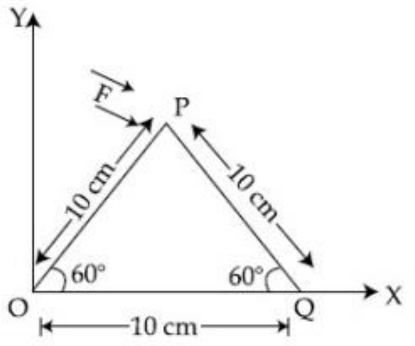
# SOLUTIONS & ANSWERS FOR JEE MAINS-2021
# 17th March Shift 1

## [PHYSICS, CHEMISTRY & MATHEMATICS]

## PART – A – PHYSICS

### SECTION A

**Q.1** A triangular plate is shown. A force $\vec{F} = 4\hat{i} - 3\hat{j}$ is applied at point P. The torque at point P with respect to point 'O' and 'Q' are :




**Options**

1. $-15-20\sqrt{3},\ 15-20\sqrt{3}$
2. $15-20\sqrt{3},\ 15+20\sqrt{3}$
3. $-15+20\sqrt{3},\ 15+20\sqrt{3}$
4. $15+20\sqrt{3},\ 15-20\sqrt{3}$

**Ans: 1**

**Sol:** $\vec{F} = 4\hat{i} - 3\hat{j},\ \vec{r}_1 = 5\hat{i} + 5\sqrt{3}\hat{j},\ \vec{r}_2 = -5\hat{i} + 5\sqrt{3}\hat{j}$

Torque about 0, $\vec{\tau}_0 = \vec{r}_1 \times \vec{F} = \left(-15-20\sqrt{3}\right)\hat{k} = \left(15+20\sqrt{3}\right)\left(-\hat{k}\right)$

Torque about Q, $\vec{\tau}_Q = \vec{r}_2 \times \vec{F} = \left(15-20\sqrt{3}\right)\hat{k}$

**Q.2** For what value of displacement the kinetic energy and potential energy of a simple harmonic oscillation become equal ?

**Options**

1. $x = \frac{A}{2}$
2. $x = \pm A$
3. $x = 0$
4. $x = \pm \frac{A}{\sqrt{2}}$

**Ans:** **4**

**Sol:** Given KE = PE

$$\frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}m\omega^2x^2$$

$$\Rightarrow A^2 - x^2 = x^2$$

$$2x^2 = A^2$$

$$x = \pm\frac{A}{\sqrt{2}}$$

**Q.3** A boy is rolling a 0.5 kg ball on the frictionless floor with the speed of 20 $ms^{-1}$. The ball gets deflected by an obstacle on the way. After deflection it moves with 5% of its initial kinetic energy. What is the speed of the ball now ?

**Options**

1. $1.00\ ms^{-1}$
2. $4.47\ ms^{-1}$
3. $14.41\ ms^{-1}$
4. $19.0\ ms^{-1}$

**Ans:** **2**

**Sol:** m = 0.5 kg, u = 20 m/s

Initial KE = $KE_i = \frac{1}{2}mu^2 = \frac{1}{2} \times 0.5 \times 20^2 = 100\ J$

After deflection it moves with 5% of initial kinetic energy

$\therefore$ Final kinetic energy, $KE_f = \frac{5}{100} \times KE_i = \frac{5}{100} \times 100 = 5J$

Let the final speed be v m/s

$\therefore\ \frac{1}{2}mv^2 = 5$

$v^2 = 20 \Rightarrow v = \sqrt{20} = 4.47$ m/s

**Q.4** A polyatomic ideal gas has 24 vibrational modes. What is the value of $\gamma$ ?

**Options**
1. 1.30
2. 10.3
3. 1.37
4. 1.03

**Ans:** 4

**Sol:** Since each vibrational mode has two degrees of freedom, total vibrational degrees of freedom = 48

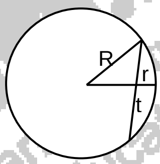
f = 3 + 3 + 48 = 54

$$\gamma = 1 + \frac{2}{f} = 1 + \frac{2}{54} = 1 + \frac{1}{27} = \frac{28}{27} = 1.03$$

**Q.5** The thickness at the centre of a plano convex lens is 3 mm and the diameter is 6 cm. If the speed of light in the material of the lens is $2 \times 10^8$ ms$^{-1}$. The focal length of the lens is __________.

**Options**
1. 15 cm
2. 30 cm
3. 1.5 cm
4. 0.30 cm

**Ans:** 2




**Sol:** $R^2 = r^2 + (R - t)^2 = r^2 + R^2 - 2Rt + t^2$

Neglecting $t^2$, we get

$$R = \frac{r^2}{2t}$$

$$\therefore \frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right) = \frac{\mu - 1}{R}$$

$$\therefore f = \frac{R}{\mu - 1} = \frac{r^2}{2t(\mu - 1)} = \frac{(3 \times 10^{-2})^2}{2 \times 3 \times 10^{-3} \times \left(\frac{3}{2} - 1\right)}$$

$$= \frac{9 \times 10^{-4}}{6 \times 10^{-3} \times 1} \times 2 = 0.3\text{m}$$

$$= 30 \text{ cm}$$

**Q.6** An AC current is given by $I = I_1 \sin\omega t + I_2 \cos\omega t$. A hot wire ammeter will give a reading :

**Options**

1. $\sqrt{\dfrac{I_1^2 - I_2^2}{2}}$

2. $\sqrt{\dfrac{I_1^2 + I_2^2}{2}}$

3. $\dfrac{I_1 + I_2}{2\sqrt{2}}$

4. $\dfrac{I_1 + I_2}{\sqrt{2}}$

**Ans: 2**

**Sol:** $I = I_1 \sin\omega t + I_2 \cos\omega t$

$\therefore I_0 = \sqrt{I_1^2 + I_2^2}$

$\therefore I_{rms} = \dfrac{I_0}{\sqrt{2}} = \sqrt{\dfrac{I_1^2 + I_2^2}{2}}$

**Q.7** Two identical metal wires of thermal conductivities $K_1$ and $K_2$ respectively are connected in series. The effective thermal conductivity of the combination is :

**Options**

1. $\dfrac{K_1 + K_2}{2K_1 K_2}$

2. $\dfrac{K_1 + K_2}{K_1 K_2}$

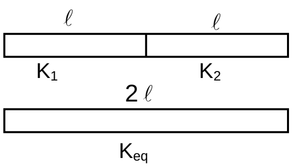
3. $\dfrac{2K_1 K_2}{K_1 + K_2}$

4. $\dfrac{K_1 K_2}{K_1 + K_2}$

**Ans: 3**

**Sol**: $R_{eff} = \dfrac{\ell}{K_1 A} + \dfrac{\ell}{K_2 A} = \dfrac{2\ell}{K_{eq} A}$

$K_{eq} = \dfrac{2K_1K_2}{K_1 + K_2}$



**Q.8** A solenoid of 1000 turns per metre has a core with relative permeability 500. Insulated windings of the solenoid carry an electric current of 5 A. The magnetic flux density produced by the solenoid is : (permeability of free space $= 4\pi \times 10^{-7}$ H/m)

**Options**

1. $\pi$ T
2. $2 \times 10^{-3}\pi$ T
3. $\frac{\pi}{5}$ T
4. $10^{-4}\pi$ T

**Ans: 1**

**Sol:** $B = \mu nI = \mu_0 \mu_r nI$

$= 4\pi \times 10^{-7} \times 500 \times 1000 \times 5$

$= \pi$ T

**Q.9** Two ideal polyatomic gases at temperatures $T_1$ and $T_2$ are mixed so that there is no loss of energy. If $F_1$ and $F_2$, $m_1$ and $m_2$, $n_1$ and $n_2$ be the degrees of freedom, masses, number of molecules of the first and second gas respectively, the temperature of mixture of these two gases is :

**Options**

1. $\dfrac{n_1F_1T_1 + n_2F_2T_2}{n_1 + n_2}$
2. $\dfrac{n_1F_1T_1 + n_2F_2T_2}{F_1 + F_2}$
3. $\dfrac{n_1T_1 + n_2T_2}{n_1 + n_2}$
4. $\dfrac{n_1F_1T_1 + n_2F_2T_2}{n_1F_1 + n_2F_2}$

**Ans: 4**

**Sol:** Let the final temperature of the mixture be x. Since there is no loss of energy, $\Delta U = 0$

$$\frac{F_1}{2} n_1 R \Delta T + \frac{F_2}{2} n_2 R \Delta T = 0$$

$$\frac{F_1}{2} n_1 R (T_1 - x) + \frac{F_2}{2} n_2 R (T_2 - x) = 0$$

$$\Rightarrow x = \frac{F_1 n_1 R T_1 + F_2 n_2 R T_2}{F_1 n_1 R + F_2 n_2 R} = \frac{F_1 n_1 T_1 + F_2 n_2 T_2}{F_1 n_1 + F_2 n_2}$$

**Q.10** The vernier scale used for measurement has a positive zero error of 0.2 mm. If while taking a measurement it was noted that '0' on the vernier scale lies between 8.5 cm and 8.6 cm, vernier coincidence is 6, then the correct value of measurement is __________ cm. (least count = 0.01 cm)

**Options**
1. 8.54 cm
2. 8.58 cm
3. 8.36 cm
4. 8.56 cm

**Ans:** 1

**Sol:** Positive zero error = 0.2 mm
Main scale reading = 8.5 cm
Vernier scale reading = 6 × 0.01 = 0.06 cm
Final reading = 8.5 + 0.06 – 0.02 = 8.54 cm

**Q.11** Which level of the single ionized carbon has the same energy as the ground state energy of hydrogen atom ?

**Options**
1. 1
2. 4
3. 6
4. 8

**Ans:** 3

**Sol:** Energy of Hydrogen atom $E = -\frac{13.6 Z^2}{n^2}$

For hydrogen atom Z = 1 and for ground state, n = 1

$\Rightarrow E = -13.6 \times \frac{1^2}{1^2} = -13.6\,\text{eV}$

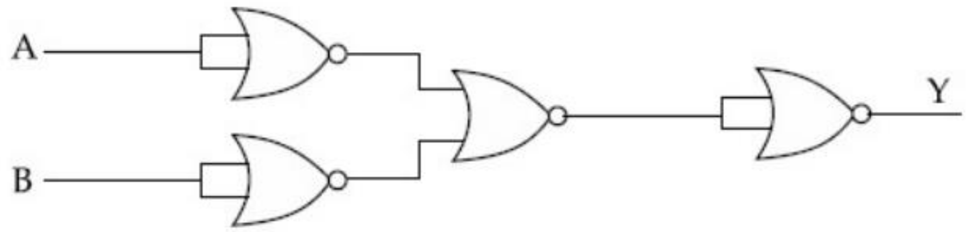
For single ionized carbon atom Z = 6

$E = -\frac{13.6 Z^2}{n^2} = -13.6$

$\Rightarrow n^2 = 6^2$

N = 6

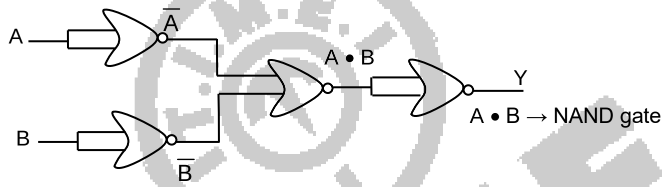
**Q.12** The output of the given combination gates represents :




