SOLUTIONS & ANSWERS FOR JEE MAINS-2021
17th March Shift 2
[PHYSICS, CHEMISTRY & MATHEMATICS]

PART – A – PHYSICS

SECTION A

Q.1 Match List - I with List - II

List - I		List - II
(a) Phase difference between current and voltage in a purely resistive AC circuit	(i)	$\frac{\pi}{2}$; current leads voltage
(b) Phase difference between current and voltage in a pure inductive AC circuit	(ii)	zero
(c) Phase difference between current and voltage in a pure capacitive AC circuit	(iii)	$\frac{\pi}{2}$; current lags voltage
(d) Phase difference between current and voltage in an LCR series circuit	(iv)	$\tan^{-1}\left(\frac{X_C - X_L}{R}\right)$

Choose the most appropriate answer from the options given below :

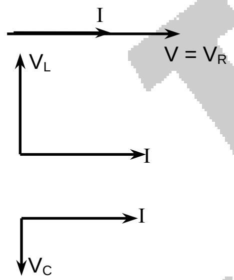
Options
1. (a)-(ii), (b)-(iii), (c)-(i), (d)-(iv)
2. (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)
3. (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)
4. (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

Ans: 1

Sol: (a)

(b)

(c)



(d) $\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$

Q.2 What happens to the inductive reactance and the current in a purely inductive circuit if the frequency is halved ?

Options
1. Inductive reactance will be doubled and current will be halved.
2. Both, inducting reactance and current will be doubled.
3. Both, inductive reactance and current will be halved.
4. Inductive reactance will be halved and current will be doubled.

Ans: 4

Sol: $X_L = \omega L$

$i = \dfrac{V_0}{\omega L}$

Q.3 A block of mass 1 kg attached to a spring is made to oscillate with an initial amplitude of 12 cm. After 2 minutes the amplitude decreases to 6 cm. Determine the value of the damping constant for this motion. (take $\ln 2 = 0.693$)

Options
1. $1.16 \times 10^2 \text{ kg s}^{-1}$
2. $5.7 \times 10^{-3} \text{ kg s}^{-1}$
3. $0.69 \times 10^2 \text{ kg s}^{-1}$
4. $3.3 \times 10^2 \text{ kg s}^{-1}$

Ans: No option Matching

Sol: $A = A_0 e^{-\gamma t}$

$\ell n 2 = \dfrac{b}{2m} \times 120 \Rightarrow b = \dfrac{0.693 \times 2 \times 1}{120} = 1.16 \times 10^{-2}$ kg/s

Q.4 A sound wave of frequency 245 Hz travels with the speed of 300 ms^{-1} along the positive x-axis. Each point of the wave moves to and fro through a total distance of 6 cm. What will be the mathematical expression of this travelling wave ?

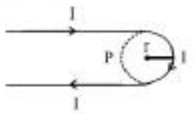
Options
1. $Y(x, t) = 0.06 [\sin 0.8x - (0.5 \times 10^3)t]$
2. $Y(x, t) = 0.03 [\sin 5.1x - (1.5 \times 10^3)t]$
3. $Y(x, t) = 0.03 [\sin 5.1x - (0.2 \times 10^3)t]$
4. $Y(x, t) = 0.06 [\sin 5.1x - (1.5 \times 10^3)t]$

Ans: 2

Sol: $\omega = 2\pi f = 1.5 \times 10^3$

$A = \dfrac{6}{2} = 3\text{cm} = 0.03\,\text{m}$

Q.5 A hairpin like shape as shown in figure is made by bending a long current carrying wire. What is the magnitude of a magnetic field at point P which lies on the centre of the semicircle ?



Options
1. $\dfrac{\mu_0 I}{2\pi r}(2 - \pi)$
2. $\dfrac{\mu_0 I}{4\pi r}(2 - \pi)$
3. $\dfrac{\mu_0 I}{4\pi r}(2 + \pi)$
4. $\dfrac{\mu_0 I}{2\pi r}(2 + \pi)$

Ans: 3

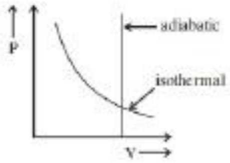
Sol: $B = 2 \times B_{st.\,wire} + B_{loop}$

$$B = 2 \times \frac{\mu_0 i}{4\pi r} + \frac{\mu_0 i}{2r} \times \frac{\pi}{2\pi}$$

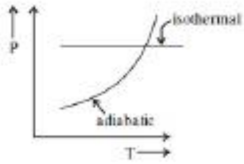
$$B = \frac{\mu_0 i}{4\pi r}(2+\pi)$$

Q.6 Which one is the correct option for the two different thermodynamic processes ?

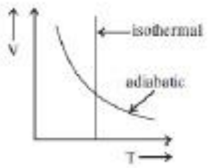
(a)



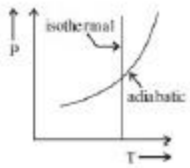
(b)



(c)



(d)



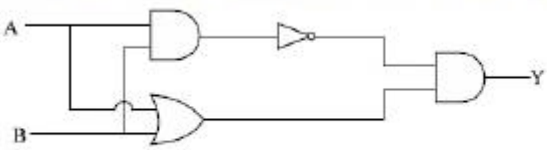
Options
1. (a) only
2. (b) and (c)
3. (c) and (d)
4. (c) and (a)

Ans: 3

Sol: In adiabatic process, $V \neq$ constant
In an isothermal process, T = constant
Option 3 (c) and (d) matches isothermal and adiabatic formula

$TV^{\gamma-1} =$ constant and $\frac{T^{\gamma}}{p^{\gamma-1}} =$ constant

Q.7 Which one of the following will be the output of the given circuit ?

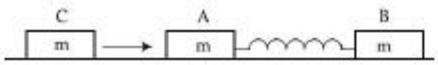


Options
1. XOR Gate
2. NAND Gate
3. AND Gate
4. NOR Gate

Ans: 1

Sol: Basic concept

Q.8 Two identical blocks A and B each of mass m resting on the smooth horizontal floor are connected by a light spring of natural length L and spring constant K. A third block C of mass m moving with a speed v along the line joining A and B collides with A. The maximum compression in the spring is



Options

1. $\sqrt{\frac{mv}{2K}}$
2. $v\sqrt{\frac{m}{2K}}$
3. $\sqrt{\frac{mv}{K}}$
4. $\sqrt{\frac{m}{2K}}$

Ans: 2

Sol: C comes to rest

V_{cm} of A and B = $\frac{V}{2} \Rightarrow \frac{1}{2}$ is $V_{net}^2 = \frac{1}{2}kx^2$

$x = \sqrt{\frac{\mu V^2}{k}} = \sqrt{\frac{m}{2k}}V$

Q.9 A carrier signal $C(t) = 25\sin(2.512 \times 10^{10}t)$ is amplitude modulated by a message signal $m(t) = 5\sin(1.57 \times 10^{8}t)$ and transmitted through an antenna. What will be the bandwidth of the modulated signal ?

Options

1. 50 MHz
2. 8 GHz
3. 2.01 GHz
4. 1987.5 MHz

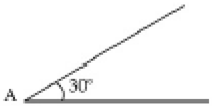
Ans: 1

Sol: Band width = 2 f_m

$\omega_m = 1.57 \times 10^8 = 2\pi f_m$

BW=2f_m = $\frac{10^8}{2}$ Hz = 50 MHz

Q.10 A sphere of mass 2 kg and radius 0.5 m is rolling with an initial speed of 1 ms^{-1} goes up an inclined plane which makes an angle of 30° with the horizontal plane, without slipping. How long will the sphere take to return to the starting point A ?



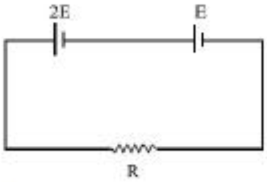
Options
1. 0.60 s
2. 0.52 s
3. 0.57 s
4. 0.80 s

Ans: 3

Sol: $a = \frac{g\sin\theta}{1+\frac{I}{mR^2}} = \frac{5}{7}\times\frac{10}{2} = \frac{25}{7}$

$t = \frac{2V_0}{a} = \frac{2\times1\times7}{25} = 0.56$

Q.11 Two cells of emf 2E and E with internal resistance r_1 and r_2 respectively are connected in series to an external resistor R (see figure). The value of R, at which the potential difference across the terminals of the first cell becomes zero is



Options
1. $r_1 - r_2$
2. $r_1 + r_2$
3. $\frac{r_1}{2} - r_2$
4. $\frac{r_1}{2} + r_2$

Ans: 3

Sol: $i = \frac{3E}{R+r_1+r_2}$

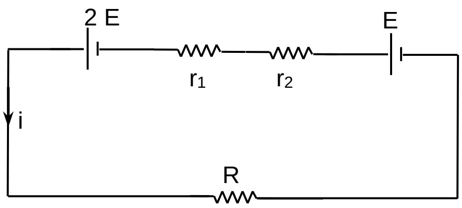
TPD = $2E - ir_1 = 0$

$2E = ir_1$

$2E = \frac{3E\times r_1}{R+r_1+r_2}$

$2R+2r_1 + 2r_2 = 3r_1$

$R = \frac{r_1}{2} - r_2$



Q.12 Two identical photocathodes receive the light of frequencies f_1 and f_2 respectively. If the velocities of the photo-electrons coming out are v_1 and v_2 respectively, then

Options

1. $v_1 - v_2 = \left[\frac{2h}{m}(f_1 - f_2)\right]^{\frac{1}{2}}$
2. $v_1^2 - v_2^2 = \frac{2h}{m}[f_1 - f_2]$
3. $v_1^2 + v_2^2 = \frac{2h}{m}[f_1 + f_2]$
4. $v_1 + v_2 = \left[\frac{2h}{m}(f_1 + f_2)\right]^{\frac{1}{2}}$

