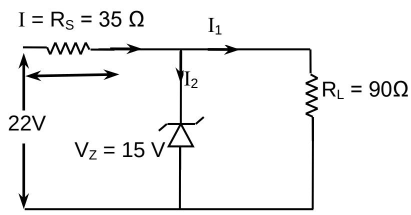
Ans: 5.00

Sol: Voltage across R_s = 22 – 15 = 7 V

Current through R_s = $I = \frac{7}{35} = \frac{1}{5}$ A

Current through 90 Ω = $I_2 = \frac{15}{90} = \frac{1}{6}$ A

Current through zener = $\frac{1}{5} - \frac{1}{6} = \frac{1}{30}$ A



Power through zener diode, P = VI $= 15\times\frac{1}{30} = 0.5\text{ W} = 5\times10^{-1}\text{W}$

Q.10 A sinusoidal voltage of peak value 250 V is applied to a series LCR circuit, in which $R = 8\ \Omega$, $L = 24$ mH and $C = 60\ \mu F$. The value of power dissipated at resonant condition is 'x' kW. The value of x to the nearest integer is ________.

Ans: **4.00**

Sol: At resonance,

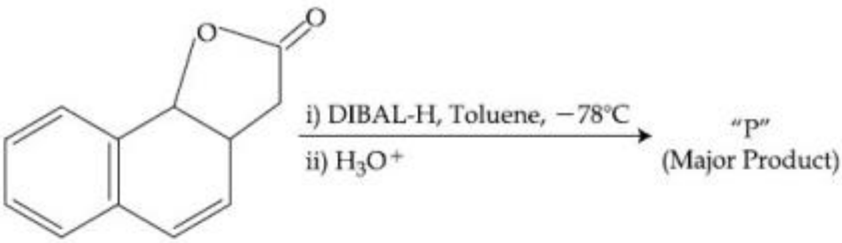
$$P = \frac{V_{rms}^2}{R} = \frac{\left(250/\sqrt{2}\right)^2}{8} = 3906.25\ W$$

$\approx 4\,KW$

PART – B – CHEMISTRY

SECTION A

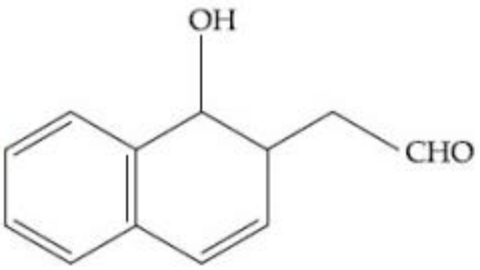
Q.1



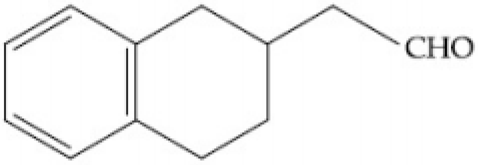
The product "P" in the above reaction is :

Options

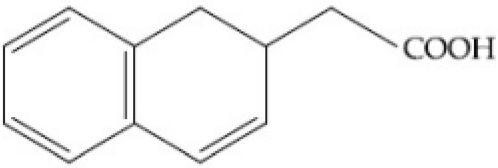
1.



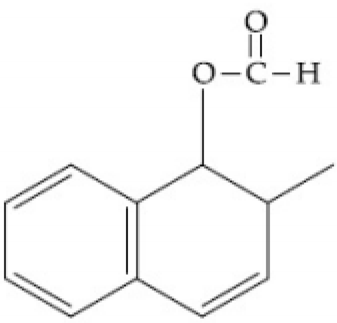
2.



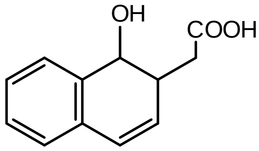
3.



4.



Ans: **1**

Sol: The product formed is 

DIBAL-H reduce the ester group into CHO group and it does not affect C=C

Q.2 Assertion A : Enol form of acetone [CH_3COCH_3] exists in < 0.1% quantity. However, the enol form of acetyl acetone [$CH_3COCH_2OCCH_3$] exists in approximately 15% quantity.

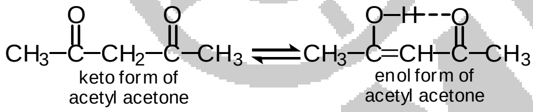
Reason R : Enol form of acetyl acetone is stabilized by intramolecular hydrogen bonding, which is not possible in enol form of acetone.

Choose the correct statement :

Options
1. **A** is false but **R** is true
2. Both **A** and **R** are true and **R** is the correct explanation of **A**
3. Both **A** and **R** are true but **R** is not the correct explanation of **A**
4. **A** is true but **R** is false

Ans: **2**

Sol: The enol form of acetone exist in less than 0.1%



The enol form of acetyl acetone dominate because of intramolecular hydrogen bonding as shown above

Q.3 Match List - I with List - II :

	List - I Name of oxo acid		List - II Oxidation state of 'P'
(a)	Hypophosphorous acid	(i)	+5
(b)	Orthophosphoric acid	(ii)	+4
(c)	Hypophosphoric acid	(iii)	+3
(d)	Orthophosphorous acid	(iv)	+2
		(v)	+1

Choose the correct answer from the options given below :

Options
1. (a)-(v), (b)-(i), (c)-(ii), (d)-(iii)
2. (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)
3. (a)-(iv), (b)-(v), (c)-(ii), (d)-(iii)
4. (a)-(v), (b)-(iv), (c)-(ii), (d)-(iii)

Ans: 1

Sol:

	Formula	Oxidation state
Hypophosphorous acid	– H_3PO_2	+1
Orthophosphorous acid	– H_3PO_4	+5
Hypophosphoric acid	– $H_4P_2O_6$	+4
Orthophosphorous acid	– H_3PO_3	+3

Q.4 Given below are two statement : one is labelled as **Assertion A** and the other is labelled as **Reason R** :

Assertion A : Size of Bk^{3+} ion is less than Np^{3+} ion.

Reason R : The above is a consequence of the lanthanoid contraction.

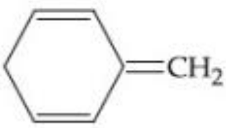
In the light of the above statements, choose the **correct** answer from the options given below :

Options
1. Both **A** and **R** are true but **R** is not the correct explanation of **A**
2. **A** is true but **R** is false
3. **A** is false but **R** is true
4. Both **A** and **R** are true and **R** is the correct explanation of **A**

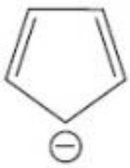
Ans: 2

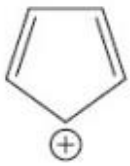
Sol: BK^{3+} (atomic no: 97) is smaller than Np^{3+} (atomic no. 93) due to actinoid contraction.

Q.5 Among the following, the aromatic compounds are :

(A) 

(B) 

(C) 

(D) 

Choose the **correct** answer from the following options :

Options
1. (B), (C) and (D) only
2. (B) and (C) only
3. (A), (B) and (C) only
4. (A) and (B) only

Ans: 2

Sol: (A) is non-aromatic, since conjugation is not complete
(B) is aromatic, obey (4n +2) rule as it contain 6 decloalised electrons
(C) is aromatic, obey (4n +2) rule with 6 delocalised electrons
(D) is antiaromatic, as it contain 4 delocalised electrons.

Q.6 Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R** :

Assertion A : The H – O – H bond angle in water molecule is 104.5°.

Reason R : The lone pair - lone pair repulsion of electrons is higher than the bond pair - bond pair repulsion.

In the light of the above statements, choose the **correct** answer from the options given below :

Options

1. Both **A** and **R** are true, but **R** is not the correct explanation of **A**
2. **A** is false but **R** is true
3. Both **A** and **R** are true, and **R** is the correct explanation of **A**
4. **A** is true but **R** is false

Ans: **3**

Sol: The hybridization of central atom in H_2O is sp^3. The bond angle reduces to 104.5° due to lone pair-lone pair repulsion

Q.7 Given below are two statements :

Statement I : H_2O_2 can act as both oxidising and reducing agent in basic medium.

Statement II : In the hydrogen economy, the energy is transmitted in the form of dihydrogen.

In the light of the above statements, choose the **correct** answer from the options given below :

Options

1. Statement I is true but statement II is false
2. Statement I is false but statement II is true
3. Both statement I and statement II are true
4. Both statement I and statement II are false

Ans: **3**

Sol: $H_2O_2 + 2Fe^{2+} \rightarrow 2Fe^{3+} + 2OH^-$ (as oxidizing agent)

$3H_2O_2 + 2MnO_4^- \rightarrow 2MnO_2 + 3O_2 + 2OH^-$ (as reducing agent)

The transportation and storage of energy in the form liquid or gaseous H_2 is the principle of hydrogen bonding

Q.8 Given below are two statements :

Statement I : The E° value for Ce^{4+}/Ce^{3+} is +1.74 V.