**Options**
1. XOR Gate
2. NAND Gate
3. NOR Gate
4. AND Gate

**Ans: 2**

**Sol:**




**Q.13** If an electron is moving in the $n^{th}$ orbit of the hydrogen atom, then its velocity ($v_n$) for the $n^{th}$ orbit is given as :

**Options**
1. $v_n \propto \frac{1}{n^2}$
2. $v_n \propto \frac{1}{n}$
3. $v_n \propto n^2$
4. $v_n \propto n$

**Ans: 2**

**Sol:** Velocity of electron in the $n^{th}$ orbit of a hydrogen atom is given by $v = \frac{2\pi K Ze^2}{nh} \propto \frac{1}{n}$

**Q.14** A car accelerates from rest at a constant rate $\alpha$ for some time after which it decelerates at a constant rate $\beta$ to come to rest. If the total time elapsed is t seconds, the total distance travelled is :

**Options**

1. $\frac{\alpha \beta}{2(\alpha + \beta)} t^2$

2. $\frac{2 \alpha \beta}{(\alpha + \beta)} t^2$

3. $\frac{4 \alpha \beta}{(\alpha + \beta)} t^2$

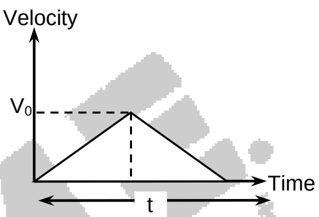
4. $\frac{\alpha \beta}{4 (\alpha + \beta)} t^2$

**Ans: 1**

**Sol:** $V_0 = \alpha t_1$ and $0 = V_0 - \beta t_2$

$\Rightarrow t_1 = \frac{V_0}{\alpha}$

$\Rightarrow V_0 = \beta t_2 \Rightarrow t_2 = \frac{V_0}{\beta}$




$t = t_1 + t_2$

$= \frac{V_0}{\alpha} + \frac{V_0}{\beta} = V_0\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$

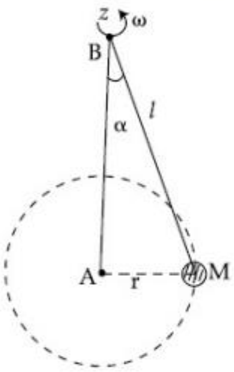
$\Rightarrow V_0 = \frac{\alpha\beta t}{\alpha + \beta}$

Distance = Area under the velocity time graph

$= \frac{1}{2} \times t \times V_0 = \frac{1}{2} \times t \times \frac{\alpha\beta t}{\alpha + \beta}$

$= \frac{\alpha\beta t^2}{2(\alpha + \beta)}$

**Q.15** A mass M hangs on a massless rod of length $l$ which rotates at a constant angular frequency. The mass M moves with steady speed in a circular path of constant radius. Assume that the system is in steady circular motion with constant angular velocity $\omega$. The angular momentum of M about point A is $L_A$ which lies in the positive $z$ direction and the angular momentum of M about point B is $L_B$. The correct statement for this system is :




**Options**

1. $L_A$ and $L_B$ are both constant in magnitude and direction
2. $L_B$ is constant, both in magnitude and direction
3. $L_A$ is constant, both in magnitude and direction
4. $L_B$ is constant in direction with varying magnitude

**Ans: 3**

**Sol:** $\vec{L} = m(\vec{r} \times \vec{v})$

With respect to A, we get direction of $\vec{L}$ along (+) ve z-axis and the constant magnitude of $\vec{L}$ as mvr. But, with respect to B, we get constant magnitude but continuously direction.

**Q.16** An electron of mass m and a photon have same energy E. The ratio of wavelength of electron to that of photon is : (c being the velocity of light)

**Options**

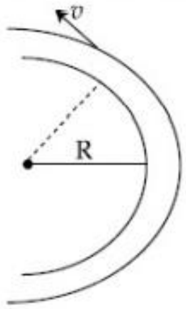
1. $c\,(2mE)^{1/2}$
2. $\left(\frac{E}{2m}\right)^{1/2}$
3. $\frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$
4. $\frac{1}{c}\left(\frac{2m}{E}\right)^{1/2}$

**Ans: 3**

**Sol:** $\lambda_1 = \frac{h}{\sqrt{2mE}}$ $\lambda_2 = \frac{hc}{E}$

$\frac{\lambda_1}{\lambda_2} = \frac{1}{c}\sqrt{\frac{E}{2m}}$

**Q.17** A modern grand - prix racing car of mass m is travelling on a flat track in a circular arc of radius R with a speed $v$. If the coefficient of static friction between the tyres and the track is $\mu_s$, then the magnitude of negative lift $F_L$ acting downwards on the car is : (Assume forces on the four tyres are identical and g = acceleration due to gravity)




**Options**

1. $m\left(\frac{v^2}{\mu_s R} - g\right)$

2. $-m\left(g + \frac{v^2}{\mu_s R}\right)$

3. $m\left(\frac{v^2}{\mu_s R} + g\right)$

4. $m\left(g - \frac{v^2}{\mu_s R}\right)$



**Ans:** 1

**Sol:** $\mu_s N = \frac{mV^2}{R}$

$N = \frac{mV^2}{\mu_s R} = mg + F_L$

$mg + F_L = \frac{mV^2}{\mu_s R}$

$\Rightarrow F_L = \frac{mV^2}{\mu_s R} - mg$

$m\left(\frac{V^2}{\mu_s R} - g\right)$

**Q.18** A Carnot's engine working between 400 K and 800 K has a work output of 1200 J per cycle. The amount of heat energy supplied to the engine from the source in each cycle is :

**Options**
1. 2400 J
2. 1600 J
3. 3200 J
4. 1800 J

**Ans:** 1

**Sol:** Efficiency of Carnot engine, $\eta = \frac{T_2}{T_1} = \frac{Q_2}{Q_1} = \frac{Q_1 - W}{Q_1} = 1 - \frac{W}{Q_1}$

$[\because W = Q_1 - Q_2]$

$\frac{400}{800} = 1 - \frac{W}{Q_1}$

$\therefore \frac{W}{Q_1} = 1 - \frac{400}{800} = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore Q_1 = 2W = 2 \times 1200 = 2400\,J$

**Q.19** A current of 10 A exists in a wire of crosssectional area of 5 $mm^2$ with a drift velocity of $2 \times 10^{-3}\ ms^{-1}$. The number of free electrons in each cubic meter of the wire is ________.

**Options**
1. $625 \times 10^{25}$
2. $1 \times 10^{23}$
3. $2 \times 10^{6}$
4. $2 \times 10^{25}$

**Ans:** 1

**Sol:** $i = 10$ A, $V_d = 2 \times 10^{-3}$ m/s, $A = 5\ mm^2 = 5 \times 10^{-6}\ m^2$

$i = n A v_d e$

$10 = n \times 5 \times 10^{-6} \times 2 \times 10^{-3} \times 1.6 \times 10^{-19}$

$\Rightarrow n = 0.625 \times 10^{28}$

$= 625 \times 10^{25}$

**Q.20** When two soap bubbles of radii a and b (b > a) coalesce, the radius of curvature of common surface is :

**Options**

1. $\frac{ab}{a+b}$

2. $\frac{ab}{b-a}$

3. $\frac{b-a}{ab}$

4. $\frac{a+b}{ab}$

**Ans:** **2**

**Sol:** Excess pressure at common surface,

$$P_{ex} = 4T\left(\frac{1}{a}-\frac{1}{b}\right) = \frac{4T}{r}$$

$$\therefore \frac{1}{r} = \frac{1}{a}-\frac{1}{b} = \frac{b-a}{ab}$$

$$\therefore r = \frac{ab}{b-a}$$

**Section B**

**1.** The following bodies,

(1) a ring

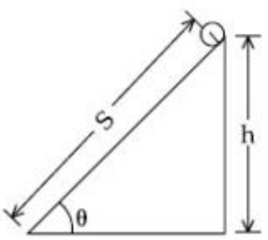
(2) a disc

(3) a solid cylinder

(4) a solid sphere,

of same mass 'm' and radius 'R' are allowed to roll down without slipping simultaneously from the top of the inclined plane. The body which will reach first at the bottom of the inclined plane is __________.

[Mark the body as per their respective numbering given in the question]




**Ans:** **4.00**

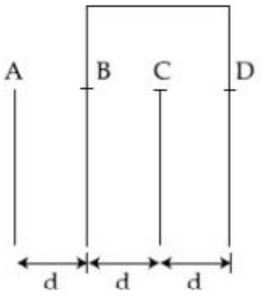
**Sol:** mg sin θ R = (mK$^2$ + mR$^2$) α

$$\Rightarrow \alpha = \frac{Rg\sin\theta}{K^2+R^2} = \frac{g\sin\theta}{\frac{K^2+R^2}{R^2}} = \frac{g\sin\theta}{1+\frac{K^2}{R^2}}$$

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2s}{g\sin\theta}\left(1+\frac{K^2}{R^2}\right)}$$

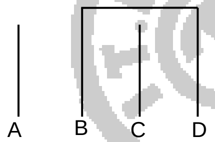
For least time, value of K should be least K is least for a solid sphere

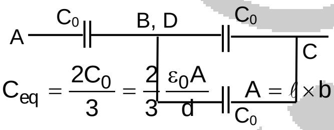
**2.** Four identical rectangular plates with length, $l = 2$ cm and breadth, $b = \frac{3}{2}$ cm are arranged as shown in figure. The equivalent capacitance between A and C is $\frac{x\varepsilon_0}{d}$. The value of $x$ is __________. (Round off to the Nearest Integer)




**Ans:** **2.00**

**Sol:**




$$C_{eq} = \frac{2C_0}{3} = \frac{2}{3}\frac{\varepsilon_0 A}{d} \quad A = l \times b = 2\times\frac{3}{2} = \frac{2\varepsilon_0}{3d}\left(2\times\frac{3}{2}\right) = 2$$