Ans: 2

Sol: $\frac{1}{2}mv_1^2 = h\nu_1 - \phi$

$\frac{1}{2}mv_2^2 = h\nu_2 - \phi$

$v_1^2 - v_2^2 = \frac{2h}{m}(\nu_1 - \nu_2)$

Q.13 The velocity of a particle is $v = v_0 + gt + Ft^2$. Its position is $x = 0$ at $t = 0$; then its displacement after time $(t = 1)$ is :

Options

1. $v_0 + 2g + 3F$
2. $v_0 + \frac{g}{2} + \frac{F}{3}$
3. $v_0 + \frac{g}{2} + F$
4. $v_0 + g + F$

Ans: 2

Sol: $V = V_0 + gt + Ft^2$

$\frac{ds}{dt} = V_0 + gt + Ft^2$

$\int ds = \int_0^1 (V_0 + gt + Ft^2)dt$

$s = \left[V_0 t + \frac{gt^2}{2} + \frac{Ft^3}{3}\right]_0^1$

$s = V_0 + \frac{g}{2} + \frac{F}{3}$

Q.14 If one mole of the polyatomic gas is having two vibrational modes and β is the ratio of molar specific heats for polyatomic gas $\left(\beta = \frac{C_P}{C_V}\right)$ then the value of β is :

Options
1. 1.25
2. 1.35
3. 1.02
4. 1.2

Ans: 4

Sol: f = 4 + 3 + 3 = 10
Assuming non linear
$$\beta = \frac{C_P}{C_V} = 1 + \frac{2}{f} = \frac{12}{10} = 1.2$$

Q.15 A rubber ball is released from a height of 5 m above the floor. It bounces back repeatedly, always rising to $\frac{81}{100}$ of the height through which it falls. Find the average speed of the ball. (Take $g = 10\ ms^{-2}$)

Options
1. $3.50\ ms^{-1}$
2. $3.0\ ms^{-1}$
3. $2.50\ ms^{-1}$
4. $2.0\ ms^{-1}$

Ans: 3

Sol:
$$V_0 = \sqrt{2gh}$$
$$V = e\sqrt{2gh} = \sqrt{2gh}$$
$$\Rightarrow e = 0.9$$
$$s = h + 2e^2h + 2e^4h + ...$$
$$t = \sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}} + 2e^2\sqrt{\frac{2h}{g}} + ...$$
$$V_{av} = \frac{s}{t} = 2.5\ m/s$$

Q.16 An object is located at 2 km beneath the surface of the water. If the fractional compression $\frac{\Delta V}{V}$ is 1.36%, the ratio of hydraulic stress to the corresponding hydraulic strain will be ______.

[Given : density of water is $1000\ kgm^{-3}$ and $g = 9.8\ ms^{-2}$.]

Options
1. $1.44 \times 10^7\ Nm^{-2}$
2. $1.96 \times 10^7\ Nm^{-2}$
3. $2.26 \times 10^9\ Nm^{-2}$
4. $1.44 \times 10^9\ Nm^{-2}$

Ans: 4

Sol: P = $h\rho g$

$$\beta = \frac{P}{(\Delta V / V)} = \frac{2\times10^3\times10^3\times9.8}{1.36\times10^{-2}} = 1.44 \times 10^9 \text{ N/m}^2$$

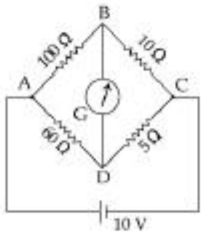
Q.17 The atomic hydrogen emits a line spectrum consisting of various series. Which series of hydrogen atomic spectra is lying in the visible region ?

Options
1. Paschen series
2. Balmer series
3. Lyman series
4. Brackett series

Ans: 2

Sol: Basic concept

Q.18 The four arms of a Wheatstone bridge have resistances as shown in the figure. A galvanometer of 15 Ω resistance is connected across BD. Calculate the current through the galvanometer when a potential difference of 10 V is maintained across AC.



Options
1. 2.44 μA
2. 2.44 mA
3. 4.87 μA
4. 4.87 mA

Ans: 4

Sol:

$$\frac{x-10}{100} + \frac{x-y}{15} + \frac{x-0}{10} = 0$$

$53x - 20y = 30$ ------------(1)

$$\frac{y-10}{60} + \frac{y-x}{15} + \frac{y-0}{5} = 0$$

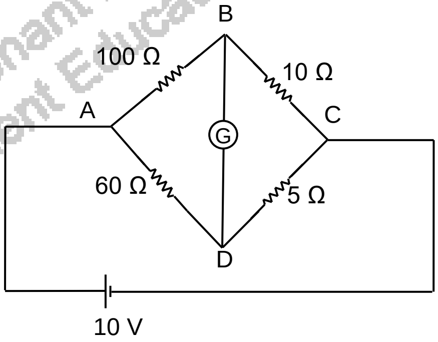
$17y - 4x = 10$ -------------------(2)

On solving (1) and (2)

$x = 0.865$

$y = 0.792$

$\Delta V = 0.73$, $R = 15\ \Omega \Rightarrow i = 4.87$ mA



Q.19 Two particles A and B of equal masses are suspended from two massless springs of spring constants K_1 and K_2 respectively. If the maximum velocities during oscillations are equal, the ratio of the amplitude of A and B is

Options
1. $\frac{K_2}{K_1}$
2. $\sqrt{\frac{K_1}{K_2}}$
3. $\frac{K_1}{K_2}$
4. $\sqrt{\frac{K_2}{K_1}}$

Ans: **4**

Sol: $A_1\omega_1 = A_2\omega_2$

$$A_1\sqrt{\frac{k_1}{m}} = A_2\sqrt{\frac{k_2}{m}}$$

$$\frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$$

Q.20 A geostationary satellite is orbiting around an arbitary planet 'P' at a height of 11R above the surface of 'P', R being the radius of 'P'. The time period of another satellite in hours at a height of 2R from the surface of 'P' is ________. 'P' has the time period of 24 hours.

Options
1. 5
2. $6\sqrt{2}$
3. $\frac{6}{\sqrt{2}}$
4. 3

Ans: **4**

Sol: $T \propto R^{3/2}$

$$\frac{24}{T} = \left(\frac{12R}{3R}\right)^{3/2} \Rightarrow T = 3\,\text{hr}$$

Section B

Q.1 Suppose you have taken a dilute solution of oleic acid in such a way that its concentration becomes 0.01 cm^3 of oleic acid per cm^3 of the solution. Then you make a thin film of this solution (monomolecular thickness) of area 4 cm^2 by considering 100 spherical drops of radius $\left(\frac{3}{40\pi}\right)^{\frac{1}{3}} \times 10^{-3}$ cm. Then the thickness of oleic acid layer will be $x \times 10^{-14}$ m. Where x is ________.

Ans: **25.00**

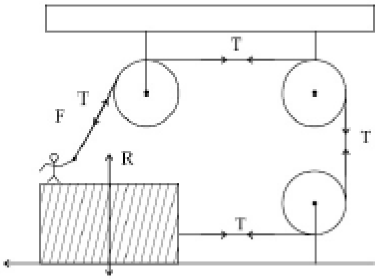
Sol: $4t_T = 100 \times \frac{4}{3}\pi r^3 = 100 \times \frac{4\pi}{3} \times \frac{3}{40\pi} \times 10^{-9} = 10^{-8}\ cm^3$

$t_T = 25 \times 10^{-10}\ cm \ = 25 \times 10^{-12} m$

$t_0 = 0.01\ t_T = 25 \times 10^{-14}\ m = 25.00$

Q.2 A boy of mass 4 kg is standing on a piece of wood having mass 5 kg. If the coefficient of friction between the wood and the floor is 0.5, the maximum force that the boy can exert on the rope so that the piece of wood does not move from its place is ________ N. (Round off to the Nearest Integer)

[Take $g = 10\ ms^{-2}$]



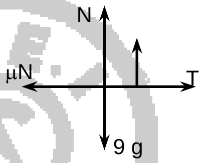
Ans: 30.00

Sol: N + T = 90

T = μN = 0.5 (90 – T)

1.5 T = 45

T = 30



Q.3 The image of an object placed in air formed by a convex refracting surface is at a distance of 10 m behind the surface. The image is real and is at $\frac{2}{3}^{rd}$ of the distance of the object from the surface. The wavelength of light inside the surface is $\frac{2}{3}$ times the wavelength in air. The radius of the curved surface is $\frac{x}{13}$ m. The value of 'x' is________.

Ans: 30.00

Sol: $\lambda_m = \frac{\lambda_a}{\mu} \Rightarrow \mu = \frac{3}{2}$

$\frac{\mu}{V} - \frac{1}{U} = \frac{\mu - 1}{R}$

$= \frac{3}{2 \times 10} + \frac{1}{15} = \frac{\frac{3}{2} - 1}{R}$

$R = \frac{30}{13}$

Ans is 30.00

Q.4 A body of mass 1 kg rests on a horizontal floor with which it has a coefficient of static friction $\frac{1}{\sqrt{3}}$. It is desired to make the body move by applying the minimum possible force F N. The value of F will be ________. (Round off to the Nearest Integer)

[Take $g = 10\ ms^{-2}$]

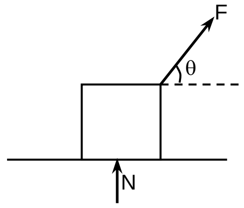
Ans: **5.00**

Sol: F cos θ = mN

F sin θ + N = mg

$F = \frac{\mu mg}{\cos\theta + \mu \sin\theta}$

$F_{min} = \frac{\mu mg}{\sqrt{1+\mu^2}} = \frac{\frac{1}{\sqrt{3}} \times 10}{\frac{2}{\sqrt{3}}} = 5$



Q.5 The electric field in a region is given by $\vec{E} = \frac{2}{5}E_0\hat{i} + \frac{3}{5}E_0\hat{j}$ with $E_0 = 4.0 \times 10^3 \frac{N}{C}$. The flux of this field through a rectangular surface area 0.4 m^2 parallel to the Y – Z plane is ________ $Nm^2\,C^{-1}$.