Statement II : Ce is more stable in Ce^{4+} state than Ce^{3+} state.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

Options

1. Both statement I and statement II are correct
2. Statement I is correct but statement II is incorrect
3. Statement I is incorrect but statement II is correct
4. Both statement I and statement II are incorrect

Ans: **2**

Sol: The E° value of Ce^{4+} / Ce^{3+} is +1.74 V, that means Ce^{4+} can easily get reduced to Ce^{3+}

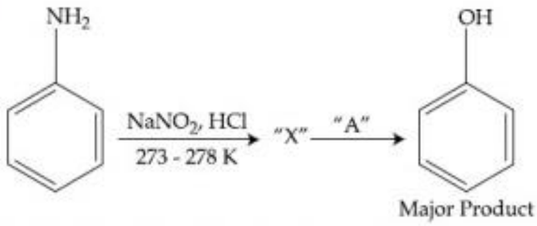
Q.9 Which among the following pairs of Vitamins is stored in our body relatively for longer duration ?

Options
1. Thiamine and Vitamin A
2. Thiamine and Ascorbic acid
3. Vitamin A and Vitamin D
4. Ascorbic acid and Vitamin D

Ans: **3**

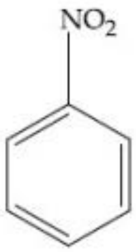
Sol: Vitamin A and Vitamin D are fat soluble vitamins

Q.10

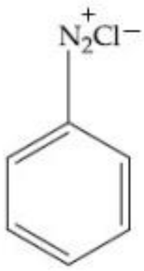


In the above chemical reaction, intermediate "X" and reagent/condition "A" are :

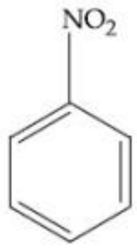
Options

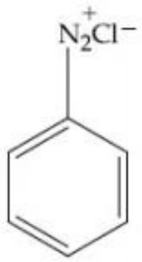
1. X - 

; A - $H_2O/NaOH$

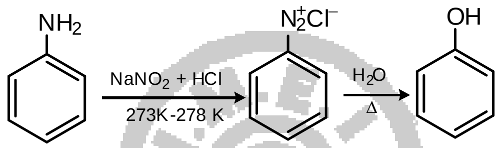
2. X - 

; A - $H_2O/NaOH$

3. X -  ; A - H_2O/Δ

4. X -  ; A - H_2O/Δ

Ans: 4

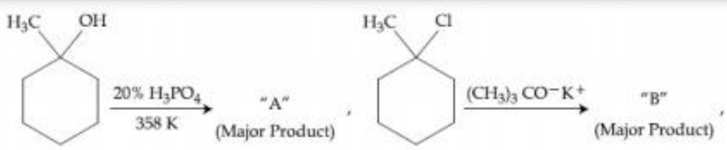
Sol: 

Q.11 In chromotography technique, the purification of compound is independent of :

Options
1. Length of the column or TLC plate
2. Mobility or flow of solvent system
3. Physical state of the pure compound
4. Solubility of the compound

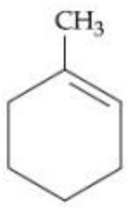
Ans: 3

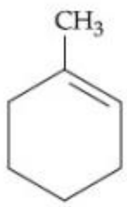
Sol: In chromatography, the purification of a compound is independent of the physical state of the physical state of the pure compound.

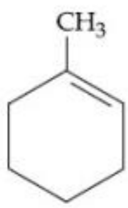
Q.12 

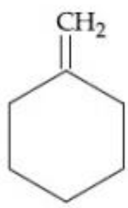
The products "A" and "B" formed in above reactions are :

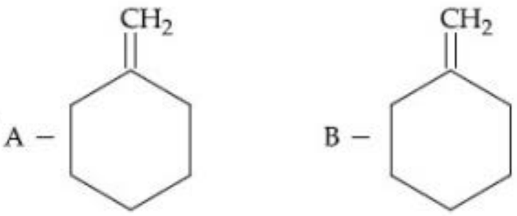
Options

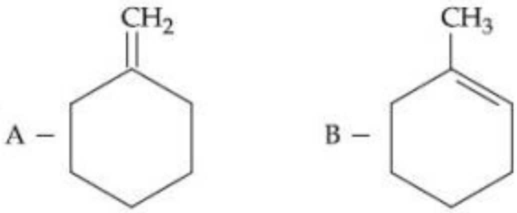
1. A – 

B – 

2. A – 

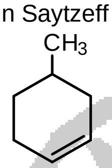
B – 

3. 

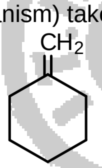
4. 

Ans: 2

Sol: When a tertiary alcohols is treated with 20% H_3PO_4, dehydration (E_1 mechanism) takes place resulting in Saytzeff product.

$\therefore$A is 

When alkyl halide is treated with tertiary butoxide (bulky base) dehydrohalogenation (E_2 mechanism) takes place resulting Hoffmann product

$\therefore$ B is 

Q.13 The process that involves the removal of sulphur from the ores is :

Options
1. Refining
2. Leaching
3. Smelting
4. Roasting

Ans: 4

Sol: During roasting sulphur is removed as SO_2

Q.14 A group 15 element, which is a metal and forms a hydride with strongest reducing power among group 15 hydrides. The element is :

Options
1. Sb
2. As
3. Bi
4. P

Ans: 3

Sol: Bismuth is a metal
The reducing character of hydrides increases down the group

Q.15 Which of the following is Lindlar catalyst ?

Options
1. Partially deactivated palladised charcoal
2. Zinc chloride and HCl
3. Cold dilute solution of $KMnO_4$
4. Sodium and Liquid NH_3

Ans: 1

Sol: Partially deactivated palladised charcoal is Lindlar catalyst

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Q.16 Given below are two statements :

Statement I : Both $CaCl_2 \cdot 6H_2O$ and $MgCl_2 \cdot 8H_2O$ undergo dehydration on heating.

Statement II : BeO is amphoteric whereas the oxides of other elements in the same group are acidic.

In the light of the above statements, choose the correct answer from the options given below :

Options
1. Statement I is true but statement II is false
2. Both statement I and statement II are true
3. Statement I is false but statement II is true
4. Both statement I and statement II are false

Ans: 4

Sol: $CaCl_2.6H_2O \xrightarrow{\Delta} CaCl_2 + 6H_2O$ (dehydration by heating is possible)

$MgCl_2.8H_2O \xrightarrow{\Delta} MgO + 2HCl + 7H_2O$ (heating results in oxide)

among the oxides of alkaline earth metals BeO is amphoteric others are basic

---

Q.17 Match List - I with List - II :

	List - I Industrial process		List - II Application
(a)	Haber's process	(i)	HNO_3 synthesis
(b)	Ostwald's process	(ii)	Aluminium extraction
(c)	Contact process	(iii)	NH_3 synthesis
(d)	Hall-Heroult process	(iv)	H_2SO_4 synthesis

Choose the correct answer from the options given below :

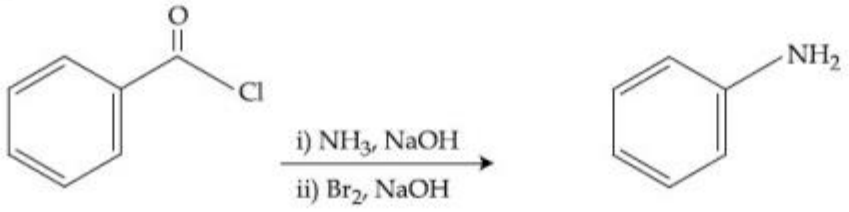
Options
1. (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)
2. (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)
3. (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)
4. (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

Ans: 2

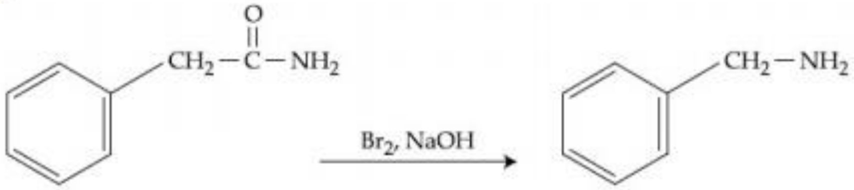
Sol: Haber's process – NH_3 synthesis
Ostwald's process – HNO_3 synthesis
Contact process – H_2SO_4 synthesis
Hall-Heroult process – Aluminium extraction

Q.18 Which of the following reaction DOES NOT involve Hoffmann bromamide degradation ?