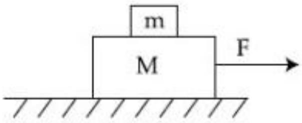
**3.** For VHF signal broadcasting, __________ $km^2$ of maximum service area will be covered by an antenna tower of height 30 m, if the receiving antenna is placed at ground. Let radius of the earth be 6400 km. (Round off to the Nearest Integer) (Take $\pi$ as 3.14)

**Ans:** **1206.00**

**Sol:** $d = \sqrt{2Rh}$

$$A = \pi d^2 = \pi \times 2Rh = 3.14 \times 2 \times 6400 \times \frac{30}{1000} = 1205.76 \text{ km}^2 \approx 1206 \text{ km}^2$$

**4.** Two blocks ($m = 0.5$ kg and $M = 4.5$ kg) are arranged on a horizontal frictionless table as shown in figure. The coefficient of static friction between the two blocks is $\frac{3}{7}$. Then the maximum horizontal force that can be applied on the larger block so that the blocks move together is _________ N. (Round off to the Nearest Integer) [Take g as 9.8 $ms^{-2}$]




**Ans: 21.00**

**Sol:** Maximum acceleration, $a_{max}$ $\mu g = \frac{3}{7} \times 9.8$

$F = (M + m)\, a_{max} = 0.5 + 4.5)\, a_{max} = 5\, a_{max}$

$= 5 \times \frac{3}{7} \times 9.8 = 21$ N

**5.** If $2.5 \times 10^{-6}$ N average force is exerted by a light wave on a non - reflecting surface of 30 $cm^2$ area during 40 minutes of time span, the energy flux of light just before it falls on the surface is _________ $W/cm^2$. (Round off to the Nearest Integer)

(Assume complete absorption and normal incidence conditions are there)

**Ans: 25.00**

**Sol:** $F = \frac{IA}{C} \Rightarrow I = \frac{FC}{A} = \frac{2.5 \times 10^{-6} \times 3 \times 10^{8}}{30} = 25\ W/cm^2$

**6.** The angular speed of truck wheel is increased from 900 rpm to 2460 rpm in 26 seconds. The number of revolutions by the truck engine during this time is _________.

(Assuming the acceleration to be uniform).

**Ans: 728.00**

**Sol:** $\theta = \left(\frac{\omega_1 + \omega_2}{2}\right)t$

Let N be the number of revolution

$\therefore 2\pi N = 2\pi\left(\frac{900 + 2460}{60 \times 2}\right) \times 26 = 728$

**7.** The radius in kilometer to which the present radius of earth ($R = 6400$ km) to be compressed so that the escape velocity is increased 10 times is _________ .

**Ans: 64.00**

**Sol**: Escape velocity, $V_e = \sqrt{\dfrac{2Gm}{R}}$ ------(1)

$$10\, V_e = \sqrt{\dfrac{2Gm}{R'}} \text{ ------(2)}$$

$$\frac{(2)}{(1)} \Rightarrow 10 = \sqrt{\frac{R}{R'}}$$

$$\frac{R}{R'} = 100$$

$$R' = \frac{R}{100} = \frac{6400}{100} = 64\,\text{km}$$

**8.** The equivalent resistance of series combination of two resistors is 's'. When they are connected in parallel, the equivalent resistance is 'p'. If $s = np$, then the minimum value for n is ________. (Round off to the Nearest Integer)

**Ans: 4.00**

**Sol:** $R_1 + R_2 = s$ ------(1)

$$\frac{R_1R_2}{R_1+R_2} = p \text{ -----(2)}$$

$R_1R_2 = p\,(R_1 + R_2) = sp$

$R_1R_2 = np^2$ [$\because s = np$]

$$R_1 + R_2 = \frac{nR_1R_2}{(R_1+R_2)}$$

$$n = \frac{(R_1+R_2)^2}{R_1R_2}$$

For minimum value of n, $R_1 = R_2 = R$

$$\therefore n = \frac{(2R)^2}{R^2} = 4$$

**9.** A parallel plate capacitor whose capacitance C is 14 pF is charged by a battery to a potential difference $V = 12$ V between its plates. The charging battery is now disconnected and a porcelin plate with $k = 7$ is inserted between the plates, then the plate would oscillate back and forth between the plates with a constant mechanical energy of ________ pJ. (Assume no friction)

**Ans: 864.00**

**Sol:** $U_i = \frac{1}{2}CV^2$

$$\frac{1}{2} \times 14 \times 12^2 = 1008 \text{ pJ}$$

$$U_f = \frac{1008}{7}\text{pJ} = 144\,\text{pJ} \quad (\because C_m = KC_0)$$

Mechanical energy = $\Delta U$ = 1008 – 144 = 864 pJ

**10.** Consider two identical springs each of spring constant k and negligible mass compared to the mass M as shown. Fig. 1 shows one of them and Fig. 2 shows their series combination. The ratios of time period of oscillation of the two SHM is $T_b/T_a = \sqrt{x}$, where value of $x$ is ________. (Round off to the Nearest Integer)

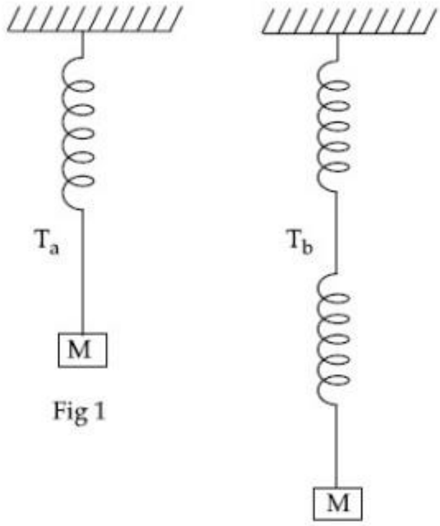



Fig 1 Fig 2

**Ans: 2.00**

**Sol:** $T_a = 2\pi\sqrt{\frac{M}{K}}$ ––––––(1)

$T_b = 2\pi\sqrt{\frac{M}{K/2}}$ ––––––(2)

$\frac{(2)}{(1)} \Rightarrow \frac{T_b}{T_a} = \sqrt{2} = \sqrt{x} \Rightarrow x = 2$

## PART – B – CHEMISTRY

### SECTION A

**Q.1** The INCORRECT statement(s) about heavy water is (are)

(A) used as a moderator in nuclear reactor

(B) obtained as a by-product in fertilizer industry

(C) used for the study of reaction mechanism

(D) has a higher dielectric constant than water

Choose the correct answer from the options given below :

**Options**

1. (B) only
2. (D) only
3. (C) only
4. (B) and (D) only

**Ans:** 2

**Sol:** The dielectric constant of normal water is higher than heavy water.

**Q.2** Reducing smog is a mixture of :

Options
1. Smoke, fog and $N_2O_3$
2. Smoke, fog and $SO_2$
3. Smoke, fog and $CH_2=CH-CHO$
4. Smoke, fog and $O_3$

**Ans:** 2

**Sol:** Reducing smog is a mixture of smoke, fog and $SO_2$

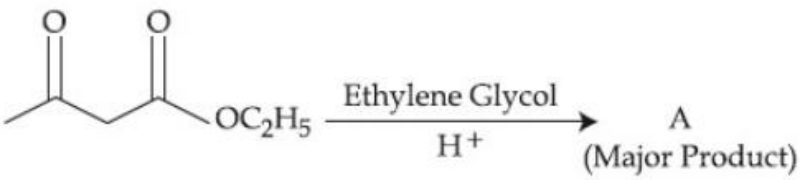
**Q.3** The correct order of conductivity of ions in water is :

Options
1. $Rb^+ > Na^+ > K^+ > Li^+$
2. $Cs^+ > Rb^+ > K^+ > Na^+$
3. $K^+ > Na^+ > Cs^+ > Rb^+$
4. $Na^+ > K^+ > Rb^+ > Cs^+$

**Ans:** 2

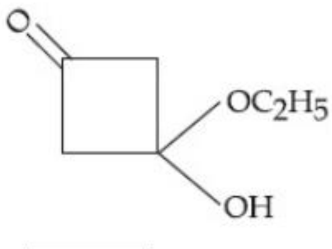
**Sol:** As the size of the ion decreases it get more hydrated ion increases and therefore the ionic mobility decreases.
$\therefore$ the correct order $Cs^+ > Rb^+ > K^+ > Na^+$

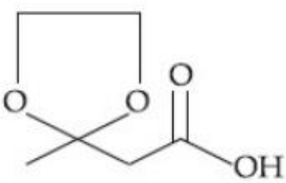
**Q.4**

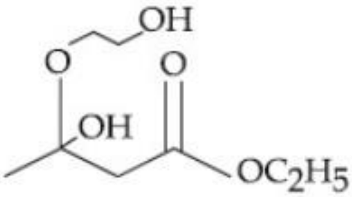


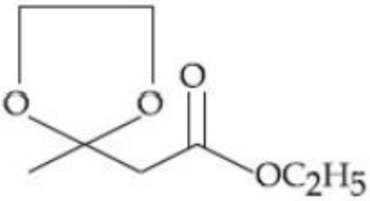

The product "A" in the above reaction is :

Options

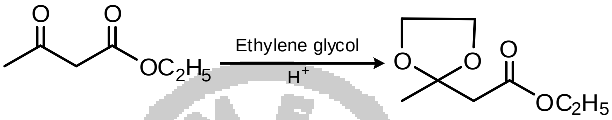
1.



2.


3. 

4. 

**Ans:** 4

**Sol:** 


**Q.5** What is the spin-only magnetic moment value (BM) of a divalent metal ion with atomic number 25, in it's aqueous solution ?

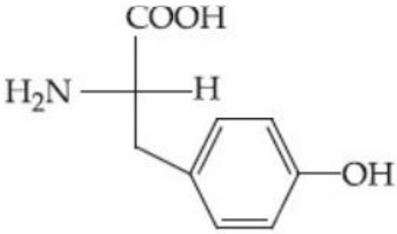
**Options**
1. 5.0
2. 5.92
3. 5.26
4. zero

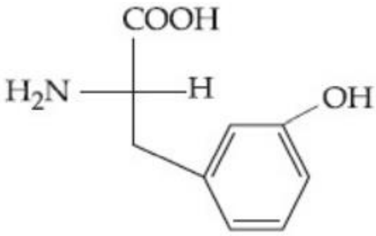
**Ans:** 2

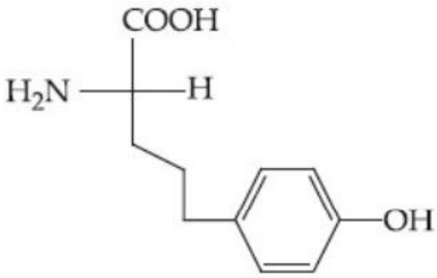
**Sol:** The electronic configuration of element with atomic number 25
[Ar] $3d^5\ 4s^2$
Total number of unpaired electrons = 5
$\mu = \sqrt{n(n+2)} = \sqrt{5 \times 7} = 5.92$

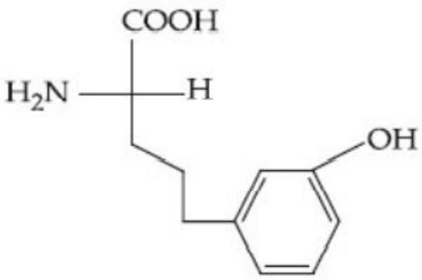
**Q.6** Which of the following is correct structure of tyrosine ?