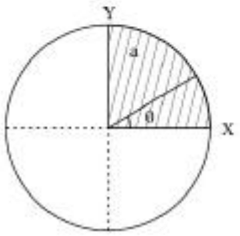
Ans: **640.00**

Sol: $\phi = E_x A$

$\Rightarrow \frac{2}{5} \times 4 \times 10^3 \times 0.4 = 640$

Q.6 The disc of mass M with uniform surface mass density σ is shown in the figure. The centre of mass of the quarter disc (the shaded area) is at the position $\frac{x}{3}\frac{a}{\pi}, \frac{x}{3}\frac{a}{\pi}$ where x is ________. (Round off to the Nearest Integer)

[a is an area as shown in the figure]



Ans: **4.00**

Sol: C.O.M of quarter disc is at $\frac{4a}{3\pi}, \frac{4a}{3\pi} = 4$

Q.7 The electric field intensity produced by the radiation coming from a 100 W bulb at a distance of 3 m is E. The electric field intensity produced by the radiation coming from 60 W at the same distance is

$\sqrt{\frac{x}{5}}E$. Where the value of x = ________.

Ans: **3.00**

Sol: $C\varepsilon_0 E^2 = \frac{100}{4\pi \times 3^2}$

$C\varepsilon_0 \left(\sqrt{\frac{x}{5}}E\right)^2 = \frac{60}{4\pi \times 3^2}$

$\Rightarrow \frac{x}{5} = \frac{3}{5}$

x = 3

Q.8 A particle of mass m moves in a circular orbit in a central potential field $U(r) = U_0 r^4$. If Bohr's quantization conditions are applied, radii of possible orbitals r_n vary with $n^{\frac{1}{\alpha}}$, where α is ________.

Ans: 3.00

Sol: $F = \frac{-dU}{dr} = -4U_0 r^3 = \frac{mV^2}{r}$

$mv^2 = 4U_0 r^4$

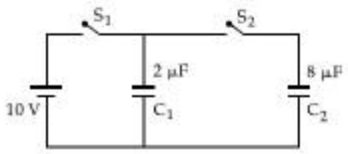
$v \propto r^2$

$mvr = \frac{nh}{2\pi}$

$r^3 \propto n$

$r \propto n^{1/3}$ =3

Q.9 A 2 μF capacitor C_1 is first charged to a potential difference of 10 V using a battery. Then the battery is removed and the capacitor is connected to an uncharged capacitor C_2 of 8 μF. The charge in C_2 on equilibrium condition is ________ μC. (Round off to the Nearest Integer)



Ans: 16.00

Sol: $20 = (C_1 + C_2)\, V$

$V = 2$ volt

$Q_2 = C_2 V = 16\ \mu C$

Q.10 Seawater at a frequency $f = 9 \times 10^2$ Hz, has permittivity $\varepsilon = 80\varepsilon_0$ and resistivity $\rho = 0.25\ \Omega m$. Imagine a parallel plate capacitor is immersed in seawater and is driven by an alternating voltage source $V(t) = V_0 \sin(2\pi f t)$. Then the conduction current density becomes 10^x times the displacement current density after time $t = \frac{1}{800}$ s. The value of x is ________.

(Given : $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ Nm^2C^{-2}$)

Ans: 6.00

Sol: $J_c = \frac{E}{\rho} = \frac{V}{\rho d}$

$J_d = \frac{1}{A}\frac{dq}{dt}$

$= \frac{C}{A}\frac{dV_c}{dt}$

$= \frac{\varepsilon}{d}\frac{dV_c}{dt}$

$\Rightarrow \frac{V_0 \sin 2\pi f t}{\rho d} = 10^x \times \frac{80\varepsilon_0}{d} V_0 (2\pi f) \cos 2\pi f t$

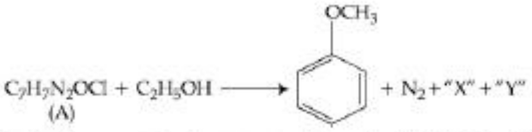
$\tan\left(2\pi \times \frac{900}{800}\right) = 10^x \times \frac{40}{9 \times 10^9} \times 900$

x = 6

PART – B – CHEMISTRY

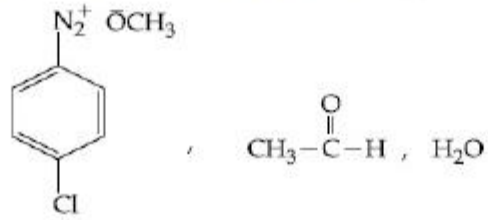
SECTION A

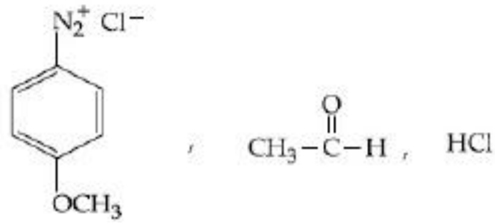
Q.1

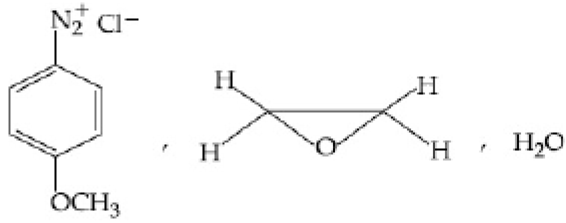


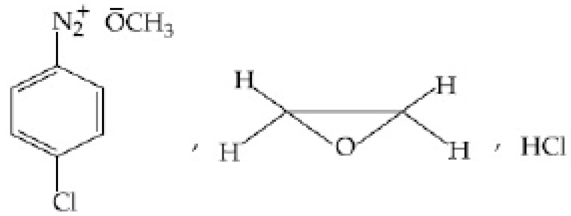
In the above reaction, the structural formula of (A), "X" and "Y" respectively are :

Options

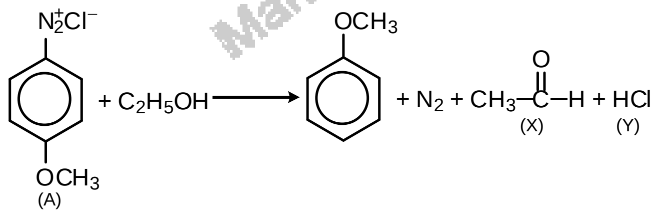
1. 

2. 

3. 

4. 

Ans: 2

Sol: 

Q.2 One of the by-products formed during the recovery of NH_3 from Solvay process is :

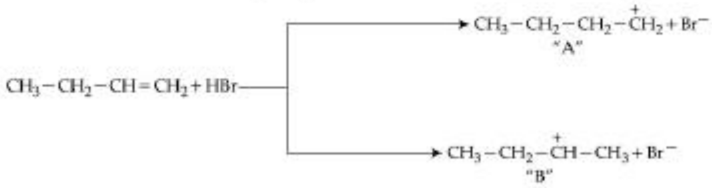
Options

1. $NaHCO_3$
2. NH_4Cl
3. $CaCl_2$
4. $Ca(OH)_2$

Ans: 3

Sol: $CaCl_2$

Q.3 Choose the correct statement regarding the formation of carbocations A and B given.



Options
1. Carbocation B is more stable and formed relatively at slow rate
2. Carbocation A is more stable and formed relatively at faster rate
3. Carbocation A is more stable and formed relatively at slow rate
4. Carbocation B is more stable and formed relatively at faster rate

Ans: 4

Sol: Order of stability of carbocation 3° > 2° > 1°. $CH_3-CH_2-\overset{+}{C}H-CH_3$ is more stable and formed relatively a faster rate

Q.4 The set of elements that differ in mutual relationship from those of the other sets is :

Options
1. Li – Na
2. B – Si
3. Li – Mg
4. Be – Al

Ans: 1

Sol: Lithium shows diagonal relationship with Mg not with Na

Q.5 Fructose is an example of :

Options
1. Aldohexose
2. Pyranose
3. Heptose
4. Ketohexose

Ans: 4

Sol: Fructose is an example of ketohexose

Q.6 The set that represents the pair of neutral oxides of nitrogen is :

Options
1. NO and NO_2
2. N_2O and NO_2
3. NO and N_2O
4. N_2O and N_2O_3

Ans: 3

Sol: Neutral oxides of nitrogen are N_2O and NO
NO_2 and N_2O_3 are acidic oxides

Q.7 The correct pair(s) of the ambident nucleophiles is (are) :

(A) AgCN/KCN
(B) RCOOAg/RCOOK
(C) $AgNO_2/KNO_2$
(D) AgI/KI

Options
1. (A) only
2. (B) and (C) only
3. (B) only
4. (A) and (C) only

Ans: 4

Sol: AgCN / KCN, $AgNO_2$ / KNO_2 are amphident nucleophiles

Q.8 The functional groups that are responsible for the ion-exchange property of cation and anion exchange resins, respectively, are :

Options
1. $-SO_3H$ and $-NH_2$
2. $-NH_2$ and $-SO_3H$
3. $-SO_3H$ and $-COOH$
4. $-NH_2$ and $-COOH$

Ans: 1

Sol: Cation exchange resin contains acidic $-SO_3H$ and anion exchange resin contains basic $-NH_2$ group

Q.9 Amongst the following, the linear species is :

Options
1. NO_2
2. N_3^-
3. O_3
4. Cl_2O

Ans: 2

Sol: N_3^- molecule has linear shape with bond angle 180°

Q.10 For the coagulation of a negative sol, the species below, that has the highest flocculating power is :

Options

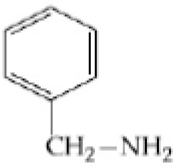
1. SO_4^{2-}
2. PO_4^{3-}
3. Na^+
4. Ba^{2+}

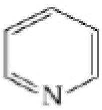
Ans: 4

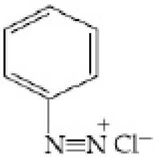
Sol: According to Hardy-Schulze rule, greater the valency greater will be the power to bring about coagulation

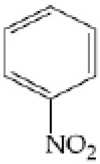
Q.11 Nitrogen can be estimated by Kjeldahl's method for which of the following compound ?