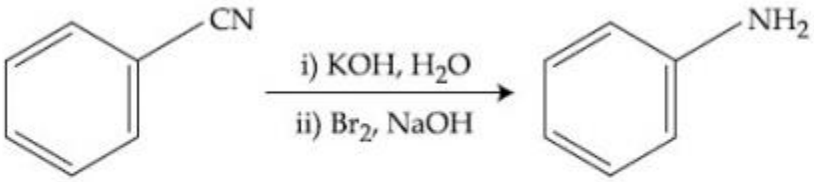
Options 1.



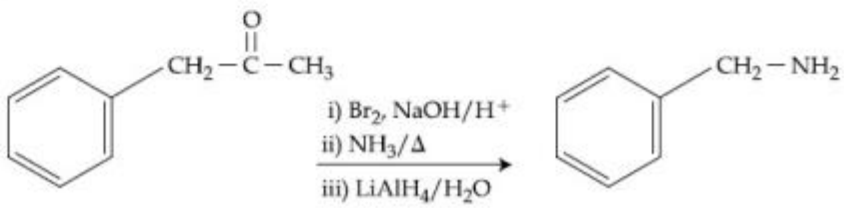
2.



3.

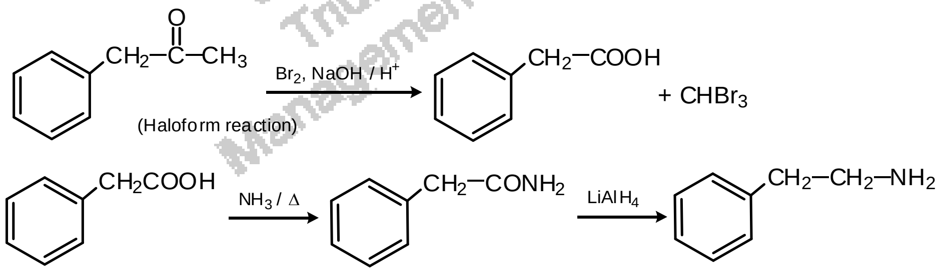


4.



Ans: 4

Sol:



Q.19 The functions of antihistamine are :

Options
1. Antiallergic and antidepressant
2. Antacid and antiallergic
3. Analgesic and antacid
4. Antiallergic and Analgesic

Ans: 2

Sol: Antihistamines are antacid and antiallergic

Q.20 The type of pollution that gets increased during the day time and in the presence of O_3 is :

Options
1. Global warming
2. Acid rain
3. Reducing smog
4. Oxidising smog

Ans: 3

Sol: In the presence of O_3 during day time (presence of sunlight) oxidizing smog get increased

SECTION A

Q.1 Complete combustion of 750 g of an organic compound provides 420 g of CO_2 and 210 g of H_2O. The percentage composition of carbon and hydrogen in organic compound is 15.3 and _________ respectively. (Round off to the Nearest Integer).

Ans: 3

Sol: $\%\ \text{Hydrogen} = \frac{2}{13} \times \frac{210}{750} \times 100 \approx 3\%$

Q.2 The decomposition of formic acid on gold surface follows first order kinetics. If the rate constant at 300 K is $1.0 \times 10^{-3}\ s^{-1}$ and the activation energy $E_a = 11.488\ kJ\ mol^{-1}$, the rate constant at 200 K is _________ $\times 10^{-5}\ s^{-1}$. (Round off to the Nearest Integer).
(Given : $R = 8.314\ J\ mol^{-1}\ K^{-1}$)

Ans: 10

Sol:

$$\log \frac{k_2}{k_1} = \frac{E_a}{2.303\,R}\left[\frac{T_2 - T_1}{T_1 T}\right]$$

$$\log \frac{k_{300}}{k_{200}} = \frac{11.48 \times 100}{2.303 \times 8.314 \times 10^{-3} \times 200 \times 300} \approx 1 \qquad \text{i.e.,} \frac{k_{300}}{k_{200}} = 10$$

$$k_{200} = \frac{1}{10} \times k_{300} = \frac{1}{10} \times 10^{-3} = 10 \times 10^{-5}$$

Q.3 Two salts A_2X and MX have the same value of solubility product of 4.0×10^{-12}. The ratio of their molar solubilities i.e. $\frac{S(A_2X)}{S(MX)}$ = ________. (Round off to the Nearest Integer).

Ans: 50

Sol: For A_2X

Solubility $K_{sp} = 4S_1^3 = 4 \times 10^{-12}$

$S_1 = 10^{-4}$

For MX

Solubility product $K_{sp} = S_2^2 = 4 \times 10^{-12}$

$S_2 = 2 \times 10^{-6}$

$\therefore \frac{S_1}{S_2} = \frac{10^{-4}}{2 \times 10^{-6}} = 50$

Q.4 A certain element crystallises in a bcc lattice of unit cell edge length 27Å. If the same element under the same conditions crystallises in the fcc lattice, the edge length of the unit cell in Å will be ________. (Round off to the Nearest Integer).

[Assume each lattice point has a single atom]

[Assume $\sqrt{3} = 1.73$, $\sqrt{2} = 1.41$]

Ans: 33

Sol: For a BCC lattice radius $r = \frac{\sqrt{3}\,a}{4} = \frac{\sqrt{3} \times 27}{4}$

Since the same element from the FCC lattice

Edge length of FCC unit cell $a = 2\sqrt{2}\,r$

$$2\sqrt{2} \times \frac{\sqrt{3} \times 27}{4} = \frac{\sqrt{3} \times 27}{\sqrt{2}} = 33.12\ \text{Å}$$

Q.5 A 6.50 molal solution of KOH (aq.) has a density of 1.89 g cm^{-3}. The molarity of the solution is ________ mol dm^{-3}. (Round off to the Nearest Integer).

[Atomic masses : K : 39.0 u; O : 16.0 u; H : 1.0 u]

Ans: 9

Sol: let the weight of solvent = 1000 g

No. of moles of KOH = 6.5

Weight of solute = 6.5 × 56 = 364 g

Weight of solution = 1000 +364 = 1364 g

Volume of solution $= \frac{\text{weight}}{\text{density}} = \frac{1364}{1.89} = 721.69$ mL

Q.6 $2\,MnO_4^- + b\,C_2O_4^{2-} + c\,H^+ \rightarrow x\,Mn^{2+} + y\,CO_2 + z\,H_2O$

If the above equation is balanced with integer coefficients, the value of c is ________. (Round off to the Nearest Integer).

Ans: 16

Sol: The balanced equation is

$2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \rightarrow 10CO_2 + 2Mn^{2+} + 8H_2O$

Q.7 AB_2 is 10% dissociated in water to A^{2+} and B^-. The boiling point of a 10.0 molal aqueous solution of AB_2 is _________°C. (Round off to the Nearest Integer).

[Given : Molal elevation constant of water $K_b = 0.5$ K kg mol^{-1} boiling point of pure water = 100°C]

Ans: **106**

Sol: $AB_2 \rightleftharpoons A^{2+} + 2B^-$

$i = 1 + (n - 1)\alpha = 1 + (3 - 1) \times 0.1 = 1.2$

$\Delta T_b = i \times K_b \times m = 1.2 \times 0.5 \times 10 = 6$

$\therefore$ Boiling point = 106°C

Q.8 For the reaction $A(g) \rightleftharpoons B(g)$ at 495 K, $\Delta_r G^\circ = -9.478$ kJ mol^{-1}.

If we start the reaction in a closed container at 495 K with 22 millimoles of A, the amount of B in the equilibrium mixture is _________ millimoles. (Round off to the Nearest Integer).

[R = 8.314 J mol^{-1} K^{-1} ; ln 10 = 2.303]

Ans: **20**

Sol: $\Delta G^\circ = -RT \ln k$

$$\ln k = \frac{-9.478 \times 1000}{-495 \times 8.314} = 2.303$$

k = 10

	$A_{(g)} \rightleftharpoons$	$B_{(g)}$
Initial	2.2	–
At eqbm	(22-x)	x

$$k = \frac{[B]}{[A]} = \frac{x}{22 - x} = 10 \qquad \text{i.e., } x = 20$$

Milli moles of B formed = 20

Q.9 When light of wavelength 248 nm falls on a metal of threshold energy 3.0 eV, the de-Broglie wavelength of emitted electrons is _________ Å. (Round off to the Nearest Integer).

[Use : $\sqrt{3} = 1.73$, $h = 6.63 \times 10^{-34}$ Js

$m_e = 9.1 \times 10^{-31}$ kg ; $c = 3.0 \times 10^8$ ms^{-1} ; 1eV = 1.6×10^{-19}J]

Ans: **9**

Sol: Energy of the reaction $= \dfrac{hc}{\lambda} = \dfrac{6.63 \times 10^{-34} \times 3 \times 10^8}{248 \times 10^{-9} \times 1.6 \times 10^{-19}} = 5\,\text{eV}$

Kinetic energy of photoelectron = 5 – 3 = 2eV

de-Broglie wavelength $\lambda = \dfrac{h}{\sqrt{2mE}} = \dfrac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 2 \times 1.6 \times 10^{-9}}}$

$= 8.70 \times 10^{-10}$ m ≈ 9 Å

Q.10 The equivalents of ethylene diamine required to replace the neutral ligands from the coordination sphere of the trans-complex of $CoCl_3 \cdot 4NH_3$ is _________. (Round off to the Nearest Integer).