**Options**

1. 

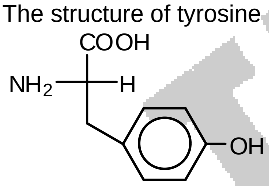
2. 

3. 

4. 

**Ans:** **1**

**Sol:** The structure of tyrosine is





**Q.7** The absolute value of the electron gain enthalpy of halogens satisfies :

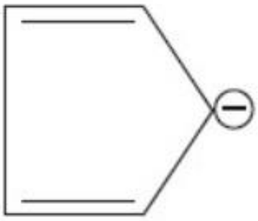
**Options**
1. F > Cl > Br > I
2. I > Br > Cl > F
3. Cl > F > Br > I
4. Cl > Br > F > I

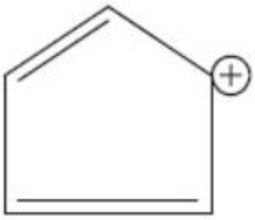
**Ans:** **3**

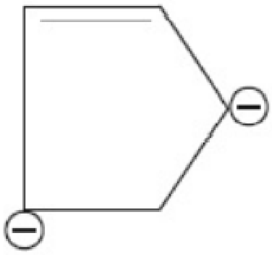
**Sol:** The correct order of absolute value of electron gain enthalpy of halogens is Cl > F > Br > I

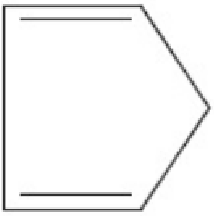
**Q.8** Which of the following is an aromatic compound ?

**Options**

1. 

2. 

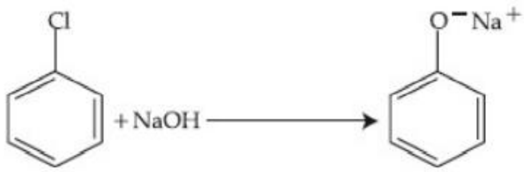
3. 

4. 

**Ans:** 1

**Sol:** Cyclopentadienyl anion is aromatic

**Q.9**




The above reaction requires which of the following reaction conditions ?

**Options**

1. 623 K, Cu, 300 atm
2. 573 K, 300 atm
3. 573 K, Cu, 300 atm
4. 623 K, 300 atm

**Ans:** 4

**Sol:** Dows process, requires 623 K temperature and 300 atm pressure

**Q.10** A central atom in a molecule has two lone pairs of electrons and forms three single bonds. The shape of this molecule is :

**Options**
1. see-saw
2. T-shaped
3. planar triangular
4. trigonal pyramidal

**Ans:** **2**

**Sol:** The shape of the molecule with 3 bond pair and 2 lone pair is 'T' shape.

**Q.11** A colloidal system consisting of a gas dispersed in a solid is called a/an :

**Options**
1. aerosol
2. solid sol
3. gel
4. foam

**Ans:** **2**

**Sol:** A system where gas is dispersed in solid is called solid sol

**Q.12** With respect to drug-enzyme interaction, identify the wrong statement.

**Options**
1. Allosteric inhibitor competes with the enzyme's active site
2. Non-Competitive inhibitor binds to the allosteric site
3. Allosteric inhibitor changes the enzyme's active site
4. Competitive inhibitor binds to the enzyme's active site

**Ans:** **3**

**Sol:** Allosteric inhibitor do not bind with the active sites of the enzyme but binds to a different site called allosteric sites which changes the shape of the active site. So that substrate cannot recognize it

**Q.13** Given below are two statements :

**Statement I :** Retardation factor ($R_f$) can be measured in meter/centimeter.

**Statement II :** $R_f$ value of a compound remains constant in all solvents.

Choose the most appropriate answer from the options given below :

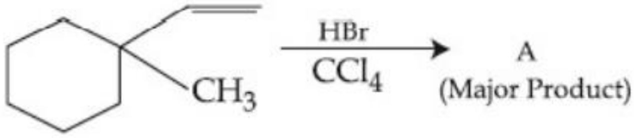
**Options**

1. Statement I is true but statement II is false
2. Both statement I and statement II are false
3. Statement I is false but statement II is true
4. Both statement I and statement II are true

**Ans: 2**

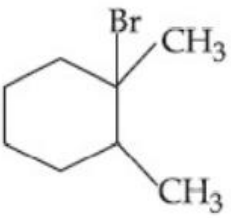
**Sol:** $R_f \text{ value} = \dfrac{\text{distance moved by the component (in cm)}}{\text{distance moved by the solvent front (in cm)}}$

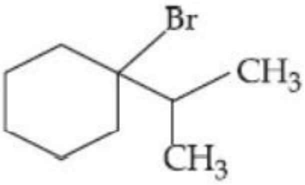
**Q.14**

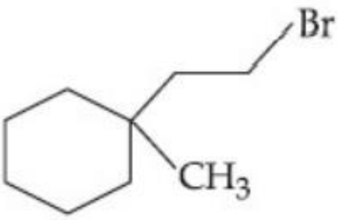


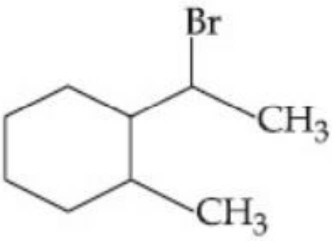

Product "A" in the above chemical reaction is :

**Options**

1. 

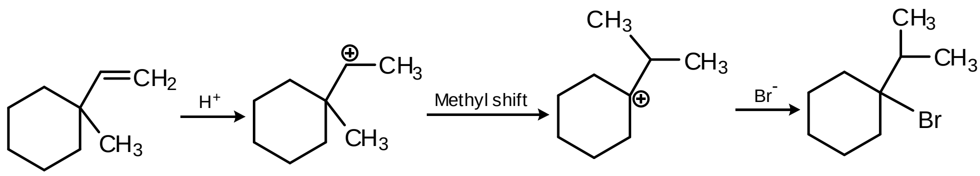

2. 


3. 


4. 

**Ans: 2**

**Sol:**




**Q.15** Which of the following compound CANNOT act as a Lewis base ?

**Options**

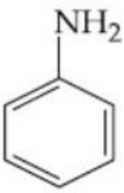
1. $SF_4$
2. $NF_3$
3. $PCl_5$
4. $ClF_3$

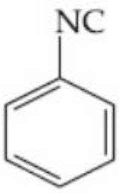
**Ans: 3**

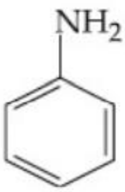
**Sol:** The central atom in $PCl_5$ doesn't carry any lone pair

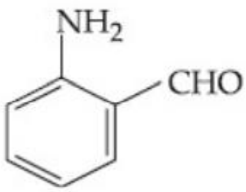
**Q.16** Hoffmann bromomide degradation of benzamide gives product A, which upon heating with $CHCl_3$ and NaOH gives product B.
The structures of A and B are :

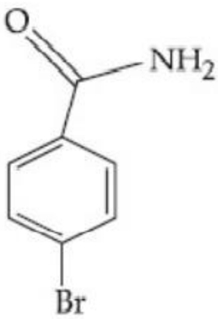
**Options**

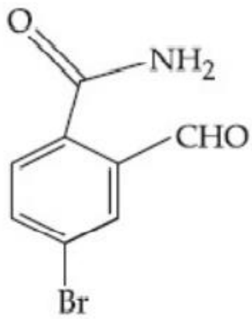
1. A – 

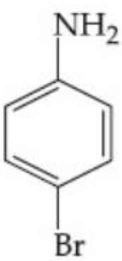
B – 

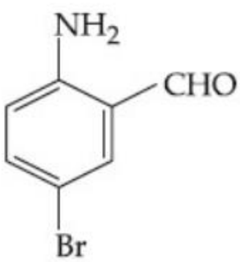

2. A – 

B – 


3. A – 

B – 


4. A – 

B – 

**Ans: 1**

**Sol:** $C_6H_5-\overset{\overset{\large O}{\|}}{C}-CNH_2 \xrightarrow{Br_2\ +\ NaOH} \underset{(A)}{C_6H_5NH_2} \xrightarrow[KOH]{CHCl_3} \underset{(B)}{C_6H_5NC}$

**Q.17** Which of the following reaction is an example of ammonolysis ?

**Options**

1. $C_6H_5CH_2CN \xrightarrow{[H]} C_6H_5CH_2CH_2NH_2$
2. $C_6H_5NH_2 \xrightarrow{HCl} C_6H_5\overset{+}{N}H_3Cl^-$
3. $C_6H_5CH_2Cl + NH_3 \longrightarrow C_6H_5CH_2NH_2$
4. $C_6H_5COCl + C_6H_5NH_2 \longrightarrow C_6H_5CONHC_6H_5$

**Ans: 3**

**Sol:** Nucleophilic substitution reaction of an alkyl halide with $NH_3$ resulting in the formation of amine is called ammonolysis

**Q.18** Mesityl oxide is a common name of :

**Options**

1. 2-Methyl cyclohexanone
2. 3-Methyl cyclohexane carbaldehyde
3. 2,4-Dimethyl pentan-3-one
4. 4-Methyl pent-3-en-2-one

**Ans: 4**

**Sol:** Methyl oxide is $CH_3-\underset{\underset{\large CH_3}{|}}{C}=CH-\overset{\overset{\large O}{\|}}{C}-CH_3$

It is the self aldol condensation product of acetone

**Q.19** Given below are two statements :

Statement I : Potassium permanganate on heating at 573 K forms potassium manganate.

Statement II : Both potassium permanganate and potassium manganate are tetrahedral and paramagnetic in nature.