Options

1. 

2. 

3. 

4. 

Ans: 1

Sol: Kjeldahl's method cannot be used for the estimation of nitrogen in nitro compounds, azo compounds and compounds containing nitrogen in the ring.

Q.12 Match List - I with List - II :

	List - I		List - II
(a)	$[Co(NH_3)_6]\,[Cr(CN)_6]$	(i)	Linkage isomerism
(b)	$[Co(NH_3)_3\,(NO_2)_3]$	(ii)	Solvate isomerism
(c)	$[Cr(H_2O)_6]Cl_3$	(iii)	Co-ordination isomerism
(d)	*cis*-$[CrCl_2(ox)_2]^{3-}$	(iv)	Optical isomerism

Choose the correct answer from the options given below :

Options
1. (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)
2. (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)
3. (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)
4. (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

Ans: 1

Sol: (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

(a) $[Co(NH_3)_6[Cr(CN)_6] \rightarrow$ Coordination isomerism
(b) $[Co(NH_3)_3(NO_2)_3] \rightarrow$Linkage isomerism
(c) $[Cr(H_2O)_6Cl_3] \rightarrow$ Solvate isomerism
(d) Cis-$[CrCl_2(ox)_2]^{3-} \rightarrow$Optical isomerism

Q.13

$$\underset{\text{Sucrose}}{C_{12}H_{22}O_{11}} + H_2O \xrightarrow{\text{Enzyme A}} \underset{\text{Glucose}}{C_6H_{12}O_6} + \underset{\text{Fructose}}{C_6H_{12}O_6}$$

$$\underset{\text{Glucose}}{C_6H_{12}O_6} \xrightarrow{\text{Enzyme B}} 2C_2H_5OH + 2CO_2$$

In the above reactions, the enzyme A and enzyme B respectively are :

Options
1. Invertase and Zymase
2. Amylase and Invertase
3. Invertase and Amylase
4. Zymase and Invertase

Ans: 1

Sol: Invertase catalyse the conversion of sucrose to glucose and fructose
Zymase catalyse the conversion of glucose to ethanol

Q.14 Match List - I with List - II :

	List - I		List - II
(a)	Haematite	(i)	$Al_2O_3 \cdot xH_2O$
(b)	Bauxite	(ii)	Fe_2O_3
(c)	Magnetite	(iii)	$CuCO_3 \cdot Cu(OH)_2$
(d)	Malachite	(iv)	Fe_3O_4

Choose the correct answer from the options given below :

Options
1. (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)
2. (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)
3. (a)-(i), (b)-(iii), (c)-(ii), (d)-(iv)
4. (a)-(ii), (b)-(iii), (c)-(i), (d)-(iv)

Ans: 2

Sol: (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

	List I		**List II**
(a)	Haematite	–	Fe_2O_3
(b)	Bauxite	–	$Al_2O_3.xH_2O$
(c)	Magnetite	–	Fe_3O_4
(d)	Malachite	–	$CuCO_3.(Cu(OH)_2$

Q.15 Which of the following statement(s) is (are) incorrect reason for eutrophication ?

(A) excess usage of fertilisers

(B) excess usage of detergents

(C) dense plant population in water bodies

(D) lack of nutrients in water bodies that prevent plant growth

Choose the most appropriate answer from the options given below :

Options
1. (B) and (D) only
2. (D) only
3. (A) only
4. (C) only

Ans: 2

Sol: Eutrophication support the plant growth but present the growth of aquatic animals

Q.16 During which of the following processes, does entropy decrease ?

(A) Freezing of water to ice at 0°C

(B) Freezing of water to ice at −10°C

(C) $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$

(D) Adsorption of CO(g) on lead surface.

(E) Dissolution of NaCl in water

Choose the correct answer from the options given below :

Options
1. (A), (C) and (E) only
2. (A) and (E) only
3. (B) and (C) only
4. (A), (B), (C) and (D) only

Ans: 4

Sol: Dissolution of NaCl is water entropy increases

Q.17 Primary, secondary and tertiary amines can be separated using :

Options
1. Benzene sulphonic acid
2. para-Toluene sulphonyl chloride
3. Acetyl amide
4. Chloroform and KOH

Ans: 2

Sol: Para-Toluene sulphonyl chloride (tosyl chloride) can be used for separating mixture of 1°, 2°, 3° amine

Q.18 Given below are two statements :

Statement I : 2-methylbutane on oxidation with $KMnO_4$ gives 2-methylbutan-2-ol.

Statement II : n-alkanes can be easily oxidised to corresponding alcohols with $KMnO_4$

Choose the correct option :

Options
1. Statement I is incorrect but statement II is correct
2. Both statement I and statement II are correct
3. Both statement I and statement II are incorrect
4. Statement I is correct but statement II is incorrect

Ans: 4

Sol: Option (4)

Q.19 The common positive oxidation states for an element with atomic number 24, are :

Options
1. $+2$ to $+6$
2. $+1$ and $+3$ to $+6$
3. $+1$ to $+6$
4. $+1$ and $+3$

Ans: 1

Sol: The element with atomic number 24 is chromium
Exhibit oxidation state from +2 to +6
$_{24}Cr$ [Ar] $3d^5\ 4s^1$

Q.20 Match List - I with List - II.

	List - I Chemical Compound		List - II Used as
(a)	Sucralose	(i)	Synthetic detergent
(b)	Glyceryl ester of stearic acid	(ii)	Artificial sweetener
(c)	Sodium benzoate	(iii)	Antiseptic
(d)	Bithionol	(iv)	Food preservative

Choose the correct match :

Options
1. (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)
2. (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)
3. (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)
4. (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

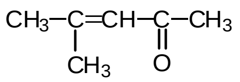
Ans: 4

Sol: (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)
(a) Sucralose – Artificial sweetener
(b) Glyceryl ester of stearic acid – Synthetic detergent
(c) Sodium benzoate – Food preservative
(d) Bithionol – Antiseptic

SECTION A

Q.1 The total number of C–C sigma bond/s in mesityl oxide ($C_6H_{10}O$) is ________. (Round off to the Nearest Integer).

Ans: 5

Sol: 

No. of C–C sigma bond = 5

Q.2 In the ground state of atomic Fe(Z = 26), the spin-only magnetic moment is ________ $\times 10^{-1}$ BM. (Round off to the Nearest Integer).

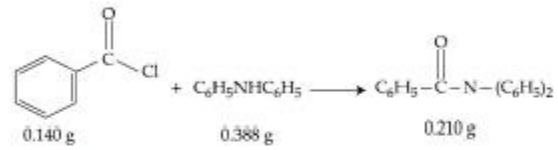
[Given : $\sqrt{3} = 1.73$, $\sqrt{2} = 1.41$]

Ans: **49**

Sol: Fe = [Ar] $3d^6\ 4s^2$

$$r = \sqrt{4(4+2)}\ \text{BM} = 4.9\ \text{BM} = 49 \times 10^{-1}\ \text{BM}$$

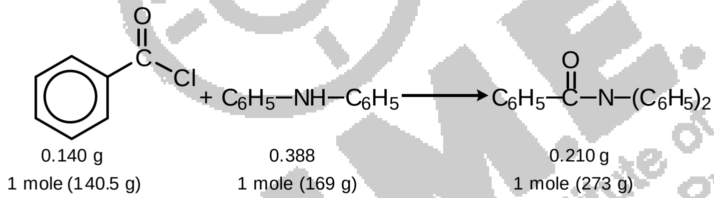
Q.3



Consider the above reaction. The percentage yield of amide product is ________. (Round off to the Nearest Integer).

(Given : Atomic mass : C : 12.0 u, H : 1.0 u, N : 14.0 u, O : 16.0 u, Cl : 35.5 u)

Ans: **77**

Sol: 

$C_6H_5-\overset{O}{\overset{\|}{C}}-Cl$ is the limiting reagent

140.5 g C_6H_5COCl produces 273 g amide

$\therefore$ 140 g C_6H_5COCl produces $= \dfrac{273 \times 1.40}{140.5} = 0.272\text{g}$ amide

% yield of amide = $\dfrac{0.210}{0.272} \times 100$

$\Rightarrow 77.2 \approx 77$

Q.4 The number of chlorine atoms in 20 mL of chlorine gas at STP is ________ 10^{21}. (Round off to the Nearest Integer).

[Assume chlorine is an ideal gas at STP

$R = 0.083\ \text{L bar mol}^{-1}\ \text{K}^{-1}$, $N_A = 6.023 \times 10^{23}$]

Ans: **1**

Sol: At STP

22400 mL of Cl_2 gas contains = 2 × N_A no. of chlorine atom

20 mL of chlorine gas at STP $= \dfrac{2 \times 6.022 \times 10^{23} \times 20}{22400}$ chlorine atom

$= 1.07 \times 10^{21} \approx 1 \times 10^{21}$

Q.5 The reaction $2A + B_2 \rightarrow 2AB$ is an elementary reaction.

For a certain quantity of reactants, if the volume of the reaction vessel is reduced by a factor of 3, the rate of the reaction increases by a factor of ________. (Round off to the Nearest Integer).