Ans: **2**

Sol: In trans-$[Co(NH_3)_4Cl_2]Cl$ there are four neutral ligands (NH_3)

Since ethylene diamine is a bidentate ligand, we need 2 ethylene diamine to replace all neutral ligands present in trans-$[Co(NH_3)_4Cl_2]Cl$

PART – C – MATHEMATICS

SECTION A

Q.1 The number of elements in the set $\{x \in \mathbf{R} : (|x| - 3)\,|x + 4| = 6\}$ is equal to :

Options
1. 1
2. 3
3. 4
4. 2

Ans: 4

Sol: $(|x| - 3)\,(|x + 4|) = 6$

Case 1

$x \geq 0$

$\Rightarrow (x - 3)(x + 4) = 6$

$\Rightarrow x^2 + x - 12 - 6 = 0$

$x^2 + x - 18 = 0$

$x = \dfrac{-1 \pm \sqrt{1 + 72}}{2} = \dfrac{-1 \pm \sqrt{73}}{2}$

$\Rightarrow x = \dfrac{-1 + \sqrt{73}}{2}$ (1 Solution)

Case 2

$-4 \leq x < 0$

$(-x - 3)\ (x + 4) = 6$

$\Rightarrow (x + 3)\ (x + 4) = -6$

$x^2 + 7x + 12 + 6 = 0 \Rightarrow x^2 + 7x + 18 = 0$

$x = \dfrac{-7 \pm \sqrt{49 - 72}}{2} \Rightarrow$ No solution

Case 3

$x \leq -4$

$(-x - 3)(-x - 4) = 6 \Rightarrow (x + 3)(x + 4) = 6$

$\Rightarrow x^2 + 7x + 6 = 0 \Rightarrow (x + 6)(x + 1) = 0$

$x = -6$ or $-1 \Rightarrow x = -6 (\because x \leq -4)$

$\Rightarrow 1$ solution

Q.2 Let $S_k = \sum_{r=1}^{k} \tan^{-1}\left(\dfrac{6^r}{2^{2r+1} + 3^{2r+1}}\right)$. Then $\lim_{k \to \infty} S_k$ is equal to :

Options
1. $\tan^{-1}(3)$
2. $\dfrac{\pi}{2}$
3. $\cot^{-1}\left(\dfrac{3}{2}\right)$
4. $\tan^{-1}\left(\dfrac{3}{2}\right)$

Ans: 3

Sol: $S_k = \sum_{r=1}^{k} \tan^{-1}\left(\frac{6^r}{2^{2r+1}+3^{2r+1}}\right)$

I^st: $\tan^{-1}\left(\frac{6}{2^3+3^3}\right)$

$= \tan^{-1}\left(\frac{6/2^3}{1+3^3/2^3}\right) = \tan^{-1}\left[\frac{\left(\frac{3}{2}\right)^2 - \frac{3}{2}}{1+\left(\frac{3}{2}\right)^2 \frac{3}{2}}\right]$

$= \tan^{-1}\left(\frac{3}{2}\right)^2 - \tan^{-1}\left(\frac{3}{2}\right)$

2 nd: $\tan^{-1}\left(\frac{6^2}{2^5+3^5}\right) = \tan^{-1}\left(\frac{6^2/2^5}{1+\left(\frac{3}{2}\right)^5}\right)$

$= \tan^{-1}\left(\frac{\left(\frac{3}{2}\right)^3 - \left(\frac{3}{2}\right)^2}{1+\left(\frac{3}{2}\right)^3\left(\frac{3}{2}\right)^2}\right) = \tan^{-1}\left(\frac{3}{2}\right)^3 - \tan^{-1}\left(\frac{3}{2}\right)^2$ etc.

k^{th} term: $\tan^{-1}\left(\frac{3}{2}\right)^{k+1} - \tan^{-1}\left(\frac{3}{2}\right)^k$

$S_k = \tan^{-1}\left(\frac{3}{2}\right)^{k+1} - \tan^{-1}\left(\frac{3}{2}\right)$

$\lim_{k\to\infty} S_k = \tan^{-1}\infty - \tan^{-1}\frac{3}{2} = \frac{\pi}{2} - \tan^{-1}\frac{3}{2} = \cot^{-1}\frac{3}{2}$

Q.3

Let a complex number z, $|z| \neq 1$, satisfy $\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right) \leq 2$. Then, the largest value of $|z|$ is equal to ________.

Options
1. 7
2. 6
3. 8
4. 5

Ans: **1**

Sol: $\log_{\left(\frac{1}{\sqrt{2}}\right)}\left(\frac{|z|+11}{(|z|-1)^2}\right) \leq 2$

$\frac{|z|+11}{(|z|-1)^2} \geq \left(\frac{1}{\sqrt{2}}\right)^2$

$\frac{|z|+11}{(|z|-1)^2} \geq \frac{1}{2}$

Let $|z| = x$

$$\frac{x+11}{x^2-2x+1}-\frac{1}{2}\geq 0$$
$$\frac{2x+22-x^2+2x-1}{x^2-2x+1}\geq 0$$
$$\frac{x^2-4x-21}{x^2-2x+1}\leq 0$$
$$\Rightarrow \frac{(x-3)(x-7)}{(x-1)^2}\leq 0$$

$$|z|_{max}=7$$

Q.4 Let a vector $\alpha\hat{i}+\beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\,\hat{i}+\hat{j}$ by an angle 45° about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices (α, β), $(0, \beta)$ and $(0, 0)$ is equal to :

Options
1. 1
2. $\frac{1}{2}$
3. $\frac{1}{\sqrt{2}}$
4. $2\sqrt{2}$

Ans: 2

Sol: $\vec{a}=\sqrt{3}\hat{i}+\hat{j}=2(\cos 30\hat{i}+\sin 30\hat{j})$

$\alpha\hat{i}+\beta\hat{j}=2(\cos(30+45)\hat{i}+\sin(30+45)\hat{j})$

$\Rightarrow \alpha=2\cos 75$ and $\beta=2\sin 75$

Area $=\frac{1}{2}\begin{vmatrix}\alpha & \beta & 1\\ 0 & \beta & 1\\ 0 & 0 & 1\end{vmatrix}=\frac{1}{2}|\alpha\beta|$

$\frac{1}{2}\times 2\cos 75\times 2\sin 75=2\sin 75\cos 75$

$=\sin(2\times 75)=\sin 150=\sin(180-30)=\frac{1}{2}$

Q.5 Let $A=\begin{bmatrix} i & -i\\ -i & i\end{bmatrix}$, $i=\sqrt{-1}$. Then, the system of linear equations $A^8\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}8\\64\end{bmatrix}$ has :

Options
1. No solution
2. Infinitely many solutions
3. Exactly two solutions
4. A unique solution

Ans: 1

Sol: $A = \hat{i}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$

$A^2 = -1\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = -\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix} = \begin{bmatrix}-2 & 2\\ 2 & -2\end{bmatrix}$

$A^4 = (A^2)^2, A^8 = (A^4)^2 A^2$

We get $A^8 = \begin{bmatrix}128 & -128\\ -128 & 128\end{bmatrix}$

$\begin{bmatrix}128 & -128\\ -128 & 128\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}8\\ 64\end{bmatrix}$

$\Rightarrow 128x - 128y = 8$

and $-128x + 128y = 64$

$(1) + (2) \Rightarrow 0 = 8 + 64$(wrong)

$\therefore$ No Solution

Q.6 Let $[x]$ denote greatest integer less than or equal to x. If for $n \in N$,

$(1 - x + x^3)^n = \sum_{j=0}^{3n} a_j x^j$, then

$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1}$ is equal to :

Options
1. n
2. 2^{n-1}
3. 1
4. 2

Ans: 3

Sol: $(1 - x + x^3)^n = a_0 + a_1x^1 + a_2x^2 + ...a_{3n}x^{3n}$

Let n = 1

$\Rightarrow a_0 = 1, a_1 = -1, a_2 = 0, a_3 = 1$

Que $\Rightarrow \sum_{j=0}^{1} a_{2j} + 4\sum_{j=0}^{1} a_{2j+1} = a_0 + a_2 + 4(a_1 + a_3) = 1 + 0 + 4(-1+1) = 1$