In the light of the above statements, choose the most appropriate answer from the options given below :

**Options**

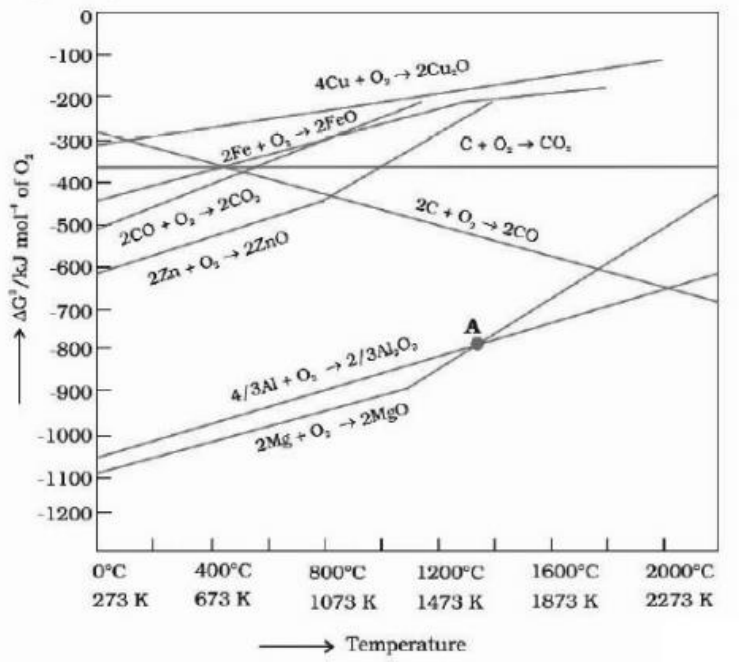
1. Statement I is false but statement II is true
2. Both statement I and statement II are true
3. Both statement I and statement II are false
4. Statement I is true but statement II is false

**Ans:** **4**

**Sol:** $\underset{\text{pot. permanganate}}{2KMnO_4} \xrightarrow{\Delta} \underset{\text{pot. manganate}}{K_2MnO_4} + MnO_2 + O_2$

Both $MnO_4^-$ and $MnO_4^{2-}$ are tetrahedral

**Q.20** The point of intersection and sudden increase in the slope, in the diagram given below, respectively, indicates :



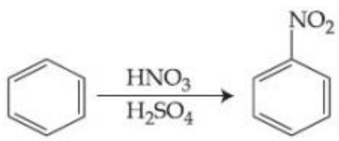

**Options**
1. $\Delta G < 0$ and decomposition of the metal oxide
2. $\Delta G > 0$ and decomposition of the metal oxide
3. $\Delta G = 0$ and melting or boiling point of the metal oxide
4. $\Delta G = 0$ and reduction of the metal oxide

**Ans:** **No Answer**

**Sol:** The point of intersection represent $\Delta G = 0$
The sudden increase in slope is due to melting or boiling point of metal

## SECTION B

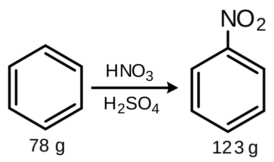
**1.**




In the above reaction, 3.9 g of benzene on nitration gives 4.92 g of nitrobenzene. The percentage yield of nitrobenzene in the above reaction is ________%. (Round off to the Nearest Integer).
(Given atomic mass : C : 12.0 u, H : 1.0 u, O : 16.0 u, N : 14.0 u)

**Ans:** **80**

**Sol:**




3.9 g of benzene gives $\frac{123}{78} \times 3.9 = 6.15$ g nitrobenzene there is 100% yield

Percentage yield $= \frac{4.92}{6.15} = 80\%$

**2.** 15 mL of aqueous solution of $Fe^{2+}$ in acidic medium completely reacted with 20 mL of 0.03 M aqueous $Cr_2O_7^{2-}$. The molarity of the $Fe^{2+}$ solution is ________ $\times 10^{-2}$ M. (Round off to the Nearest Integer).

**Ans:** **24**

**Sol:** 15 × M ($Fe^{2+}$) × 1 = 20 × 0.03 × 6
Molarity of $Fe^{2+}$ = 0.24 M = $24 \times 10^{-2}$

**3.** For a certain first order reaction 32% of the reactant is left after 570 s. The rate constant of this reaction is ________ $\times 10^{-3}$ $s^{-1}$. (Round off to the Nearest Integer).
[Given : $\log_{10}2 = 0.301$, $\ln 10 = 2.303$]

**Ans:** **2**

**Sol:** $k = \frac{2.303}{t} \log \frac{R_0}{R_t} = \frac{2.303}{570} \times \log \frac{100}{32}$

$k = \frac{2.303}{570} \times [\log 100 - 5 \times \log 2] = 2 \times 10^{-3}$ $s^{-1}$

**4.** The oxygen dissolved in water exerts a partial pressure of 20 kPa in the vapour above water. The molar solubility of oxygen in water is ________ $\times 10^{-5}$ mol $dm^{-3}$.
(Round off to the Nearest Integer).
[Given : Henry's law constant = $K_H = 8.0 \times 10^4$ kPa for $O_2$.
Density of water with dissolved oxygen = 1.0 kg $dm^{-3}$]

**Ans:** **1389**

**Sol:** $x_{O_2} = \frac{p_{O_2}}{K_H} = \frac{20}{8 \times 10^4} = 2.5 \times 10^{-4}$

$n_{O_2} = 2.5 \times 10^{-4} \times n_{H_2O}$

Let the volume of water be 100 mL

then $n_{H_2O} = \frac{1000}{18}$ $\because$ density $= 1$ kg $dm^{-3}$

Molar solubility of $O_2$ $= 2.5 \times 10^{-4} \times \frac{1000}{18} = 1388.8 \times 10^{-5}$

**5.** The reaction of white phosphorus on boiling with alkali in inert atmosphere resulted in the formation of product 'A'. The reaction of 1 mol of 'A' with excess of $AgNO_3$ in aqueous medium gives ________ mol(s) of Ag. (Round off to the Nearest Integer).

**Ans:** 4

**Sol:** $P_4 + 3NaOH + 3H_2O \rightarrow PH_3 + 3NaH_2PO_2$

$\underset{1\text{ mole}}{NaH_2PO_2} + 4AgNO_3 + 2H_2O \rightarrow \underset{4\text{ mole}}{4Ag} + H_3PO_3 + 3HNO_3$

**6.** The standard enthalpies of formation of $Al_2O_3$ and CaO are $-1675$ kJ $mol^{-1}$ and $-635$ kJ $mol^{-1}$ respectively.
For the reaction
$3CaO + 2Al \rightarrow 3Ca + Al_2O_3$ the standard reaction enthalpy $\Delta_r H^0 =$ ________ kJ.
(Round off to the Nearest Integer).

**Ans:** 230

**Sol:** $3caO + Al \rightarrow Al_2O_3 + 3Ca$

$\Delta_r H^\circ = \sum(\Delta H_f)_P - \sum(\Delta H_f)_R$

$= (-1675) - 3 \times (-635) = 230$ kJ $mol^{-1}$

**7.** A certain orbital has $n = 4$ and $m_L = -3$. The number of radial nodes in this orbital is ________. (Round off to the Nearest Integer).

**Ans:** 0

**Sol:** Since $m_\ell = -3$, $\ell = 3$
No. of radial nodes $= n - \ell - 1 = 4 - 3 - 1 = 0$

**8.** The mole fraction of a solute in a 100 molal aqueous solution is ________ $\times 10^{-2}$.
(Round off to the Nearest Integer).
[Given : Atomic masses : H : 1.0 u, O : 16.0 u]

**Ans:** 64

**Sol:** For aqueous solution $\chi_{solute} = \dfrac{m}{m + \dfrac{1000}{18}} = \dfrac{100}{100 + 55.5} = 64 \times 10^{-2}$

**9.** The pressure exerted by a non-reactive gaseous mixture of 6.4 g of methane and 8.8 g of carbon dioxide in a 10 L vessel at 27°C is ________ kPa.
(Round off to the Nearest Integer).
[Assume gases are ideal, $R = 8.314$ J $mol^{-1}$ $K^{-1}$
Atomic masses : C : 12.0 u, H : 1.0 u, O : 16.0 u]

**Ans:** 150

**Sol:** $n_{CH_4} = \dfrac{6.4}{16} = 0.4$ $\quad n_{CO_2} = \dfrac{8.8}{44} = 0.2$

$n_{total} = 0.6 \quad P = \dfrac{nRT}{V} = \dfrac{0.6 \times 8.314 \times 300}{10 \times 10^{-3}} = 149652 \text{ Pa} \approx 150\, kP_a$

**10.** 0.01 moles of a weak acid HA ($K_a = 2.0 \times 10^{-6}$) is dissolved in 1.0 L of 0.1 M HCl solution. The degree of dissociation of HA is ________ $\times 10^{-5}$ (Round off to the Nearest Integer).
[Neglect volume change on adding HA.
Assume degree of dissociation <<1]

**Ans:** 2

**Sol:**

| | HA | $\rightleftharpoons$ | $H^+$ | + | $A^-$ |
|---|---|---|---|---|---|
| Initial | 0.01 | | - | | - |
| Eq | (0.01-x) | | x | | x |

| | HCl | $\rightarrow$ | $H^+$ | + | $Cl^-$ |
|---|---|---|---|---|---|
| Initial | 0.1 | | - | | - |
| Eq | - | | 0.1 | | 0.1 |

$[H^+] = x + 0.1 \approx 0.1$

$$K_a = \frac{[H^+][A^-]}{[HA]} = \frac{0.1 \times x}{0.01} = 2 \times 10^{-6}$$

$x = 2 \times 10^{-7}$

Since $x = C\alpha$

$$\alpha = \frac{2 \times 10^{-7}}{0.01} = 2 \times 10^{-5}$$

# PART – C – MATHEMATICS

## SECTION A

**Q.1** Team 'A' consists of 7 boys and n girls and Team 'B' has 4 boys and 6 girls. If a total of 52 single matches can be arranged between these two teams when a boy plays against a boy and a girl plays against a girl, then n is equal to :

**Options**
1. 2
2. 5
3. 4
4. 6

**Ans:** 3



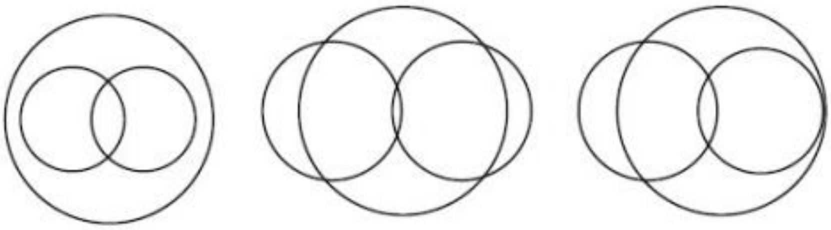
**Sol:** $A \rightarrow$ 7 boys, n girls