Ans: 27

Sol: Rate = $k[A]^2 [B_2]^1$

Volume reduced by factor 3. Then

$\text{Rate} = k\left[\frac{1}{3}A\right]^2 \left[\frac{1}{3}B_2\right]^1 \qquad \text{Rate} = \left(\frac{1}{3}\right)^3 k[A]^2[B_2]^1$

$\therefore$ Rate increases by the factor 27

Q.6 Consider the reaction $N_2O_4(g) \rightleftharpoons 2NO_2(g)$. The temperature at which $K_C = 20.4$ and $K_P = 600.1$, is ________ K. (Round off to the Nearest Integer).

[Assume all gases are ideal and $R = 0.0831$ L bar K^{-1} mol^{-1}]

Ans: 354

Sol: $K_p = K_c (RT)^{\Delta n}$

$K_p = K_c (RT)^1$ For the reaction $N_2O_{4(g)} \rightleftharpoons 2NO_{2(g)}$

$$T = \frac{600.1}{0.0831 \times 20.4} = 353.99 \approx 354$$

Q.7 A 1 molal $K_4Fe(CN)_6$ solution has a degree of dissociation of 0.4. Its boiling point is equal to that of another solution which contains 18.1 weight percent of a non electrolytic solute A. The molar mass of A is ________ u. (Round off to the Nearest Integer).

[Density of water = 1.0 g cm^{-3}]

Ans: 85

Sol:

	$K_4[Fe(CN)_6] \longrightarrow$	$4K^+$	$+ [Fe(CN)_6]^{4-}$
	1	0	0
	1-α	4α	α
α = 40%	0.6	1.6	0.4

Final molality = 1.6 + 0.4 + 0.6 = 2.6

For solution with same boiling point have same molality

The molecular mass of solute with weight percent 18.1 $= \frac{18.1 \times 1000}{2.6 \times (100 - 18.1)} = 85$

Q.8 On complete reaction of $FeCl_3$ with oxalic acid in aqueous solution containing KOH, resulted in the formation of product A. The secondary valency of Fe in the product A is ________. (Round off to the Nearest Integer).

Ans: 6

Sol: $FeCl_3 + 6KOH + 3H_2C_2O_4 \rightarrow K_3[Fe(C_2O_4)_3] + 3KCl + 6H_2O$

Secondary valency of Fe in the complex $K_3[Fe(C_2O_4)_3]$ is '6'

Q.9 KBr is doped with 10^{-5} mole percent of $SrBr_2$. The number of cationic vacancies in 1 g of KBr crystal is ________ 10^{14}. (Round off to the Nearest Integer).

[Atomic Mass : K : 39.1 u, Br : 79.9 u

$N_A = 6.023 \times 10^{23}$]

Ans: 5

Sol: 1 mole KBr produces $= \frac{10^{-5}}{100}$ mol cation vacancy when dopped with $SrBr_2$

The number of cation vacancy of 1 g KBr $= \frac{6.023 \times 10^{23}}{119} \times \frac{10^{-5}}{100} = 0.050 \times 10^{16} = 5.0 \times 10^{14}$

Q.10 A KCl solution of conductivity 0.14 S m^{-1} shows a resistance of 4.19 Ω in a conductivity cell. If the same cell is filled with an HCl solution, the resistance drops to 1.03 Ω. The conductivity of the HCl solution is __________ $\times 10^{-2}$ S m^{-1}. (Round off to the Nearest Integer).

Ans: 57

Sol: $k = \frac{1}{R}\left(\ell/a\right)$

For the same conductivity cell $\left(\ell/a\right)$ a constant

$\frac{K_{HCl}}{K_{KCl}} = \frac{R_{KCl}}{R_{HCl}}$ $\Rightarrow \frac{K_{HCl}}{0.14} = \frac{4.19}{1.03}$

$K_{HCl} = \frac{4.19 \times 0.14}{1.03} = 0.569 \approx 57 \times 10^{-2}$

PART – C – MATHEMATICS

SECTION A

Q.1 Let $y = y(x)$ be the solution of the differential equation

$\cos x(3\sin x + \cos x + 3)\,dy = (1 + y\sin x(3\sin x + \cos x + 3))dx$, $0 \le x \le \frac{\pi}{2}$, $y(0) = 0$. Then, $y\left(\frac{\pi}{3}\right)$ is equal to :

Options

1. $2\log_e\left(\frac{2\sqrt{3} + 10}{11}\right)$
2. $2\log_e\left(\frac{2\sqrt{3} + 9}{6}\right)$
3. $2\log_e\left(\frac{3\sqrt{3} - 8}{4}\right)$
4. $2\log_e\left(\frac{\sqrt{3} + 7}{2}\right)$

Ans: 1

Sol: cosx (3sinx+cosx+3)dy = (1+y sin (3sinx+cosx+3) dx

$\div$ by $\cos x(3\sin x + \cos x + 3)$

$dy = \left(\frac{1}{3\sin x + \cos x + 3} + y\tan x\right)dx$

Rearranging

$$\frac{dy}{dx} - y\tan x = \frac{1}{(3\sin x + \cos x + 3)\cos x}$$

$$\text{IF.} = e^{-\int \tan x} = e^{\log \cos x} = \cos x$$

Solution is

$$y \cos x \int \cos x \frac{1}{(3\sin x + \cos x + 3)\cos x} dx$$

put t=tan $x/2$ then $dx = \frac{2dt}{1+t^2}$; $\sin x = \frac{2t}{1+t^2}$; $\cos x = \frac{1-t^2}{1+t^2}$

$$y \cos x = \int \frac{dt}{(t+1)(t+2)} = \ln\left|\frac{t+1}{t+2}\right| + c = \ln\left|\frac{\tan x/2 + 1}{\tan x/2 + 2}\right| + c$$

Now y (0) = 0

$$\Rightarrow 0 = \log\frac{1}{2} + c \Rightarrow c = \ln 2$$

$$\therefore y \cos x = \ln\left|\frac{\tan x/2 + 1}{\tan x/2 + 2}\right| = \ln 2$$

When $x = \pi/3$

$$y \cdot \frac{1}{2} = \ln \frac{2\left(1 + \frac{1}{\sqrt{3}}\right)}{2 + \frac{1}{\sqrt{3}}}$$

$$\ln \frac{2\sqrt{3}+10}{11} \Rightarrow y = 2\ln\left(\frac{2\sqrt{3}+10}{11}\right)$$

Q.2 If the equation of plane passing through the mirror image of a point (2, 3, 1) with respect to line $\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+2}{-1}$ and containing the line $\frac{x-2}{3} = \frac{1-y}{2} = \frac{z+1}{1}$ is $\alpha x + \beta y + \gamma z = 24$, then $\alpha + \beta + \gamma$ is equal to :

Options
1. 20
2. 21
3. 19
4. 18

Ans: 3

Sol: foot of the $\perp^r$ is given by

$$\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+2}{-1} = \frac{2(3)+1(0)-1(3)}{4+1+1}$$

$$\Rightarrow \frac{x+1}{2} = \frac{y-3}{1} = \frac{z+2}{-1} = \frac{1}{2}$$

Foot of the $\perp^r = \left(0, \frac{7}{2}, -5/2\right)$

Required plane is

$$\begin{vmatrix} x-2 & y-1 & z+1 \\ 3 & -2 & 1 \\ 4 & -3 & 5 \end{vmatrix} = 0$$

$\Rightarrow 7x + 11y + z = 24$

$\alpha = 7, \quad \beta = 11, \quad \gamma = 1$

$\alpha + \beta + \gamma = 7 + 11 + 1 = 19$

Q.3 Let L be a tangent line to the parabola $y^2 = 4x - 20$ at (6, 2). If L is also a tangent to the ellipse $\frac{x^2}{2} + \frac{y^2}{b} = 1$, then the value of b is equal to :

Options
1. 11
2. 16
3. 14
4. 20

Ans: 3

Sol: $y^2 = 4x - 20$

$2y\frac{dy}{dx} = 4 \Rightarrow \frac{dy}{dx} = \frac{2}{y}$

At (6,2)

$\frac{dy}{dx} = 1$

Equation of tangent

$y - 2 = 1(x - 6)$

$y = x - 4$

m=1 c= -4

Given ellipse is $\frac{x^2}{2} + \frac{y^2}{b} = 1$

Since $y = x - 4$ is a tangent to the ellipse

$16 = 2 \times 1 + b$

$\Rightarrow b = 14$

Q.4 If the curve $y = y(x)$ is the solution of the differential equation

$$2(x^2 + x^{5/4})\,dy - y(x + x^{1/4})\,dx = 2x^{9/4}\,dx,\ x > 0$$ which passes through the point $\left(1, 1 - \frac{4}{3}\log_e 2\right)$, then the value of $y(16)$ is equal to :

Options

1. $\left(\frac{31}{3} + \frac{8}{3}\log_e 3\right)$

2. $4\left(\frac{31}{3} - \frac{8}{3}\log_e 3\right)$

3. $4\left(\frac{31}{3} + \frac{8}{3}\log_e 3\right)$

4. $\left(\frac{31}{3} - \frac{8}{3}\log_e 3\right)$

Ans: 2

Sol: Given eq is

$$2\left(x^2 + x^{5/4}\right)dy - y\left(x + x^{1/4}\right)dx = 2x^{9/4}dx$$

$\div$ by $\left(x + x^{1/4}\right)$, we get

$$2x\,dy - y\,dx = \frac{2x^{9/4}}{\left(x + x^{1/4}\right)}dx$$

$\div$ by $2x\,dx$

$$\frac{dy}{dx} - \frac{y}{2x} = \frac{x^{5/4}}{x + x^{1/4}}$$

$$\text{I.F} = e^{-\int 2x} = e^{\frac{-1}{2}\log x} = \frac{1}{\sqrt{x}}$$

Solution is

$$\frac{y}{\sqrt{x}} = \int \frac{1}{x^{1/2}} \cdot \frac{x^{5/4}}{\left(x + x^{1/4}\right)}dx = \int \frac{x^{3/4}}{x + x^{1/4}}dx$$