(1), (2) and (3) matching

Let n = 2

$(1 - x + x^3)^2 = 1 + 2(-x + x^3) + (-x + x^3)^2$

$= 1 - 2x + 2x^3 + x^2 - 2x^4 + x^6 = 1 - 2x + x^2 + 2x^3 - 2x^4 + x^6$

$a_0 = 1, a_1 = -2, a_2 = 1, a_3 = 2, \quad a_4 = -2, \quad a_5 = 0, \ a_6 = 1$

Que $\Rightarrow \sum_{j=0}^{3} a_{2j} + 4\sum_{j=0}^{2} a_{2j+1}$

$a_0 + a_2 + a_4 + a_6 + 4(a_1 + a_3 + a_5) = (1 + 1 + -2 + 1) + 4(-2 + 2 + 0) = 1$

Q.7 If $y = y(x)$ is the solution of the differential equation, $\frac{dy}{dx} + 2y\tan x = \sin x$, $y\left(\frac{\pi}{3}\right) = 0$, then the maximum value of the function $y(x)$ over $\mathbb{R}$ is equal to :

Options

1. $\frac{1}{8}$
2. $\frac{1}{2}$
3. 8
4. $-\frac{15}{4}$

Ans: 1

Sol: $\frac{dy}{dx} + 2y\tan x = \sin x$

$I.F = e^{\int 2\tan x\, dx} \quad e^{2\log(\sec x)} = e^{\log(\sec x)^2} = \sec^2 x$

$y.\sec^2 x = \int \sin x.\sec^2 x dx = \int \sec x \tan x dx = \sec x + C$

$y = \cos x + C\ \cos^2 x$

$0 = \cos\frac{\pi}{3} + C.\ \cos^2\frac{\pi}{3} \Rightarrow 0 = \frac{1}{2} + C\frac{1}{4} \Rightarrow C = -2$

$\therefore y = \cos x - 2\cos^2 x$

$y = -2\left(\cos^2 x - \frac{1}{2}\cos x\right)$

$= -2\left[\left(\cos x - \frac{1}{4}\right)^2 - \frac{1}{16}\right] = -2\left(\cos x - \frac{1}{4}\right)^2 + \frac{1}{8}$

$\therefore\ y_{max} = \frac{1}{8}$

Q.8 The number of roots of the equation,

$(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$

in the interval $[0, \pi]$ is equal to :

Options

1. 2
2. 3
3. 8
4. 4

Ans: 4

Sol: Let $81^{\sin^2 x} = t$

$\Rightarrow 81^{\sin^2 x} = 81^{(1-\sin^2 x)} = \frac{81}{t}$

$\therefore t + \frac{81}{t} = 30 \text{ where } 0 \le t \le 81$

$t^2 + 81 = 30t \Rightarrow t^2 - 30t + 81 = 0$

$\Rightarrow (t-27)(t-3)=0 \Rightarrow t=27$ or 3

$3^{4\sin^2 x}=3^3$ or $3^{4\sin^2 x}=3^1$

$\sin^2 x=\frac{3}{4}$ or $\frac{1}{4}$

$\sin x=\frac{\sqrt{3}}{2}$ or $\frac{1}{2}$ $(\because x\in[0,\pi])$

$x=60^0,\ 180-60,\ 30, 180-30$

Q.9 The locus of the midpoints of the chord of the circle, $x^2+y^2=25$ which is tangent to the hyperbola, $\frac{x^2}{9}-\frac{y^2}{16}=1$ is :

Options
1. $(x^2+y^2)^2-9x^2+16y^2=0$
2. $(x^2+y^2)^2-16x^2+9y^2=0$
3. $(x^2+y^2)^2-9x^2-16y^2=0$
4. $(x^2+y^2)^2-9x^2+144y^2=0$

Ans: 1

Sol: Let (h,k) be the mid point of the chord . Then $x.h+yk=h^2+k^2$ which is tangent to the hyperbola $\frac{x^2}{9}-\frac{y^2}{16}=1$

$\Rightarrow yk=-hx+(h^2+k^2)$

$y=\frac{-h}{k}x+\left(\frac{h^2+k^2}{k}\right)\Rightarrow m=\frac{-h}{k}, c=\frac{h^2+k^2}{k}$

$a^2=9, b^2=16$

$c^2=a^2m^2-b^2$

$\left(\frac{h^2+k^2}{k}\right)^2=9\left(\frac{-h}{k}\right)^2-16$

$\Rightarrow (h^2+k^2)^2=9h^2-16k^2$

$(x^2+y^2)^2-9x^2+16y^2=0$

Q.10 Let the functions $f:\mathbf{R}\to\mathbf{R}$ and $g:\mathbf{R}\to\mathbf{R}$ be defined as :

$$f(x)=\begin{cases}x+2, & x<0\\ x^2, & x\geq 0\end{cases} \quad \text{and} \quad g(x)=\begin{cases}x^3, & x<1\\ 3x-2, & x\geq 1\end{cases}$$

Then, the number of points in **R** where $(fog)(x)$ is NOT differentiable is equal to :

Options
1. 3
2. 0
3. 2
4. 1

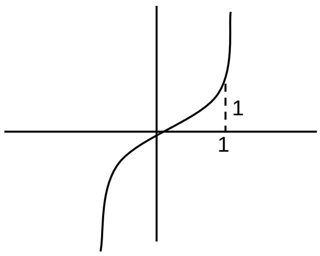
Ans: 4

Sol: $f(x) = \begin{cases} x+2, x<0 \\ x^2 \quad , x \geq 0 \end{cases}$

$g(x) = \begin{cases} x^3, \quad x<1 \\ 3x-2, \; x \geq 1 \end{cases}$

$fog(x) = f[g(x)] = \begin{cases} g(x)+2, \quad g(x)<0 \\ [g(x)^2] \;, \; g(x) \geq 0 \end{cases}$

Graph of g(x) is



Clearly $g(x) < 0 \Rightarrow x < 0$ and $g(x) \geq 0 \Rightarrow x \geq 0$

$\because fog(x) = \begin{cases} x^3+2, \quad x<0 \\ x^3+2, \quad 0 \leq x < 1 \\ (3x-2)^2 \quad x \geq 1 \end{cases}$ or

$fog(x) = \begin{cases} x^3+2 \quad \text{if} \quad x<1 \\ (3x-2)^2 \quad \text{if} \; x \geq 1 \end{cases}$

Clearly fog(x) is not continuous at x=1

$\Rightarrow$ Not differentiable at x=1

Q.11 If the three normals drawn to the parabola, $y^2 = 2x$ pass through the point (a, 0) $a \neq 0$, then 'a' must be greater than :

Options
1. 1
2. $-\frac{1}{2}$
3. $\frac{1}{2}$
4. -1

Ans: 1

Sol: $y^2 = 4\left(\frac{1}{2}\right)x$

$A = 1/2$

Normal **:** $y = 3x - 2Am - Am^3$

$\Rightarrow y = mx - m - \frac{1}{2}m^3$ passes through (a,0)

$0 = ma - m - \frac{1}{2}m^3$

$\Rightarrow \frac{1}{2}m^3 + m - ma = 0$

$\Rightarrow m^3 + 2m - 2ma = 0$

$m(m^2 + 2 - 2a) = 0$

$m = 0$ or

$m = \pm\sqrt{2(a-1)}$

Clearly a>1

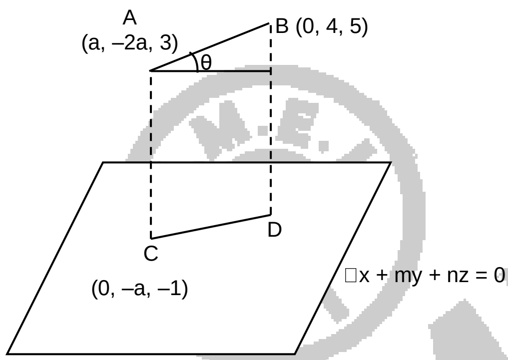
Q.12 If for a > 0, the feet of perpendiculars from the points A(a, −2a, 3) and B(0, 4, 5) on the plane $lx + my + nz = 0$ are points C(0, −a, −1) and D respectively, then the length of line segment CD is equal to :

Options

1. $\sqrt{55}$
2. $\sqrt{41}$
3. $\sqrt{66}$
4. $\sqrt{31}$

Ans: 3

Sol:



$\overrightarrow{CA} = a\hat{i} - a\hat{j} + 4\hat{k} \;||\; \ell\hat{i} + m\hat{j} + n\hat{k}$

$\Rightarrow \frac{a}{\ell} = -\frac{a}{m} = \frac{4}{n}$ and

$\ell(0) + m(-a) + n(-1) = 0$

$\Rightarrow am + n = 0$

$\Rightarrow a + \frac{n}{m} = 0 \Rightarrow a + \frac{4}{-a} = 0$

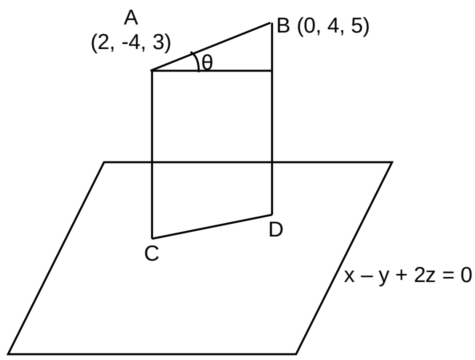
$\Rightarrow a = \frac{4}{a} \Rightarrow a = 2$