$B \rightarrow$ 4 boys, 6 girls

Number of matches = $({}^7C_1 \times {}^4C_1) + ({}^nC_1 \times {}^6C_1) = 52$

$\Rightarrow 25 + 6n = 52 \Rightarrow 6n = 24 \Rightarrow n = 4$

**Q.2** In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagrams can justify the above statement ?



P Q R

Options
1. None of these
2. P and Q
3. Q and R
4. P and R

**Ans:** 1

**Sol:** Option (1)

**Q.3** The system of equations $kx+y+z=1$, $x+ky+z=k$ and $x+y+zk=k^2$ has no solution if k is equal to :

Options
1. $1$
2. $-2$
3. $0$
4. $-1$

**Ans:** 2

**Sol:** kx + y + z = 1 ---- (1)
x + ky + z = k ---- (2)
x + y + zk = $k^2$ ---- (3)
(1) – (2) ⇒ (k –1)x + (1 – k)y = (1 – k) --- (A)
k × (2) – (3) ⇒ (k – 1)x + ($k^2$ – 1y) = 0 --- (B)
Clearly k ≠ 1 since if k = 1, the equations (1), (2) and (3) are identical hence infinite number of solution
∴ (A) ⇒ x – y = –1 and
(B) ⇒ x + (k + 1) y
(A) ⇒ –(k + 1)y – y = –1 ⇒ (–k – 1 –1)y = –1
$$\Rightarrow y = \frac{-1}{-k-2} = \frac{1}{k+2}$$
If k = –2 then the equations have no solution

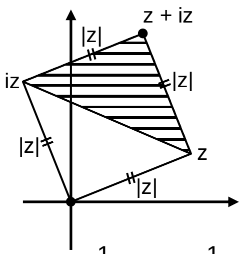


**Q.4** The area of the triangle with vertices A($z$), B($iz$) and C($z+iz$) is :

Options
1. $\frac{1}{2}|z|^2$
2. $1$
3. $\frac{1}{2}|z+iz|^2$
4. $\frac{1}{2}$

**Ans:** 1

**Sol:**




$$\text{Area} = \frac{1}{2}|z| \times |z| = \frac{1}{2}|z|^2$$

**Q.5** Two dices are rolled. If both dices have six faces numbered 1, 2, 3, 5, 7 and 11, then the probability that the sum of the numbers on the top faces is less than or equal to 8 is :

**Options**

1. $\frac{5}{12}$
2. $\frac{4}{9}$
3. $\frac{17}{36}$
4. $\frac{1}{2}$

**Ans:** 3

**Sol:** Die A : 1, 2, 3, 5, 7, 11
Die B : 1, 2, 3, 5, 7, 11
S = 2 $\Rightarrow$ (1, 1)
S = 3 $\Rightarrow$ (1, 2), (2, 1)
S = 4 $\Rightarrow$ (1, 3) (3, 1) (2, 2)
S = 5 $\Rightarrow$ (2, 3) (3, 2)
S = 6 $\Rightarrow$ (1, 5), (5, 1), (3, 3)
S = 7 $\Rightarrow$ (2, 5), (5, 2)
S = 8 $\Rightarrow$ (1, 7), (7, 1), (3, 5), (5, 3)
$\therefore P = \frac{17}{36}$



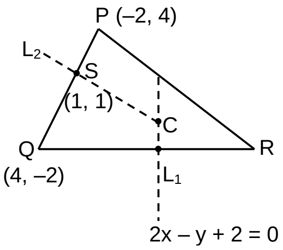
**Q.6** In a triangle PQR, the co-ordinates of the points P and Q are $(-2, 4)$ and $(4, -2)$ respectively. If the equation of the perpendicular bisector of PR is $2x - y + 2 = 0$, then the centre of the circumcircle of the $\Delta PQR$ is :

**Options**

1. $(1, 4)$
2. $(0, 2)$
3. $(-1, 0)$
4. $(-2, -2)$

**Ans:** 4

**Sol:**




S is the mid point of PQ

$\Rightarrow S\left(\frac{-2+4}{2}, \frac{4+-2}{2}\right) = (1,1)$

$\text{Slope of } PQ = \frac{4+2}{-2+4} = \frac{6}{-6} = -1$

Slope of $L_2$ is $\frac{-1}{-1} = 1$

$\therefore L_2 \Rightarrow y - 1 = 1\ (x - 1)$

$\Rightarrow y - 1 = x - 1 \Rightarrow x - y = 0$

C is the point of intersection of $L_1$ and $L_2$

Solve : $2x - y = -2$ --- (1) and

$x - y = 0$ ---- (2)

$x = -2,\ y = -2$

$\therefore C\ (-2, -2)$

**Q.7** The inverse of $y = 5^{\log x}$ is :

**Options**

1. $x = y^{\log 5}$
2. $x = 5^{\frac{1}{\log y}}$
3. $x = y^{\frac{1}{\log 5}}$
4. $x = 5^{\log y}$

**Ans: 3**

**Sol:** $y = 5^{\log x} \Rightarrow \log y = \log x \,.\, \log 5$

$\Rightarrow \log x = \frac{\log y}{\log 5} = \log\left[y^{1/\log 5}\right]$

$\Rightarrow x = y^{1/\log 5}$

**Q.8** The equation of the plane which contains the $y$-axis and passes through the point (1, 2, 3) is :

**Options** 1. $3x + z = 6$

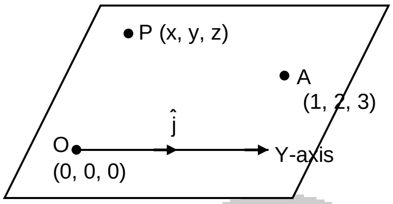
2. $x + 3z = 0$

3. $3x - z = 0$

4. $x + 3z = 10$

**Ans:** 3

**Sol:**




$\vec{AP} = (x-1)\hat{i} + (y-2)\hat{j} + (z-3)\hat{k}$

$\vec{OA} = 1\hat{i} + i\hat{j} + 3\hat{k}$

y axis $\Rightarrow 0\,\hat{i} + 1\,\hat{j} + 0\,\hat{k}$

The above three vectors are coplanar

$\therefore \begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 2 & 3 \\ 0 & 1 & 0 \end{vmatrix} = 0 \Rightarrow -1(3x - 3 - z + 3) = 0$

$\Rightarrow 3x - z = 0$

**Q.9** Choose the incorrect statement about the two circles whose equations are given below :

$x^2 + y^2 - 10x - 10y + 41 = 0$ and

$x^2 + y^2 - 16x - 10y + 80 = 0$

**Options** 1. Both circles' centres lie inside region of one another.

2. Both circles pass through the centre of each other.

3. Distance between two centres is the average of radii of both the circles.

4. Circles have two intersection points.

**Ans:** 1

**Sol:** $C_1 : (5, 5), r_1 = \sqrt{25 + 25 - 41} = 3$

$C_1 : (8, 5), r_2 \sqrt{64 + 25 - 80} = 3$

$C_1\ C_2 = \sqrt{(8-5)^2 + (5-5)^2} = 3$

**Q.10** The value of $4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{4+.....\infty}}}}$ is :

**Ans: 1**

**Sol:** $x=4+\cfrac{1}{5+\cfrac{1}{x}}$

$\Rightarrow x-4=\frac{1}{(5x+1)/x}=\frac{x}{5x+1}$

$(x-4)(5x+1)=x\Rightarrow 5x^2+x-20x-4-x=0$

$5x^2-20x-4=0\Rightarrow x=\frac{20\pm\sqrt{400+80}}{10}$

$\frac{20\pm4\sqrt{30}}{10}=2+\frac{2\sqrt{30}}{5}(\because x>0)$

---

**Q.11** If $A=\begin{pmatrix}0 & \sin\alpha\\ \sin\alpha & 0\end{pmatrix}$ and $\det\left(A^2-\frac{1}{2}I\right)=0$, then a possible value of $\alpha$ is :

**Options**

1. $\frac{\pi}{6}$

2. $\frac{\pi}{3}$

3. $\frac{\pi}{2}$

4. $\frac{\pi}{4}$

**Ans: 4**

**Sol:** $A=\begin{bmatrix}0 & \sin\alpha\\ \sin\alpha & 0\end{bmatrix}$

$A^2=\begin{bmatrix}0 & \sin\alpha\\ \sin\alpha & 0\end{bmatrix}\begin{bmatrix}0 & \sin\alpha\\ \sin\alpha & 0\end{bmatrix}=\begin{bmatrix}\sin^2\alpha & 0\\ 0 & \sin^2\alpha\end{bmatrix}$

**Given** $A^2=\frac{1}{2}I\Rightarrow\begin{bmatrix}\sin^2\alpha & 0\\ 0 & \sin^2\alpha\end{bmatrix}=\begin{bmatrix}1/2 & 0\\ 0 & 1/2\end{bmatrix}$

$\Rightarrow\sin^2\alpha=\frac{1}{2}\Rightarrow\sin\alpha=\pm\frac{1}{\sqrt{2}}$

One of the value of α is $\pi/4$

**Q.12** If $\cot^{-1}(\alpha)=\cot^{-1}2+\cot^{-1}8+\cot^{-1}18+\cot^{-1}32+.....$ upto 100 terms, then $\alpha$ is :

**Options**
1. 1.03
2. 1.02
3. 1.00
4. 1.01

**Ans: 4**

**Sol:** $\cot^{-1}\alpha=\cot^{-1}2+\cot^{-1}8+\cot^{-1}18+\cot^{-1}32+.....\text{to } 100 \text{ terms}$

RHS $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{8}\tan^{-1}\frac{1}{18}+\tan^{-1}\frac{1}{32}+......\text{to } 100 \text{ terms}$

$=\tan^{-1}\frac{1}{2(1)^2}+\tan^{-1}\left(\frac{1}{2(2)^2}\right)+\tan^{-1}\left(\frac{1}{2(3^2)}\right)+\tan^{-1}\left(\frac{1}{2(4^2)}\right)+......\text{to 100 terms}$

General term $=\tan^{-1}\left(\frac{1}{2r^2}\right)=\tan^{-1}\left[\frac{2}{4r^2}\right]$

$=\tan^{-1}\left(\frac{2}{(4r^2-1)+1}\right)=\tan^{-1}\left[\frac{(2r+1)-(2r-1)}{1+(2r+1)(2r-1)}\right]=\tan^{-1}(2r+1)-\tan^{-1}(2r-1)$

1st : $\tan^{-1}3-\tan^{-1}1$

2nd : $\tan^{-1}5-\tan^{-1}3$

3rd : $\tan^{-1}7-\tan^{-1}5$

--------------------------------------------------------

100th : $\tan^{-1}201-\tan^{-1}199$

Sum = $\tan^{-1}201-\tan^{-1}1=\tan^{-1}\left(\frac{201-1}{1+(201)(1)}\right)=\tan^{-1}\left(\frac{200}{202}\right)$

$\therefore \quad =\cot^{-1}\left(\frac{202}{200}\right)=\cot^{-1}\left(\frac{101}{100}\right)=\cot^{-1}\alpha$