Put $x = t^4$

$dx = 4t^3dt$

$$\frac{y}{\sqrt{x}} = \int \frac{t^3\ .\ 4t^3\ dt}{t^4 + t}$$

$$= 4\int t^2dt - 4\int \frac{t^2}{t^3 + 1}dx$$

$$\frac{y}{\sqrt{x}} = \frac{4}{3}t^3 - \frac{4}{3}\ln\left(t^3 + 1\right) + c$$

Since it passes through $\left(1, 1-\frac{4}{3}\log_e 2\right)$ we get

$1-\frac{4}{3}\log_e 2 = \frac{4}{3}-\frac{4}{3}\log_e 2 + c$

$\Rightarrow c = \frac{-1}{3}$

$y = \frac{4}{3}x^{5/4} - \frac{4}{3}\sqrt{x}\ln\left(x^{3/4}+1\right) - \frac{\sqrt{x}}{3}$

$y(16) = \frac{4}{3}\times 32 - \frac{4}{3}\times 4\ln 9 - \frac{4}{3}$

$= 4\left(\frac{31}{3} - \frac{8}{3}\ln 3\right)$

Q.5 Let O be the origin. Let $\overrightarrow{OP} = x\hat{i} + y\hat{j} - \hat{k}$ and $\overrightarrow{OQ} = -\hat{i} + 2\hat{j} + 3x\hat{k}$, $x, y \in R$, $x > 0$, be such that $|\overrightarrow{PQ}| = \sqrt{20}$ and the vector $\overrightarrow{OP}$ is perpendicular to $\overrightarrow{OQ}$. If $\overrightarrow{OR} = 3\hat{i} + z\hat{j} - 7\hat{k}$, $z \in R$, is coplanar with $\overrightarrow{OP}$ and $\overrightarrow{OQ}$, then the value of $x^2 + y^2 + z^2$ is equal to :

Options
1. 9
2. 1
3. 7
4. 2

Ans: **1**

Sol: $|PQ| = 20$

$\Rightarrow (x+1)^2 + (y-2)^2 + (1+3x)^2 = 20$

$\Rightarrow x = 1$

Op is $\perp^r$ to OQ

$OP.OQ = 0 \Rightarrow -x + 2y - 3x = 0$

$\Rightarrow y = 2x$

Also $\overline{OP}$, $\overline{OQ}$, $\overline{OR}$ are coplanar

$$\begin{vmatrix} x & y & 1 \\ -1 & 2 & 3x \\ 3 & z & -7 \end{vmatrix} = 0$$

Sub x and y and solving, we get z=-2

$\therefore x^2 + y^2 + z^2 = 1^2 + 2^2 + (-2)^2 = 9$

Q.6

The value of $\sum_{r=0}^{6} \left({}^6C_r \cdot {}^6C_{6-r}\right)$ is equal to :

Options
1. 924
2. 1024
3. 1124
4. 1324

Ans: 1

Sol: $\sum_{r20}^{6} {}^6C_1 \ {}^6C_{6-r}$

$= {}^6C_0 . \ {}^6C_6 . + {}^6C_1 . \ {}^6C_5 . + \ldots\ldots\ldots \ {}^6C_6 . \ \ {}^6C_0 .$, which is equivalent to the coeff of x^6 is $(1+x)^{12}$

Since $(1+x)^{12} = (1+x)^6 (1+x)^6$

$= \left(6C_0 + 6C_1x \ldots\ldots + 6C_6x^6\right)\left(6C_0 + 6C_1x + 6C_2x^2 + \ldots\right)$ comparing coeff of x^6 on both sides

Required answer $= 12C_6 = 924$

Q.7

Two tangents are drawn from a point P to the circle $x^2 + y^2 - 2x - 4y + 4 = 0$, such that the angle between these tangents is $\tan^{-1}\left(\frac{12}{5}\right)$, where $\tan^{-1}\left(\frac{12}{5}\right) \in (0, \pi)$. If the centre of the circle is denoted by C and these tangents touch the circle at points A and B, then the ratio of the areas of ΔPAB and ΔCAB is :

Options
1. 2 : 1
2. 3 : 1
3. 11 : 4
4. 9 : 4

Ans: 4

Sol: Given tan $\theta = \frac{12}{5} \Rightarrow \cos\theta = \frac{5}{13}$

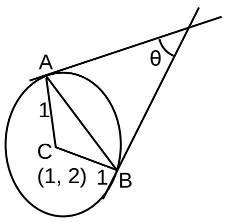
From ΔPAC, $PA = \cot \theta/2$

Area of $\Delta PAB = \frac{1}{2}(PA)^2 \sin\theta$

$= \frac{1}{2}\cot^2 \theta/2 \, \text{Sin}\theta$

$\frac{1}{2} \frac{\cos^2 \theta/2}{\sin^2 \theta/2} . \sin\theta$

$\frac{1}{2}\left(\frac{1+\cos\theta}{1-\cos\theta}\right)\sin\theta = \frac{27}{26}$



Area of $\Delta CAB = \frac{1}{2}1.1.\sin\theta = \frac{1}{2}\frac{12}{13} = \frac{6}{13}$

$\therefore$ Required ratio $= \frac{27}{26}\times\frac{13}{6} = \frac{9}{4} \Rightarrow 9:4$

Q.8 The number of solutions of the equation $\sin^{-1}\left[x^2+\frac{1}{3}\right]+\cos^{-1}\left[x^2-\frac{2}{3}\right]=x^2$, for $x\epsilon[-1, 1]$, and $[x]$ denotes the greatest integer less than or equal to x, is :

Options

1. 2
2. Infinite
3. 0
4. 4

Ans: 3

Sol: $\sin^{-1}\left(x^2+\frac{1}{3}\right)+\cos^{-1}\left(x-\frac{2}{3}\right)=x^2$

$\sin^{-1}\left(x^2+\frac{1}{3}\right)$ is defined as $\left[0, 5/3\right)$ and $\cos^{-1}\left(x^2-2/3\right)$ is defined as $\left[0, 8/3\right)$

$\therefore$ the sum is defined as $\left[0, 5/3\right]$

When $0 \le x^2 < 2/3$

$\sin^{-1}(0)+\cos^{-1}(-1)=x^2$

$\Rightarrow 0+\pi = x^2$

$\Rightarrow$ but $\pi \notin \left[0, 2/3\right] \Rightarrow$ no solution is this interval

When $2/3 \le x^2 < 5/3$

$\sin^{-1}(1)+\cos^{-1}(0)=x^2$

$\Rightarrow \frac{\pi}{2}+\frac{\pi}{2}=x^2$

$\Rightarrow x^2=\pi$ but $\pi \notin \left[2/3, 5/3\right)$

no solution is this interval

No: of solutions = 0

Q.9 If the Boolean expression $(p\wedge q)\circledast(p\otimes q)$ is a tautology, then $\circledast$ and $\otimes$ are respectively given by :

Options

1. $\wedge, \rightarrow$
2. $\wedge, \vee$
3. $\vee, \rightarrow$
4. $\rightarrow, \rightarrow$

Ans: 4

Sol:

p	q	$p \wedge q$	$p \to q$	$(p \wedge q) \to (p \to q)$	$(p \vee q)$	$(p \wedge q) \vee (p \wedge q)$	$(p \wedge q) \vee (p \to q)$	$(p \wedge q) \wedge (p \to q)$
T	F	F	F	T	T	T	F	F
T	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	F
F	F	F	T	T	F	F	T	F

Q.10 Let a computer program generate only the digits 0 and 1 to form a string of binary numbers with probability of occurrence of 0 at even places be $\frac{1}{2}$ and probability of occurrence of 0 at the odd place be $\frac{1}{3}$. Then the probability that '10' is followed by '01' is equal to :

Options

1. $\frac{1}{6}$
2. $\frac{1}{18}$
3. $\frac{1}{3}$
4. $\frac{1}{9}$

Ans: 4

Sol: Required Probability $= \left(\underset{O}{\frac{1}{2}}.\underset{E}{\frac{1}{3}}.\underset{O}{\frac{1}{2}}.\underset{E}{\frac{2}{3}}\right) + \left(\underset{E}{\frac{2}{2}}.\underset{O}{\frac{1}{2}}.\underset{E}{\frac{1}{3}};\underset{O}{\frac{1}{2}}\right) = \frac{1}{9}$

Q.11 The value of

$$\lim_{n\to\infty} \frac{[r] + [2r] + \ldots + [nr]}{n^2},$$

where r is a non-zero real number and [r] denotes the greatest integer less than or equal to r, is equal to :

Options

1. $\frac{r}{2}$
2. $2r$
3. r
4. 0

Ans: 1

Sol: $r-1+2r-1+\ldots\ldots+nr-1<[r]+[2r]+\ldots..+[nr]\le r+2r+\ldots.nr$

$r\left(\frac{n(n+1)}{2}\right)-n,<[r]+[2r]+\ldots.+[nr]=n\frac{(n+1)}{2}r$

Applying limits and sandwich theorem,

$$\lim_{n\to\alpha}\frac{[r]+[2r]+\ldots+[nr]}{n^2}=\frac{r}{2}$$

Q.12 Let $f: \mathbb{R} \to \mathbb{R}$ be defined as $f(x)=e^{-x}\sin x$. If $F: [0, 1] \to \mathbb{R}$ is a differentiable function such that $F(x)=\int_0^x f(t)\,dt$, then the value of $\int_0^1 (F'(x)+f(x))e^x dx$ lies in the interval