$\therefore \frac{2}{\ell} = \frac{-2}{m} = \frac{4}{n} \Rightarrow \ell = \frac{2}{\lambda},\; m = -\frac{2}{\lambda}, n = \frac{4}{\lambda}$

Plane : $\frac{2}{\lambda}x - \frac{2}{\lambda}y + \frac{4}{\lambda}z = 0$

$\Rightarrow\; 2x - 2y + 4z = 0$

$\Rightarrow\; x - y + 2z = 0$



$CD = AB\cos\theta$

$AB = \sqrt{4+64+4} = \sqrt{72}$

$\vec{AB} = -2\hat{i} + 8\hat{j} + 2\hat{k}$

$\vec{n} = \hat{i} - \hat{j} + 2\hat{k}$

$$\sin\theta = \frac{\vec{b}.\vec{n}}{|\vec{b}||\vec{n}|} = \frac{|-2-8+4|}{\sqrt{4+64+4}\sqrt{1+1+4}}$$

$$\frac{6}{\sqrt{72}.\sqrt{76}} = \frac{\sqrt{6}}{\sqrt{72}}$$

$$\cos\theta = \sqrt{1-\frac{6}{72}} = \sqrt{\frac{66}{72}}$$

$$\therefore CD = \sqrt{72}.\frac{\sqrt{66}}{\sqrt{72}} = \sqrt{66}$$

Q.13 If n is the number of irrational terms in the expansion of $\left(3^{1/4} + 5^{1/8}\right)^{60}$, then $(n-1)$ is divisible by :

Options
1. 7
2. 26
3. 30
4. 8

Ans: 2

Sol: $T_{r+1} = {}^{60}C_r\left(3^{1/4}\right)^{60-r}.\left(5^{1/8}\right)^{r} = {}^{60}C_r\, 3^{\left(15-\frac{r}{4}\right)}.5^{\left(r/8\right)}$

$\Rightarrow$ r must be divisible by 8

$\therefore$ r = 0,8,16,24,........56.

Number of rational terms =8

Total Number of terms=60+1=61

Number of irrational terms =61-8=53

$n = 53 \Rightarrow n-1 = 52$

Q.14 Consider three observations a, b and c such that $b = a + c$. If the standard deviation of $a+2$, $b+2$, $c+2$ is d, then which of the following is true ?

Options
1. $b^2 = a^2 + c^2 + 3d^2$
2. $b^2 = 3(a^2 + c^2) - 9d^2$
3. $b^2 = 3(a^2 + c^2 + d^2)$
4. $b^2 = 3(a^2 + c^2) + 9d^2$

Ans: 2

Sol: $\sigma(a+2,\ b+2,\ c+2) = \sigma(a,\ b,\ c)$

$$= \sqrt{\frac{\sum x_i^2}{n} - \frac{\left(\sum x_i\right)^2}{n}} = \sqrt{\frac{a^2+b^2+c^2}{3} - \frac{(a+b+c)^2}{3}} = d$$

$$\Rightarrow \frac{a^2+b^2+c^2}{3} - \left(\frac{2b}{3}\right)^2 = d^2$$

$$3a^2 + 3b^2 + 3c^2 - 4b^2 = 9d^2$$

$$3\left(a^2 + c^2\right) - b^2 = 9d^2$$

$$\Rightarrow 3\left(a^2 + c^2\right) - 9d^2 = b^2$$

Q.15 A pack of cards has one card missing. Two cards are drawn randomly and are found to be spades. The probability that the missing card is not a spade, is :

Options

1. $\frac{3}{4}$
2. $\frac{39}{50}$
3. $\frac{22}{425}$
4. $\frac{52}{867}$

Ans: 2

Sol: A → missing card is not spade

B → Getting 2 spade cards

$$P\left(A/B\right) = \frac{P(A).P\left(B/A\right)}{P(A).P\left(B/A\right) + P(A').P\left(B/A'\right)} \therefore = \frac{\frac{39}{52}\cdot\frac{{}^{13}C_2}{{}^{51}C_2}}{\frac{39}{52}\cdot\frac{{}^{13}C_2}{{}^{51}C_2} + \frac{13}{52}\cdot\frac{{}^{12}C_2}{{}^{15}C_2}} = \frac{39}{50}$$

Q.16 If for $x \in \left(0, \frac{\pi}{2}\right)$, $\log_{10}\sin x + \log_{10}\cos x = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$, $n > 0$,

then the value of n is equal to :

Options

1. 16
2. 20
3. 9
4. 12

Ans: 4

Sol: $\log_{10}(\sin x \cos x) = -1 \Rightarrow \sin x \cos x = \frac{1}{10} - - - (1)$

$$2\log_{10}(\sin x + \cos x)^2 = \log_{10} n - 1$$

$$\Rightarrow^{-} \log_{10}(\sin x + \cos x)^2 + \log_{10} n = 1$$

$$\Rightarrow \log_{10}\left[\frac{n}{(\sin x + \cos x)^2}\right] = 1 \Rightarrow \frac{n}{1 + 25\sin x\cos x} = 10$$

$$\Rightarrow \frac{n}{1+2\left(\frac{1}{10}\right)} = 10 \quad \Rightarrow n = 12$$

Q.17 The range of $a \in \mathbf{R}$ for which the function

$f(x) = (4a-3)(x+\log_e 5) + 2(a-7)\cot\left(\frac{x}{2}\right)\sin^2\left(\frac{x}{2}\right)$, $x \neq 2n\pi$, $n \in N$ has critical points, is :

Options
1. $(-3, 1)$
2. $\left[-\frac{4}{3}, 2\right]$
3. $[1, \infty)$
4. $(-\infty, -1]$

Ans: 2

Sol: $f(x) \quad (4a-3)(x+\log_e 5)+2(a-7)\cot\frac{x}{2}.5\sin^2\left(\frac{x}{2}\right)$

$f'(x) = (4a-3)(x+\log 5)+(a-7).\sin x$

$f'(x) = (4a-3)(1)+(a-7)\cos x = 0$

$\cos x = \frac{3-4a}{a-7}$

$-1 \leq \frac{3-4a}{a-7} \leq 1$

$\frac{3-4a}{a-7} \geq 1 \Rightarrow \frac{3-4a}{a-7} + 1 \geq 0$

$\Rightarrow \frac{-4-3a}{a-7} \geq 0 \Rightarrow \frac{3a+4}{a-7} \leq 0$

$\Rightarrow \frac{-4}{3} \leq a < 7 --- (1)$ and

$\frac{3-4a}{a-7} \leq 1 \Rightarrow \frac{3-4a}{a-7} \ -1 \leq 0$

$\frac{10-5a}{a-7} \leq 0 \Rightarrow \frac{5a-10}{a-7} \geq 0$

$a < 2$ or $a > 7$

$\Rightarrow a \in \left[-\frac{4}{3}, 2\right]$

Q.18 Let the position vectors of two points P and Q be $3\hat{i} - \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} - 4\hat{k}$, respectively. Let R and S be two points such that the direction ratios of lines PR and QS are (4, −1, 2) and (−2, 1, −2), respectively. Let lines PR and QS intersect at T. If the vector $\overrightarrow{TA}$ is perpendicular to both $\overrightarrow{PR}$ and $\overrightarrow{QS}$ and the length of vector $\overrightarrow{TA}$ is $\sqrt{5}$ units, then the modulus of a position vector of A is :

Options

1. $\sqrt{227}$
2. $\sqrt{171}$
3. $\sqrt{5}$
4. $\sqrt{482}$

Ans: 2

Sol: Equation of PR : $\frac{x-3}{4} = \frac{y+1}{-1} = \frac{z-2}{2}$

Equation of QS : $\frac{x-1}{-2} = \frac{y-2}{1} = \frac{z+4}{-2}$

(1) $-x+3 = 4y+4 \Rightarrow x+4y = -1$

(2) $x-1 = -2y+4 \Rightarrow x+2y = 5$

Solving, $2y = -6 \Rightarrow y = -3 \Rightarrow x-12 = -1 \Rightarrow x = 11$

$\frac{11-3}{4} = \frac{z-2}{2} \Rightarrow \frac{z-2}{2} = 2 \Rightarrow z = 4+2 = 6$

$\therefore T(11,-3,6)$

$$\overrightarrow{TA} = \lambda\left(\overrightarrow{PR} \times \overrightarrow{QS}\right) = \lambda\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 2 \\ -2 & 1 & -2 \end{vmatrix}$$