$\therefore \alpha=1.01$

**Q.13** If the fourth term in the expansion of $(x+x^{\log_2 x})^7$ is 4480, then the value of $x$ where $x \in \mathbf{N}$ is equal to :

**Options**
1. 1
2. 3
3. 2
4. 4

**Ans: 3**

**Sol:** $\left(x+x^{\log_2 x}\right)^7$

$T_4=T_{3+1}=7\,C_3\ x^4\ \left(x^{\log x}\right)^3=\frac{7.6.5}{1.2.3}\ x^4\ .\ x^{3\log_2 x}$

$=35\ x^{(4+3\log_2 x)}=4480$

$\Rightarrow x^{(4+3\log_2 x)}=128$

$\Rightarrow x=2$ (By back substitution)

**Q.14** If the Boolean expression $(p \Rightarrow q) \Leftrightarrow (q * (\sim p))$ is a tautology, then the Boolean expression $p * (\sim q)$ is equivalent to :

**Options**
1. $\sim q \Rightarrow p$
2. $p \Rightarrow \sim q$
3. $q \Rightarrow p$
4. $p \Rightarrow q$

**Ans: 3**

**Sol:**

| p | q | $p \Rightarrow q$ | $q \wedge \sim p$ | $(p \Rightarrow q) \Leftrightarrow (q \wedge \sim p)$ |
|---|---|---|---|---|
| T | T | T | F | F |
| T | F | F | F | T |
| F | T | T | T | T |
| F | F | T | F | F |

| p | q | $p \wedge \sim q$ | $q \Rightarrow p$ |
|---|---|---|---|
| T | F | T | T |
| F | T | F | F |

$\therefore\ p \wedge \sim q = (q \Rightarrow p)$

If $(p \Rightarrow q) \Leftrightarrow (q \wedge \sim p)$

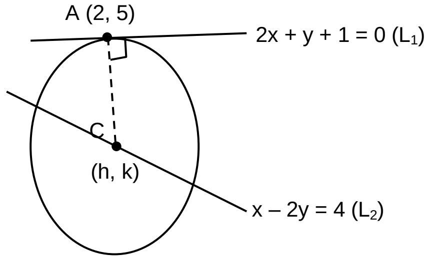
Is a tautology

**Q.15** The line $2x - y + 1 = 0$ is a tangent to the circle at the point (2, 5) and the centre of the circle lies on $x - 2y = 4$. Then, the radius of the circle is :

**Options**
1. $5\sqrt{4}$
2. $3\sqrt{5}$
3. $5\sqrt{3}$
4. $4\sqrt{5}$

**Ans: 2**

**Sol:**




$m_{AC} = \frac{k-5}{h-2}$, $m_{L_1} = \frac{-2}{-1} = 2$

$m_{AC} \text{ x } m_{L_1} = -1 \Rightarrow \left(\frac{k-5}{h-2}\right)(2) = -1$

$\Rightarrow 2k - 10 = -h + 2$

$\Rightarrow h + 2k = 12$ _________ (1)

(h, k) satisfying the equation x-2y=4

$\Rightarrow h - 2k = 4$ ______ (2)

$(1) + (2) \Rightarrow 2h = 16 \Rightarrow h = 8,\ k = 2$

$\therefore C\ (8,2)$

$r = AC = \sqrt{(8-2)^{21} + (2-5)^2} = \sqrt{6^2 + (-3)^2} = \sqrt{45} = 3\sqrt{5}$

**Q.16** Which of the following is true for $y(x)$ that satisfies the differential equation $\frac{dy}{dx} = xy - 1 + x - y;\ y(0) = 0$ :

**Options**

1. $y(1) = e^{\frac{1}{2}} - 1$
2. $y(1) = e^{-\frac{1}{2}} - 1$
3. $y(1) = 1$
4. $y(1) = e^{\frac{1}{2}} - e^{-\frac{1}{2}}$

**Ans: 2**

**Sol:** $\frac{dy}{dx} = xy - 1 + x - y = (x-1)(y+1)$

$\Rightarrow \int \frac{dy}{y+1} = \int (x-1)\ dx \Rightarrow \log(y+1) = \frac{x^2}{2} - x + C$

$\Rightarrow \log(0+1) = \frac{0^2}{2} - 0 + C \Rightarrow C = 0$

$\therefore\ \log(y+1) = \frac{x^2}{2} - x \Rightarrow y = e^{\left(\frac{x^2}{2} - x\right)} - 1$

$y\ (1) = e^{1/2 - 1} - 1 = e^{-1/2} - 1$



**Q.17** The sum of possible values of $x$ for $\tan^{-1}(x+1) + \cot^{-1}\left(\frac{1}{x-1}\right) = \tan^{-1}\left(\frac{8}{31}\right)$ is :

**Options**

1. $-\frac{33}{4}$
2. $-\frac{31}{4}$
3. $-\frac{32}{4}$
4. $-\frac{30}{4}$

**Ans:** 3

**Sol:** $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\left(\frac{8}{31}\right)$

$$\tan^{-1}\left(\frac{2x}{1-\left(x^2-1\right)}\right]=\tan^{-1}\frac{8}{31}$$

$$\frac{2x}{2-x^2}=\frac{8}{31}\Rightarrow\frac{x}{2-x^2}=\frac{4}{31}$$

$$8-4x^2=31x\Rightarrow 4x^2+31x-8=0$$

$$x=\frac{-31\pm\sqrt{1089}}{8}=\frac{1}{4}\text{ or }-8$$

x = –8 satisfies the given equation but $x=\frac{1}{4}$ does not satisfied

∴ only root is x = –8

$$\text{Sum}=-8=\frac{-32}{4}$$

**Q.18** Which of the following statements is incorrect for the function $g(\alpha)$ for $\alpha \in \mathbf{R}$ such that

$$g(\alpha)=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sin^{\alpha}x}{\cos^{\alpha}x+\sin^{\alpha}x}\,dx$$

**Options**
1. $g(\alpha)$ is a strictly decreasing function
2. $g(\alpha)$ is a strictly increasing function
3. $g(\alpha)$ has an inflection point at $\alpha=-\frac{1}{2}$
4. $g(\alpha)$ is an even function

**Ans:** 4

**Sol:** $g(\alpha)=\int_{\pi/6}^{\pi/3}\frac{\sin^{\alpha}x}{\cos^{\alpha}x+\sin^{\alpha}x}=\int_{\pi/6}^{\pi/3}\frac{\sin^{\alpha}\left(\frac{\pi}{2}-x\right)dx}{\cos^{\alpha}\left(\frac{a}{2}-x\right)+\sin^{\alpha}\left(\frac{\pi}{2}-x\right)}$

$$\therefore\int_a^b f(x)dx=\int_a^b(a+b-x)dx\quad=\int_{\pi/6}^{\pi/3}\frac{\cos^{\alpha}x\,dx}{\sin^{\alpha}x+\cos^{\alpha}x}$$

$$2.g(\alpha)=\int_{\pi/6}^{\pi/3}1dx=\pi/6\Rightarrow g(\alpha)=\frac{\pi}{12}$$

$g(-\alpha)=\pi/12\Rightarrow g(\alpha)$ is an even function

**Q.19** Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = 7\hat{i} + \hat{j} - 6\hat{k}$.

If $\vec{r} \times \vec{a} = \vec{r} \times \vec{b}$, $\vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = -3$, then $\vec{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k})$ is equal to :

**Options**
1. 8
2. 13
3. 12
4. 10

**Ans: 3**

**Sol:** $\left(\vec{r} \times \vec{a}\right) - \left(\vec{r} \times \vec{b}\right) = 0$

$\Rightarrow \vec{r} \times \left(\vec{a} - \vec{b}\right) = 0 \Rightarrow \vec{r} = \lambda\left(\vec{a} - \vec{b}\right)$

$\Rightarrow \vec{r} = \lambda(-5\hat{i} - 4\hat{j} + 10\hat{k})$

$\vec{r} \, . \, (\hat{i} + 2\hat{j} + \hat{k}) = \lambda(-5 - 8 + 10) = -3\lambda = -3 \Rightarrow \lambda = 1$

$\therefore \vec{r} \, . \, (2\hat{i} - 3\hat{j} + \hat{k}) = 1(-10 + 12 + 10) = 12$

**Q.20** The value of $\lim_{x \to 0^+} \frac{\cos^{-1}(x - [x]^2) \cdot \sin^{-1}(x - [x]^2)}{x - x^3}$, where $[x]$ denotes the greatest integer $\le x$ is :

**Options**
1. $\pi$
2. $\frac{\pi}{4}$
3. $\frac{\pi}{2}$
4. 0

**Ans: 3**

**Sol:** As $x \to 0^+$, $h \to 0$ and x = h where h>0

The given limit = $\lim_{h \to 0} \frac{\{\cos^{-1}(h - [h]^2) \; . \; \sin^{-1}(h - [h]^2)\}}{h - h^3}$

as $h \to 0$, [h]=0 $\quad \because h > 0$

$$\lim_{h \to 0}\left\{\frac{\cos^{-1}h \, . \, \sin^{-1}h}{h - h^3}\right\} = \frac{\pi}{2} . \lim_{h \to 0}\left(\frac{\sin^{-1}h}{h - h^3}\right) = \frac{\pi}{2} . \lim_{h \to 0}\left\{\frac{\frac{1}{\sqrt{1 - h^2}}}{1 - 3h^2}\right\} = \frac{\pi}{2}$$

## SECTION B

**1.** Let there be three independent events $E_1$, $E_2$ and $E_3$. The probability that only $E_1$ occurs is $\alpha$, only $E_2$ occurs is $\beta$ and only $E_3$ occurs is $\gamma$. Let 'p' denote the probability of none of events occurs that satisfies the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1).

Then, $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$ is equal to ________.