Options

1. $\left[\frac{335}{360}, \frac{336}{360}\right]$
2. $\left[\frac{327}{360}, \frac{329}{360}\right]$
3. $\left[\frac{331}{360}, \frac{334}{360}\right]$
4. $\left[\frac{330}{360}, \frac{331}{360}\right]$

Ans: **4**

Sol: $f(x)=e^{-x}\sin x$

$F(x)=\int_0^x f(x) \Rightarrow F'(x)=f(x)$

$I=\int_0^1 (F'(x)+f(x))e^x = 2\int_0^1 f(x)e^x$

$=2\int_0^1 \sin x\, dx = (-2\cos x)_0^1$

$2(1-\cos 1)$

$I=2\left(1-\left(1-\frac{1}{2!},+\frac{1}{4!}-\frac{1}{6!}+\frac{1}{8!}\cdots\right)\right)$

$I=1-\frac{2}{4!}+\frac{2}{6!}+\cdots \Rightarrow I<2\left(\frac{1}{2}-\frac{1}{24}+\frac{1}{720}\right)$

$\Rightarrow I<\frac{331}{360}$

Also $2\left(\frac{1}{2}-\frac{1}{24}\right),<I$

$\Rightarrow \frac{11}{2}<I \Rightarrow \frac{330}{360}<I$

$\Rightarrow I\in\left[\frac{330}{360},\frac{331}{360}\right]2$

Q.13 Let S_1, S_2 and S_3 be three sets defined as

$S_1 = \{z \in \mathbb{C} : |z-1| \le \sqrt{2}\}$

$S_2 = \{z \in \mathbb{C} : \text{Re}((1-i)z) \ge 1\}$

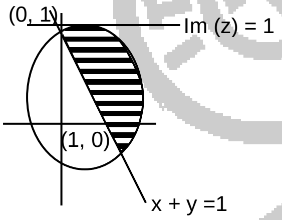
$S_3 = \{z \in \mathbb{C} : \text{Im}(z) \le 1\}$

Then the set $S_1 \cap S_2 \cap S_3$

Options
1. is a singleton
2. has exactly two elements
3. has exactly three elements
4. has infinitely many elements

Ans:

Sol:



Required region is the shaded region
$\Rightarrow$ infinitely many elements

Q.14 The number of solutions of the equation $x + 2\tan x = \frac{\pi}{2}$ in the interval $[0, 2\pi]$ is :

Options
1. 4
2. 3
3. 2
4. 5

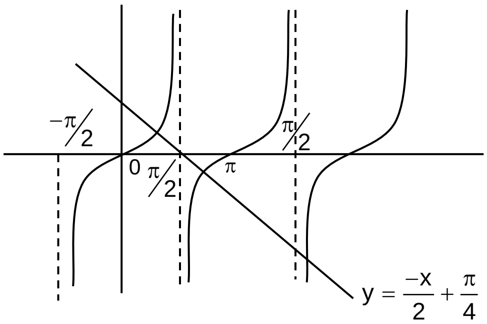
Ans: 2

Sol: $x + 2\tan x = \frac{\pi}{2}$

$\Rightarrow 2\tan x = \frac{\pi}{2} + x$

$\Rightarrow \tan x = \frac{\pi}{4} + \frac{x}{2}$

Form the figure, 3 solutions



Q.15 If x, y, z are in arithmetic progression with common difference d, $x \neq 3d$, and the determinant

of the matrix $\begin{bmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{bmatrix}$ is zero, then the value of k^2 is :

Options
1. 12
2. 36
3. 72
4. 6

Ans: 3

Sol: $2y = x + z$

$$\begin{vmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{vmatrix} = 0$$

$$R_2 \to R_2 - \frac{R_1 + R_3}{2}$$

$$\Rightarrow \begin{vmatrix} 3 & 4\sqrt{2} & x \\ 0 & 5\sqrt{2} - \frac{(4\sqrt{2}+k)}{2} & 0 \\ 5 & k & z \end{vmatrix} = 0$$

$$\Rightarrow 5\sqrt{2} - \left(\frac{4\sqrt{2}+k}{2}\right) = 0$$

$$6\sqrt{2} = k$$

$$\Rightarrow k^2 = 72$$

Q.16 If the integral $\int_0^{10} \frac{[\sin 2\pi x]}{e^{x-[x]}} dx = \alpha e^{-1} + \beta e^{-\frac{1}{2}} + \gamma$, where α, β, γ are integers and $[x]$ denotes the greatest integer less than or equal to x, then the value of $\alpha + \beta + \gamma$ is equal to :

Options
1. 25
2. 10
3. 20
4. 0

Ans: 4

Sol: $\int_0^{10} \frac{[\sin[2\pi x]}{e^{x-[x]}} dx = \int_0^{10} \frac{[\sin[2\pi x]}{e^{\{x\}}} dx$

$= 10\int_0^{1} \frac{[\sin 2\pi x]}{e^{x}} dx$ [since the function is periodic with period 1]

$$10\int_0^{1/2}\frac{[\sin 2\pi x]}{e^x}+\int_{1/2}^{1}\frac{[\sin 2\pi x]}{e^x}$$

$$10\left(e^{-1}-e^{1/2}\right)$$

Comparing with $\alpha\, e^{-1}+\beta e^{-1/2}+\gamma,$

$\alpha = 10,\ \beta = -10,\ \gamma = 0$

$\alpha+\beta+\gamma = 0$

Q.17

Consider the function $f: R \to R$ defined by $f(x) = \begin{cases}\left(2-\sin\left(\frac{1}{x}\right)\right)|x|, & x \neq 0 \\ 0 &, x = 0\end{cases}$. Then f is :

Options
1. monotonic on $(-\infty, 0)$ only
2. not monotonic on $(-\infty, 0)$ and $(0, \infty)$
3. monotonic on $(0, \infty)$ only
4. monotonic on $(-\infty, 0) \cup (0, \infty)$

Ans: 2

Sol: $$f(x) = \begin{cases} -x\left(2-\sin\frac{1}{x}\right) & x<0 \\ 0 & x=0 \\ x\left(2-\sin\left(\frac{1}{x}\right)\right) & x>0\end{cases}$$

$$f'(x) = \begin{cases} -2+\sin\frac{1}{x}-\frac{1}{x}\cos\frac{1}{x} & x<0 \\ 2-\sin\frac{1}{x}+\frac{1}{x}\cos\frac{1}{x} & x>0\end{cases}$$

$\Rightarrow f(x)$ is non monotonic in $(-\infty, 0) \cup (0, \infty)$

Q.18 Let the tangent to the circle $x^2+y^2=25$ at the point $R(3, 4)$ meet x-axis and y-axis at points P and Q, respectively. If r is the radius of the circle passing through the origin O and having centre at the incentre of the triangle OPQ, then r^2 is equal to :

Options
1. $\frac{585}{66}$
2. $\frac{625}{72}$
3. $\frac{125}{72}$
4. $\frac{529}{64}$

Ans: 2

Sol: Given circle is $x^2 + y^2 = 25$

Equation of the tangent is 3x+4y=25

Putting x = 0, Q = $\left(0, \frac{25}{4}\right)$

Putting y=0, P=$\left(\frac{25}{3}, 0\right)$

Coordinates of in center=$\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right) = \left(\frac{25}{12}, \frac{25}{12}\right)$

$r^2 = \left(\frac{25}{12}\right)^2 + \left(\frac{25}{12}\right)^2 = \frac{625}{72}$

Q.19 If the sides AB, BC and CA of a triangle ABC have 3, 5 and 6 interior points respectively, then the total number of triangles that can be constructed using these points as vertices, is equal to :

Options

1. 360
2. 333
3. 240
4. 364

Ans: 2

Sol: **No: of triangles** $= 14C_3 - 3C_3 - 5C_3 - 6C_3$ =333

Q.20 The value of the limit $\lim_{\theta \to 0} \frac{\tan(\pi \cos^2\theta)}{\sin(2\pi \sin^2\theta)}$ is equal to :

Options

1. $-\frac{1}{2}$
2. $\frac{1}{4}$
3. $-\frac{1}{4}$
4. 0

Ans: 486

Sol: $\lim_{\theta \to 0} \frac{\tan(\pi\cos^2\theta)}{\sin(2\pi\sin^2\theta)} \left(\frac{0}{0}\right)$

$\lim_{\theta \to 0} \frac{\tan(\pi - \pi\sin^2\theta)}{\sin(2\pi\sin^2\theta)} = \lim_{\theta \to 0} \frac{-\tan(\pi\sin^2\theta)}{\pi\sin^2\theta} \cdot \frac{\pi\sin^2\theta}{\theta^2} \cdot \frac{\theta^2 2\pi\sin^2\theta}{\sin(2\pi\sin^2\theta)\ 2\pi\sin^2\theta} = \frac{-1}{2}$

SECTION B

Q.1 Consider a set of 3n numbers having variance 4. In this set, the mean of first 2n numbers is 6 and the mean of the remaining n numbers is 3. A new set is constructed by adding 1 into each of first 2n numbers, and subtracting 1 from each of the remaining n numbers. If the variance of the new set is k, then 9k is equal to ________.