$= \lambda\left[\hat{i}(2-2) - \hat{j}(-8+4) + \hat{k}(4-2)\right]$

$= \lambda\left(0\hat{i} + 4\hat{j} + 2\hat{k}\right)$

$\left|\overrightarrow{TA}\right| = |\lambda|.\sqrt{16+4} = \sqrt{5} \Rightarrow |\lambda|.2\sqrt{5} = \sqrt{5} \Rightarrow \lambda = \frac{\pm 1}{2}$

$\therefore \overrightarrow{TA} = \frac{\pm 1}{2}\left(4\hat{j} + 2\hat{k}\right) = \pm\left[2\hat{j} + \hat{k}\right]$

<u>Case 1</u> $\left(\text{if } \overrightarrow{TA} = 2\hat{j} + \hat{k}\right)$

$\overrightarrow{OA} - \overrightarrow{OT} = 2\hat{j} + \hat{k} \Rightarrow \overrightarrow{OA} = 11\hat{i} + 3\hat{j} + 6\hat{k} + 2\hat{j} + \hat{k} = 11\hat{i} - \hat{j} - 7\hat{k}$

$\left|\overrightarrow{OA}\right| = \sqrt{|z| + 1 + 49} = \sqrt{171}$

Q.19 Which of the following Boolean expression is a tautology ?

Options
1. $(p \wedge q) \vee (p \vee q)$
2. $(p \wedge q) \wedge (p \rightarrow q)$
3. $(p \wedge q) \rightarrow (p \rightarrow q)$
4. $(p \wedge q) \vee (p \rightarrow q)$

Ans: 3

Sol:

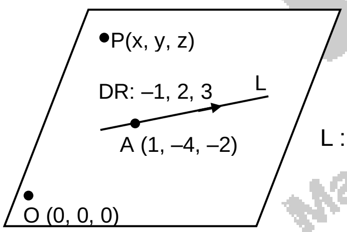
p	q	p ^ q	p ⇒ q	(p^q) ⇒(p ⇒ q)
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

Q.20 Let P be a plane $lx + my + nz = 0$ containing the line, $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$. If plane P divides the line segment AB joining points A(−3, −6, 1) and B(2, 4, −3) in ratio k : 1 then the value of k is equal to :

Options
1. 1.5
2. 3
3. 4
4. 2

Ans: 4

Sol:



$L : \frac{x-1}{-1} = \frac{y-(-4)}{2} = \frac{z-(-2)}{3}$

$\vec{OP} = x\hat{i} + y\hat{j} + 2\hat{k},\ \ \vec{OA} = 1\hat{i} - 4\hat{j} - 2\hat{k}$

Parallel vector : $-1\hat{i} + 2\hat{j} + 3\hat{k}$

$$\begin{vmatrix} x & y & z \\ 1 & -4 & -2 \\ -1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow x(-12+4) - y(3-2) + z(2-4) = 0$$

$$-8x - y - 2z = 0$$
$$8x + y + 2z = 0$$

Required ratio= $\left(\frac{-(8(-3)+(-6)+2(1))}{8(2)+4+2(-3)}\right) = -\left(\frac{-24-6+2}{16+4-6}\right) = \frac{28}{14} = 2 \Rightarrow k = 2$

SECTION B

Q.1 Consider an arithmetic series and a geometric series having four initial terms from the set {11, 8, 21, 16, 26, 32, 4}. If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to ________.

Ans: 3

Sol: GP : $2^2, 2^3, 2^4, 2^5, 2^6, 2^7, 2^8, 2^9, 2^{10}, 2^{11}, 2^{12}, 2^{13}$
AP : 11, 16, 21, 26, ...
General term = 11 + (n – 1) 5 = 5n + 6
(last digit is 1 or 6)
The terms in GP with last digit 1 or 6
are 2^4, 2^8 and 2^{12}

Q.2 Let $P = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix}$ and $A = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega+1 \end{bmatrix}$ where $\omega = \frac{-1+i\sqrt{3}}{2}$, and I_3 be the identity matrix of order 3. If the determinant of the matrix $(P^{-1}AP-I_3)^2$ is $\alpha\omega^2$, then the value of α is equal to ________.

Ans: 36

Sol: $|P^{-1}AP - I| = |P^{-1}AP - P^{-1}P| = |P^{-1}|\,|AP - P| = |P^{-1}|\,|A - I|\,|P| = \frac{1}{|P|}.|A-I|.|P| = |A-I|$

$$A - I = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega+1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 7 & \omega^2 \\ -1 & -\omega-1 & 1 \\ 0 & -\omega & -\omega \end{bmatrix} = \begin{bmatrix} 1 & 7 & \omega^2 \\ -1 & \omega^2 & 1 \\ 0 & -\omega & -\omega \end{bmatrix}$$

$|A - I| = 1(-\omega^3 + \omega) - 7(\omega - 0) + \omega^2(\omega - 0)$
$= -\omega^2 + \omega - 7\omega + \omega^3 = -6\omega$
$|P^{-1}AP - I|^2 = (-6\omega)^2 = 36\,\omega^2$
$\alpha = 36$

Q.3 If the normal to the curve $y(x) = \int_0^x (2t^2 - 15t + 10)\,dt$ at a point (a, b) is parallel to the line $x + 3y = -5$, $a > 1$, then the value of $|a + 6b|$ is equal to ________.

Given --

Ans: 406

Sol: $y = \int_0^x (2t^2 - 15t + 10)\,dt$

$\frac{dy}{dx} = 2x^2 - 15x + 10$

$\frac{-1}{dy/dx} = \frac{-1}{2x^2 - 15x + 10} = \frac{-1}{3}$

$2x^2 - 15x + 10 = 3$
$2x^2 - 15x + 7 = 0$

$x = \frac{15 \pm \sqrt{225 - 56}}{4} = \frac{15 \pm 13}{4} = 7$ or $\frac{1}{2}$

$\therefore a = 7 \Rightarrow b = \int_0^7 (2t^2 - 15t + 10)\,dt = \left[\frac{2t^3}{3} - \frac{15t^2}{2} + 10t\right]_0^7 = \frac{-413}{6}$

$\therefore |a + 6b| = |7 - 413| = 406$

Q.4

If $\lim\limits_{x\to 0} \dfrac{ae^{x} - b\cos x + ce^{-x}}{x\sin x} = 2$, then a + b + c is equal to ________.

Ans: 4

Sol: $\lim\limits_{x\to 0}\left(\dfrac{ae^{x} - b\cos x + ce^{-x}}{x\sin x}\right)$

$$\lim_{x\to 0}\left[\frac{a\left(1+x+\frac{x^2}{2!}+.....\right)-b\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-.....\right)+c\left(1-x+\frac{x^2}{2!}+......\right)}{x\left(x-\frac{x^3}{3!}+......\right)}\right]$$

$$\lim_{x\to 0}\left[\frac{(a-b+c)+(a-c)x+\left(\frac{a}{2}+\frac{b}{2}+\frac{c}{2}\right)x^2+......}{x^2-\frac{x^4}{3!}+......}\right]$$

$\Rightarrow a - b + c = 0$ ---- (1)

$a - c = 0$ ---- (2)

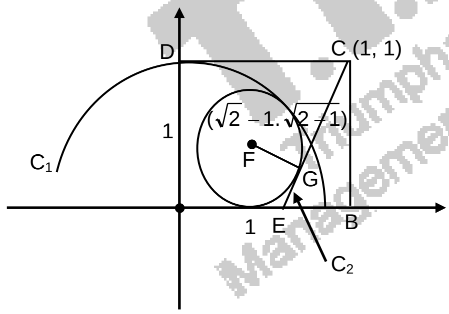
$\dfrac{a}{2}+\dfrac{b}{2}+\dfrac{c}{2} = 2 \Rightarrow a+b+c = 4$

Q.5

Let ABCD be a square of side of unit length. Let a circle C_1 centered at A with unit radius is drawn. Another circle C_2 which touches C_1 and the lines AD and AB are tangent to it, is also drawn. Let a tangent line from the point C to the circle C_2 meet the side AB at E. If the length of EB is $\alpha + \sqrt{3}\,\beta$, where α, β are integers, then $\alpha + \beta$ is equal to ________.