**Ans: 6**

**Sol:** Let $P(E_1) = x, P(E_2) = y$ and $P(E_3) = z$

$\Rightarrow x(1-y)(1-z) = \alpha,\ y(1-z)(1-x) = \beta$ and

$z(1-x)(1-y) = r$

$P = \frac{\alpha\beta}{\alpha - 2\beta} \Rightarrow (1-x)(1-y)(1-z) = \frac{\alpha\beta}{\alpha - 2\beta}$ and

$(1-x)(1-y)(1-z) = \frac{2\beta r}{(\beta - 3r)}$

$\Rightarrow (1-x)(1-y)(1-z) = \frac{xy(1-y)(1-x)(1-z)^2}{x(1-y)(1-z) - 2y(1-z)(1-x)}$

$= \frac{xy(1-x)(1-y)(1-z)^2}{(1-z)[x - xy - 2y + 2xy]} = \frac{xy(1-x)(1-y)(1-z)}{x + xy - 2y}$

$\Rightarrow x + xy - 2y = xy \Rightarrow x = 2y$ ________(1)

$(1-x)(1-y)(1-z) = \frac{2yz(1-y)(1-z)(1-x)^2}{y(1-z)(1-x) - 3z(1-x)(1-y)} = \frac{2yz(1-y)(1-z)(1-x)^2}{(1-x)(y - yz - 3z + 3yz)} \Rightarrow \frac{2yz}{(y - 3z + 2yz)} = 1$

$\Rightarrow y - 3z + 2yz = 2yz \Rightarrow y = 3z$ ________(2)

$\therefore x = 2(3z) \Rightarrow x = 6z \Rightarrow \frac{x}{z} = 6$

**2.** If $f(x) = \sin\left(\cos^{-1}\left(\frac{1 - 2^{2x}}{1 + 2^{2x}}\right)\right)$ and its first derivative with respect to $x$ is $-\frac{b}{a}\log_e 2$ when $x = 1$, where a and b are integers, then the minimum value of $|a^2 - b^2|$ is ________.

**Ans: 481**

**Sol:** $\frac{dy}{dx} = \cos\left[\cos^{-1}\left(\frac{1-2^{2x}}{1+2^{2x}}\right)\right] \times \frac{-1}{\sqrt{1 - \left(\frac{1-2^{2x}}{1+2^{2x}}\right)^2}}$

$\frac{(1+2^{2x}).\ (-4^x.\log 4) - (1-2^{2x})\ \ 4^x \log 4}{(1+2^{2x})^2}$

If $x = 1$, $\frac{dy}{dx} = \frac{-3}{5} \times \frac{-1}{\sqrt{1 - \frac{9}{25}}} \times \left[\frac{(5)(-4\log 4) - (-3)(4\log 4)}{25}\right]$

$= \frac{-3}{5} \times \frac{5}{4} \times \frac{-8\log 4}{25} = \frac{6}{25}\log 4 = -\left(\frac{-12}{25}\right)\log 2$

$\Rightarrow a = 25$ and $b = -12$

$|a^2 - b^2| = 481$

**3.** If the equation of the plane passing through the line of intersection of the planes $2x-7y+4z-3=0$, $3x-5y+4z+11=0$ and the point $(-2, 1, 3)$ is $ax+by+cz-7=0$, then the value of $2a+b+c-7$ is __________.

**Ans: 4**

**Sol:** The required plane is $(2x-7y+4z-3)+\lambda(3x-5y+4z+11)=0$ which passes through the point (-2, 1, 3)

$$(-4-7+12-3)+\lambda(-6-5+12+11)=0$$

$$-2+12\lambda=0 \Rightarrow \lambda=\frac{1}{6}$$

$$\Rightarrow 12x-42y+24z-18+3x-5y+4z+11=0$$

$$\Rightarrow 15x-47y+28z-7=0 \Rightarrow 2a+b+c-7=30-47+28-7=4$$

**4.** If $[\cdot]$ represents the greatest integer function, then the value of

$$\left|\int_0^{\sqrt{\frac{\pi}{2}}}\left[[x^2]-\cos x\right]dx\right|$$ is __________.

**Ans: 1**

**Sol:** $$\int_0^{\sqrt{\pi/2}}\left[[x^2]-\cos x\right]dx=\int_0^1\left[[x^2]-\cos x\right]dx+\int_1^{\sqrt{\pi/2}}\left[[x^2]-\cos x\right]dx$$

$$\int_0^1[0-\cos x]dx+\int_1^{\sqrt{\pi/2}}[1-\cos x]\,dx$$

$$\int_0^1-1.\,dx+\int_1^{\sqrt{\pi/2}}0dx=-[x]_0^1=-1$$

**5.** If the function $f(x)=\dfrac{\cos(\sin x)-\cos x}{x^4}$ is continuous at each point in its domain and

$f(0)=\dfrac{1}{k}$, then k is __________.

**Ans: 6**

**Sol:** $$\frac{1}{k}=\lim_{x\to0}\left(\frac{\cos(\sin x)-\cos x}{x^4}\right)=\lim_{x\to0}\left[\frac{-\sin(\sin x).\ \cos x+\sin x}{4x^3}\right]$$

$$=\lim_{x\to0}\left[\frac{-\cos(\sin x).\cos^2x+-\sin(\sin x).\ (-\sin x)+\cos x}{12x^2}\right]$$

$$\lim_{x\to0}\left[\frac{\sin(\sin x).\cos^3x+-\cos(\sin x).\ 2\cos x(-\sin x)+\cos(\sin x).\cos x.\ (\sin x)+\sin(\sin x).\cos x-\sin x}{24x}\right]$$

$$=\frac{1}{24}+\frac{2}{24}+\frac{1}{24}+\frac{1}{24}-\frac{1}{24}=\frac{1}{6}\Rightarrow k=6$$

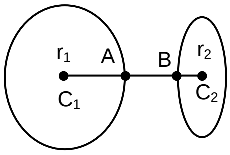
**6.** The minimum distance between any two points $P_1$ and $P_2$ while considering point $P_1$ on one circle and point $P_2$ on the other circle for the given circles' equations

$x^2+y^2-10x-10y+41=0$

$x^2+y^2-24x-10y+160=0$ is ________.

**Ans: 1**

**Sol:**




$AB = C_1C_2 - (r_1+r_2)$

$C_1:(5,5),\ C_2:(12,5),\ r_1=\sqrt{25+25-41}=3$

$r_2=\sqrt{144+25-160=3}$

$C_1C_2=\sqrt{(5-12)^2+(5-5)^2=7}$

$r_1+r_2=6 \Rightarrow AB=1$

**7.** If $\vec{a}=\alpha\hat{i}+\beta\hat{j}+3\hat{k}$,

$\vec{b}=-\beta\hat{i}-\alpha\hat{j}-\hat{k}$ and

$\vec{c}=\hat{i}-2\hat{j}-\hat{k}$

such that $\vec{a}\cdot\vec{b}=1$ and $\vec{b}\cdot\vec{c}=-3$, then $\frac{1}{3}\left((\vec{a}\times\vec{b})\cdot\vec{c}\right)$ is equal to ________.

**Ans: 2**

**Sol:** $\vec{a}.\vec{b}=-\alpha\beta-\alpha\beta-3=1$

$\Rightarrow 2\alpha\beta=-4 \Rightarrow \alpha\beta=-2$ ______(1)

$\vec{b}.\vec{c}=-\beta+2\alpha+1=-3$

$\Rightarrow 2\alpha-\beta=-4$ ______(2)

From (1) and (2)

$\alpha=-1, \beta=2$

$$\frac{1}{3}\left(\vec{a}\times\vec{b}\right).\vec{c}=\frac{1}{3}\begin{vmatrix}-1 & 2 & 3\\ -2 & 1 & -1\\ 1 & -2 & -1\end{vmatrix}$$

$$=\frac{1}{3}[-1](-1-2)-2(2+1)+3(4-1)=\frac{1}{3}(3-6+9)=2$$

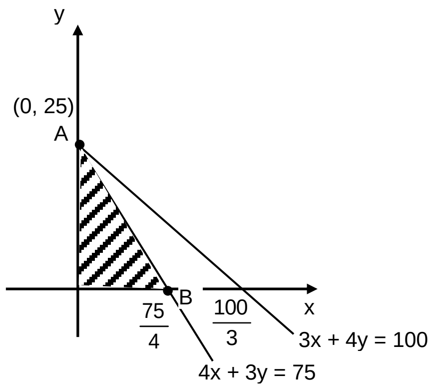
**8.** The maximum value of $z$ in the following equation $z=6xy+y^2$, where $3x+4y\le 100$ and $4x+3y\le 75$ for $x\ge 0$ and $y\ge 0$ is ________.

**Ans: 904**

**Sol:** $3x + 4y = 100$ ________ (1)

$4x + 3y = 75$ _______ (2)

Solving x=0, y=25




$Z = 6xy + y^2$ has minimum value at (0,0). As x or y increases, the value of Z increases

$\therefore$ Z gets maximum value at some point on the line segment AB.

Equation of AB is 4x+3y=75

$$\Rightarrow x = \frac{75-3y}{4} = 3\left(\frac{25-y}{4}\right)$$

$$\therefore z = 6 \times 3\frac{(25-y).y}{4} + y^2 = \frac{9}{2}(25-y)y + y^2$$

$$= \frac{9}{2}\left(25y - y^2\right) + y^2 = \frac{225y}{2} - \frac{7}{2}y^2$$

$$\frac{dz}{dy} = \frac{225}{2} - 7y = 0 \Rightarrow y = \frac{225}{14}$$

$$\frac{d^2z}{dy^2} = -7 < 0 \left(\text{max at } y = \frac{225}{14}\right)$$

$$\therefore Z_{max} = \frac{225}{2} \cdot \frac{225}{14} - \frac{7}{2}\left(\frac{225}{14}\right)^2$$

$$= \frac{(225)^2}{2 \times 14}\left[1 - \frac{1}{2}\right] = \frac{(225)^2}{4 \times 14} = 904 \text{ (approx)}$$

**9.** If $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$, then the value of $\det(A^4) + \det\left(A^{10} - (\text{Adj}(2A))^{10}\right)$ is equal to ________.

**Ans: 16**

**Sol:** $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$

$A^2 = A.A\,A^4 = A^2.A$ etc,

We get $A^{10} = \begin{bmatrix} 1024 & 1023 \\ 0 & 1 \end{bmatrix}$

$2A = \begin{bmatrix} 4 & 6 \\ 0 & -2 \end{bmatrix}$

$[\text{adj}(2A)]^{10} = \begin{bmatrix} -2 & -6 \\ 0 & -2 \end{bmatrix}^{10} = \begin{bmatrix} 1024 & 30720 \\ 0 & 1024 \end{bmatrix}$

$\therefore A^{10} - [adj(2A)]^{10} = \begin{bmatrix} 0 & -29697 \\ 0 & -1023 \end{bmatrix}$

$\Rightarrow \left| A^{10} - [adj(2A)]^{10} \right| = 0$

$\left| A^4 \right| = |A|^4 = (-2)^4 = 16$

**10.** If $(2021)^{3762}$ is divided by 17, then the remainder is _________.

**Ans:** **4**

**Sol:** $(2021)^{3762} = [(17 \times 119) - 2]^{3762}$

$= 17\lambda + 2^{3762}$ [Binomial theorem]

and $2^{3762} = (2^4)^{940} \cdot 2^2 = 4(17-1)^{940}$

$= 4[17\mu + 1] = (4)(17)\mu + 4$

$\therefore (2021)^{3762} = 17\lambda + (4)(17)\mu + 4$

$\therefore$ Reminder =4