Ans: **68**

Sol: Let $x_1, x_2, \ldots\ldots\ldots x_{2n} \ldots . x_{3n}$ be the 3n observations

$$\sum_{i=1}^{3n} \frac{xi^2}{3n} - (\bar{x})^2 = 4$$

Now $\frac{2n \,.\, 6 + n \,.\, 3}{3n} = 5$ is the combined mean $= \bar{x}$

$$\sum \frac{xi^2}{3n} - 25 = 4$$

$$\sum \frac{x^2}{3n} = 29 \Rightarrow \sum xi^2 = 87n$$

`

New set of numbers is given by

$x_1 + 1,\ x_2 + 1, \ldots x_{2n} + 1, x_{2n+1} - 1 \cdots x_{3n} - 1$

New mean $= \frac{2n(7) + n.2}{3n} = \frac{16}{3}$

$$k = \sum \frac{xi^2}{3n} - \frac{256}{9}$$

$$= \frac{1}{3n}\left[\sum x_i^2 + 2\sum_{i=1}^{2n} xi - 2\sum_{i=2n+1}^{3n} xi + 3n\right] - \frac{256}{9}$$

$$= \frac{1}{3n}[87n + 24n - 6n + 3n] - \frac{256}{9}$$

$$= 36 - \frac{256}{9} = \frac{68}{9}$$

$$\Rightarrow 9k = 68$$

Q.2 Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ such that $AB = B$ and $a + d = 2021$, then the value of $ad - bc$ is equal to ________.

Ans: **2020**

Sol: AB=B

$$\begin{bmatrix} a\alpha + b\beta \\ c\alpha + d\beta \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$(a-1)\alpha + b\beta = 0$

$c\alpha + (d-1)\beta = 0$

Since $B \neq 0$

$$\begin{vmatrix} a-1 & b \\ ad & d-1 \end{vmatrix} = 0$$

$\Rightarrow ad - a - d - bc + 1 = 0$

$\Rightarrow ad - bc = a + d - 1 = 2021 - 1 = 2020$

Q.3 Let $f: [-1, 1] \to R$ be defined as $f(x) = ax^2 + bx + c$ for all $x \in [-1, 1]$, where a, b, c $\in$ R such that $f(-1) = 2$, $f'(-1) = 1$ and for $x \in (-1, 1)$ the maximum value of $f''(x)$ is $\frac{1}{2}$. If $f(x) \le \alpha$, $x \in [-1, 1]$, then the least value of α is equal to ________.

Ans: **5**

Sol:
$f(x) = ax^2 + bx + c$
$f(-1) = 2 \Rightarrow a - b + c = 2$
$f'(x) = 2ax + b$
$f'(-1) = 1$
$\Rightarrow -2a + b = 1$
$f''(x) = 2a$
Max value of 2a= $1/2$
$\Rightarrow b = 3/2,\ a = 1/4$ and $c = \frac{13}{4}$
$\Rightarrow f(x) = \frac{x^2}{4} + \frac{3}{2}x + \frac{13}{4}$
For $x \in [-1, 1]$, $f(x) \le 5$
$\Rightarrow \alpha = 5$

Q.4 Let $\tan\alpha$, $\tan\beta$ and $\tan\gamma$; $\alpha, \beta, \gamma \ne \frac{(2n-1)\pi}{2}$, $n \in N$ be the slopes of three line segments OA, OB and OC, respectively, where O is origin. If circumcentre of ΔABC coincides with origin and its orthocentre lies on y-axis, then the value of $\left(\frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha\cos\beta\cos\gamma}\right)^2$ is equal to ________.

Ans: **12**

Sol: Since orthocenter and circum centre lie on y axis, centroid also lies on y axis
$\Rightarrow \cos\alpha + \cos\beta + \cos\gamma = 0$
$\cos^3\alpha + \cos^3\beta + \cos^3\gamma = 3\cos\alpha\cos\beta\cos\gamma$
Now $\frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha\cos\beta\cos\gamma}$
$= \frac{4(\cos^3\alpha + \cos^3\beta + \cos^3\gamma) - 3(\cos\alpha + \cos\beta + \cos\gamma)}{\cos\alpha\cos\beta\cos\gamma}$ =12

Q.5 Let $I_n = \int_1^e x^{19} (\log|x|)^n\, dx$, where $n \in N$. If $(20)I_{10} = \alpha I_9 + \beta I_8$, for natural numbers α and β, then $\alpha - \beta$ equals to ________.

Ans: **1**

Sol:
$I_n = \int_1^e x^{19} (\log|x|)^n dx$
$I_{10} = \int_1^e x^{19} (\log|x|)^{10} dx = \left((\log|x|)^{10} \frac{x^{20}}{20}\right)_1^e - \int_1^e 10(\log|x|)^9 \frac{x^{19}}{20} dx$

$20I_{10} = e^{20} - 10\, I_9$ ______ (1)

$I_9 = \int_1^e x^{19} (\log|x|)^9 \, dx$

$\left((\log|x|)^9 \dfrac{x^{20}}{20} \right)_1^e - \int_1^e 9 (\log|x|)^8 \dfrac{x^{19}}{20} dx$

$20I_9 = e^{20} - 9I_8$

$e^{20} = 20I_9 + 9I_8$ ____(20)

Sub is (1)

$20I_{10} = 20I_9 + 9I_8 - 10I_9 = 10\, I_9 + 9\, I_8$

Comparing with $\alpha I_9 + \beta I_8$

$\alpha = 10 \quad \beta = +9$

$\alpha - \beta = 1$

Q.6 Let the coefficients of third, fourth and fifth terms in the expansion of $\left(x + \dfrac{a}{x^2}\right)^n, x \neq 0$, be in the ratio 12 : 8 : 3. Then the term independent of x in the expansion, is equal to ________.

Ans: 4

Sol: $T_{r+1} = {}^nC_r \, x^{n-r} \left(\dfrac{a}{x^2}\right)^r = {}^nC_r \, x^{n-3r} a^r$

Coeff of 3rd term = ${}^nC_2 a^2 = 12$

Coeff of 4th term = ${}^nC_3 a^3 = 8$

Coeff of 5th term = ${}^nC_4 a^4 = 3$

Solving n =6 a = $1/2$ r=2

Coeff of term independent of x is given by r =2

$\therefore 6C_2 (1/2)^2 = 15.\dfrac{1}{4} = 3.75 \approx 4$

Q.7 Let $f : [-3, 1] \to \mathbb{R}$ be given as

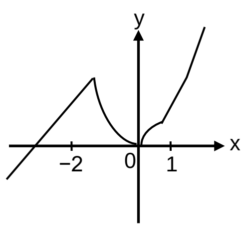
$$f(x) = \begin{cases} \min \{(x+6), x^2\}, & -3 \le x \le 0 \\ \max \{\sqrt{x}, x^2\}, & 0 \le x \le 1. \end{cases}$$

If the area bounded by $y = f(x)$ and x-axis is A, then the value of 6A is equal to ________.

Ans: 41

Sol: Area $= \int_{-3}^{-2} (x+6)dx + \int_{-2}^{0} x^2 dx + \int_0^1 \sqrt{x}\, dx$

$= \left(\dfrac{x^2}{2} + 6x\right)_{-3}^{-2} + \left(\dfrac{x^3}{3}\right)_{-2}^{0} + \left(\dfrac{2x^{3/2}}{3}\right)_0^1 = \dfrac{41}{6}$



6A=41

Q.8 If 1, $\log_{10}(4^x-2)$ and $\log_{10}\left(4^x+\frac{18}{5}\right)$ are in arithmetic progression for a real number x, then

the value of the determinant $\begin{vmatrix} 2\left(x-\frac{1}{2}\right) & x-1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{vmatrix}$ is equal to :

Ans: 2

Sol: $\log_{10}\left(4^x+\frac{18}{5}\right)+1=2\log_{10}\left(4^x-2\right)$

$\Rightarrow \left(4^x-2\right)^2=10\left(4^x+\frac{18}{5}\right)$

$\Rightarrow 4^x=16$

$\Rightarrow x=2$

$\Rightarrow \begin{vmatrix} 3 & 1 & 4 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{vmatrix}=2$

Q.9 Let P be an arbitrary point having sum of the squares of the distances from the planes $x+y+z=0$, $lx-nz=0$ and $x-2y+z=0$, equal to 9. If the locus of the point P is $x^2+y^2+z^2=9$, then the value of $l-n$ is equal to ________.

Given

Ans: 0

Sol: Let P be (a, b, c)

$\left(\frac{a+b+c}{\sqrt{3}}\right)^2+\left(\frac{la-nc}{\sqrt{l^2+n^2}}\right)^2+\left(\frac{a-2b+c}{\sqrt{6}}\right)^2=9$

$\Rightarrow$ locus is

$\left(\frac{x+y+z}{\sqrt{3}}\right)^2+\left(\frac{lx-nz}{\sqrt{l^2+n^2}}\right)+\left(\frac{x-2y+z}{6}\right)^2=9$

Solving we get l = n

$\Rightarrow l-n=0$

Q.10 Let $\vec{x}$ be a vector in the plane containing vectors $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$. If the vector $\vec{x}$ is perpendicular to $\left(3\hat{i}+2\hat{j}-\hat{k}\right)$ and its projection on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$, then the value of $\left|\vec{x}\right|^2$ is equal to ________.

Ans: 486

Sol: Let $\bar{x}=p\bar{a}+q\bar{b}$

$\Rightarrow \bar{x}=(2p+q)i+(-p+2q)j+(p-q)k$

$\bar{x}\perp 3i+2j+k$

$\Rightarrow x.\,(3i+2j-k)=0$

$\Rightarrow 6p+3q-2p+4q-p+q=0$

$\Rightarrow 3p + 8q = 0$ ____ (0)

Projection of $\bar{x}$ as $\bar{a}$ is $\frac{17\sqrt{6}}{2}$

$$\Rightarrow \frac{\bar{x}..\bar{a}}{|a|} = \frac{17\sqrt{6}}{2}$$

$$\frac{4p + 2q + p - 2q + p - q =}{\sqrt{6}} \frac{17\sqrt{6}}{2}$$

$\Rightarrow 6p - q = 51$ _____ (2)

Solving p=8 q = –3

$\Rightarrow \bar{x} = 13i - 14j + 11k$

$|x|^2 = 169 + 196 + 121 = 486$