Ans: 1

Sol:



$C_1 : x^2 + y^2 = 1$

<u>For C_2</u>

Centre : (h, h)

radius : h

Distance between centres = $\sqrt{h^2 + h^2} = \sqrt{2}\,h$

$|r_1 - r_2| = |h - 1|$

$|h - 1| = \sqrt{2}\,h \Rightarrow h - 1 = \sqrt{2}\,h$ (Case 1)

$\left(1-\sqrt{2}\right)h = 1$ (Not possible as h > 0)

$h - 1 = \sqrt{2}h \Rightarrow \left(1+\sqrt{2}\right)h = 1 \Rightarrow h = \sqrt{2} - 1$

CE : $y - 1 = m(x - 1) \Rightarrow mx - y + (1 - m) = 0$

$$FG = \frac{\left|m(\sqrt{2}-1)-(\sqrt{2}-1)+1-m\right|}{\sqrt{m^2+1}} = \sqrt{2}-1$$

$$\left|\sqrt{2}m - m - \sqrt{2}+1+1-m\right| = (\sqrt{2}-1)\left(\sqrt{m^2+1}\right)$$

$$\left|\sqrt{2}m - 2m + 2 - \sqrt{2}\right| = (\sqrt{2}-1)\left(\sqrt{m^2+1}\right)$$

$$\left|-(2-\sqrt{2})m + (2-\sqrt{2}) = (\sqrt{2}-1)\left(\sqrt{m^2+1}\right)\right|$$

$2(1 - m)^2 = 1 + m^2$
$2(1 - 2m + m^2) = 1 + m^2$
$2m^2 - 4m + 2 - 1 - m^2 = 0$
$m^2 - 4m + 1 = 0$

$$m = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$$

Case 1

$y - 1 = (2+\sqrt{3})(x-1)$

$$\Rightarrow x - 1 = \frac{y-1}{2+\sqrt{3}}$$

$$y = 0 \Rightarrow x - 1 = \frac{-1}{2+\sqrt{3}} = -(2-\sqrt{3}) = -2+\sqrt{3}$$

$x = -1 + \sqrt{3}$

$EB = 1 - (-1+\sqrt{3}) = 2 - \sqrt{3}$

$\Rightarrow \alpha + \beta = 1$

Case 2

$$y - 1 = (2-\sqrt{3})(x-1) \Rightarrow x - 1 = \frac{y-1}{2-\sqrt{3}}$$

$$y = 0 \Rightarrow x - 1 = \frac{-1}{2-\sqrt{3}} = -(2+\sqrt{3}) = -2-\sqrt{3}$$

$x = -1 - \sqrt{3}$ (Not possible)

Q.6 Let $f : (0, 2) \to \mathbb{R}$ be defined as $f(x) = \log_2\left(1 + \tan\left(\frac{\pi x}{4}\right)\right)$.

Then, $\lim_{n\to\infty} \frac{2}{n}\left(f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + \ldots + f(1)\right)$ is equal to ________.

Ans: 1

Sol: $$\lim_{n\to\infty} \frac{2}{n}\left[f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + \ldots\ldots + f(1)\right]$$

$$\lim_{n\to\infty} 2\frac{1}{n}\sum_{r=1}^{n} f\left(\frac{r}{n}\right)$$

$$\frac{r}{n} = x,\ \frac{1}{n} = dx$$

$$\Rightarrow 2\int_0^1 f(x)\,dx = 2\int_0^1 \log_2\left[1 + \tan\frac{\pi x}{4}\right]dx (= I)$$

$$\Rightarrow 2\int_0^1 \log_2\left[1 + \tan\frac{\pi}{4}(1-x)\right]dx$$

$$\Rightarrow 2\int_0^1 \log_2\left[1 + \tan\left(\frac{\pi}{4} - \frac{\pi}{4}x\right)\right]dx$$

$$\Rightarrow 2\int_0^1 \log_2\left[1+\frac{1-\tan\frac{\pi x}{4}}{1+\tan\frac{\pi x}{4}}\right]dx = 2\int_0^1 \log_2\left(\frac{2}{1+\tan\frac{\pi x}{4}}\right) (= I)$$

$$2I = 2\int_0^1 \log_2 2\,dx = 2\int_0^1 1.dx = 2$$

I = 1

Q.7 Let the curve $y = y(x)$ be the solution of the differential equation, $\frac{dy}{dx} = 2(x+1)$. If the numerical value of area bounded by the curve $y = y(x)$ and x-axis is $\frac{4\sqrt{8}}{3}$, then the value of $y(1)$ is equal to ________.

Ans: **2**

Sol: $y = \int(2x+2)\,dx = x^2 + 2x + c$

$$A = \int_\alpha^\beta (x^2+2x+c)\,dx \quad (\alpha+\beta) = \left[\frac{x^3}{3} + x^2 + cx\right]_\alpha^\beta = \frac{1}{3}(\beta^3-\alpha^3) + (\beta^2-\alpha^2) + c(\beta-\alpha)$$

$\alpha + \beta = -2,\ \alpha\beta = c$

$$|\beta^2-\alpha^2| = \sqrt{4-4c} \quad \left(\because |\beta-\alpha| = \sqrt{(\alpha+\beta)^2-4\alpha\beta}\right)$$

$$\frac{1}{3}\left[\sqrt{(4-4c)}(4-c) + (-2)\sqrt{(4-4c)} + c\left(\sqrt{4-4c}\right)\right] = \frac{4\sqrt{8}}{3}$$

$\Rightarrow$ c = –1 (by back substitution)

$\therefore y = x^2 + 2x - 1$

$y(1) = 1 + 2 - 1 = 2$

Q.8 Let $f: R \to R$ be a continuous function such that $f(x) + f(x+1) = 2$, for all $x \in R$. If $I_1 = \int_0^8 f(x)dx$ and $I_2 = \int_{-1}^3 f(x)dx$, then the value of $I_1 + 2I_2$ is equal to ________.

Ans: **16**

Sol: $I_1 + 2I_2 = \int_0^8 f(x)\,dx + 2\int_{-1}^3 f(x)\,dx$

Consider I_2

$x = t - 1 \Rightarrow dx = dt \quad x = 1 \Rightarrow t = 0,\ x = 3 \Rightarrow t = 4$

$$I_2 = \int_0^4 f(t-1)\,dt = \int_0^4 [2-f(t)]\,dt = 8 - \int_0^4 f(t)\,dt + 8 - \int_0^4 f(x)\,dx$$

$$I_1 + 2I_2 = \int_0^8 f(x)\,dx + 16 - 2\int_0^4 f(x)\,dx = 16 + \left[\int_0^8 f(x)\,dx - 2\int_0^4 f(x)\,dx\right]$$

Clearly f(x) is a periodic function with period 2

[□ f(x) + f (x + 1) = 2 -- (1), f(x + 1) + f(x + 2) = 2 ---- (2) $\Rightarrow$ f(x) = f(x + 2)]

$$\therefore \int_0^8 f(x)\,dx = 2\int_0^4 f(x)\,dx$$

Q.9 Let z and w be two complex numbers such that $w = z\bar{z} - 2z + 2$, $\left|\frac{z+i}{z-3i}\right| = 1$ and Re (w) has minimum value. Then, the minimum value of $n \in N$ for which w^n is real, is equal to ________.

Ans: 4

Sol: $|z + i| = |z - 3i|$

$|(x + yi) + i| = |x + yi - 3i|$

$\sqrt{x^2 + (y+1)^2} = \sqrt{x^2 + (y-3)^2}$

$\Rightarrow 2y + 1 = -6y + 9 \Rightarrow 8y = 8 \Rightarrow y = 1$

$z = x + i$

$\omega = |z|^2 - 2z + 2$

$\omega = x^2 + 1 - 2(x + i) + 2$

$\omega = (x^2 - 2x + 3) - 2i$

$\text{Re}(\omega) = x^2 - 2x + 3$

Minimum value $= \frac{-D}{4a} = \frac{-(4-12)}{4} = 2$

and it is obtained at $x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = 1$

$\omega = 2 - 2i = 2(1 - i)$

$\omega^n = 2^n (1 - i)^n \Rightarrow n = 4$

Q.10 The total number of 3×3 matrices A having entries from the set {0, 1, 2, 3} such that the sum of all the diagonal entries of AA^T is 9, is equal to ________.

Ans: 766

Sol: $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}, A^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$

$AA^T = \begin{bmatrix} a^2+b^2+c^2 & - & - \\ - & d^2+e^2+f^2 & \\ - & - & g^2+h^2+i^2 \end{bmatrix}$

$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 9$

All elements 1 $\Rightarrow$ 1 case

$a^2, b^2, c^2, \ldots\ldots i^2 \in \{0, 1, 4, 9\}$

Case 1

1, 1, 1, 1, 1, 1, 1, 1, 1 $\to$ 1 way

Case 2

4, 1, 1, 1, 1, 1, 0, 0, 0 $\Rightarrow \frac{9!}{5!.3!}$

Case 3

4, 4, 1, 0, 0, 0, 0, 0, 0 $\Rightarrow \frac{9!}{2!.6!}$

Case 4

9 0, 0, 0, 0, 0, 0, 0, 0 $\Rightarrow \frac{9!}{8!}$

Ans $= 1 + \frac{9.8.7.6}{1.2.3} + \frac{9.8.7}{1.2} + 9 = 766$