Solutions for CAT 2017 QA - Morning (Slot-1)

67. Let Barun's age be 10x. Arun's age is 4x. The difference of these ages in 6x, a constant. When Arun's age is 50% of Barun's age, this difference also would be 50% ie Barun's age, at that stage would be 12x. It would be increase by 20%.

. Ans: (20)

68. Let the number of days required to complete the job be n.
1 person works on day 1, 2 on day 2, 3 on day 3, …. n on day n.
Each person has the same efficiency.

Work = $1\left(\frac{1}{120}\right)+2\left(\frac{1}{120}\right)+3\left(\frac{1}{120}\right).....+n\left(\frac{1}{120}\right)$.

This is also equal to 1.

$$\frac{1}{120}+\frac{2}{120}+\frac{3}{120}+.......+\frac{n}{120}=1$$

$\Sigma n = 120$
n = 15. Ans: (15)

69. Number of people in the group cannot exceed $\frac{630}{53}$ i.e., 11.8.
Maximum possible number of people in the group = 11. Ans: (11)

70. The speed in the second case is 5/4 times the speed in the first case. Therefore, the time would be 4/5 times the time, i.e., 1/5 less. This one fifth is 20 min. Therefore, the time taken in the first case is 100 min.

The distance = $(12)\left(\frac{5}{3}\right)$ km = 20 km

Ans: (20)

71. Let the total monthly savings be S.

Investment in FD = $\frac{50}{100}$S.

Investment in stocks = $\frac{30}{100}\left(S-\frac{50}{100}S\right)=\frac{15}{100}S$

Investment in savings bank account = $\frac{35}{100}S$

$$\frac{35}{100}S+\frac{50}{100}S=59500$$

S = 70000 Ans: (70000)

72. Let the retail price be 100.
Discount = 15
Selling price = 85

Cost price = $\frac{85}{1.02}=\frac{500}{6}$

In order to make a profit of 20%, the selling price

$= \frac{500}{6}(1.2)=100$

The seller must sell at the retail price
Choice (4)

73. Let the speed of the boat in still water and the speed of the river be u and v respectively.

$$\frac{d}{2x+y}+\frac{d}{2x-y}=\frac{1}{4}\left(\frac{d}{x+y}+\frac{d}{x-y}\right)$$

$$\frac{d(4x)}{4x^2-y^2}=\frac{1}{4}\left(\frac{d(2x)}{x^2-y^2}\right)$$

$8(x^2 – y^2) = 4x^2 – y^2$

$$\frac{x^2}{y^2}=\frac{7}{4}$$

$$\frac{x}{y}=\frac{\sqrt{7}}{2}$$ Choice (2)

74. The data is given below

C1	C2	C3	C4	C5
9	10	8		
	18		19	20
81	90	72	95	100

C5 – C1 = 19. The numbers above are the actual profits (and not just the ratio). The total profit = 438 crore.
Choice (1)

75. Let the number of boys appearing for the admission test be b.
Percentage of candidates who get admission =

$$\frac{\frac{30}{100}(2b)+\frac{45}{100}b}{2b+b}(100)\%=35\%$$

65% of the candidates do not get admission.
Choice (4)

76. Let the total number of popcorn packets in stock be T.
Total number of chips packets in stock = T

Required ratio = $\frac{16}{40}T:\frac{14}{35}T=1:1$

Choice (1)

77. Let the price of each good mango be g.

Price of each medium quality mango = $\frac{g}{2}$.

Total cost price = 80g + $40\left(\frac{g}{2}\right)$ = 100g

Total selling price = 120(0.9g) = 108g
Overall profit = 8%
Choice (2)

78. Let the printed price be p.
If 40% discount is given, selling price
= 0.6 (60p) = 36 p
20% profit is then made.

Total cost price = $\frac{36p}{1.2}$ = 30p.

Ten toys are destroyed in the fire.

The remaining toys are sold at a price such that the same amount of profit is made as in the conditional case.
Profit made on remaining toys = 6p
Total selling price of remaining toys = 36p
Discount that should be given = 50p – 36p = 14p
Discount% = 28%

Choice (4)

79. $\left(\frac{a+3}{b}\right)^2 = 9$ and $\left(\frac{a-1}{b-1}\right)^2 = 4$. We get 4 cases

a + 3 = 3b a – 1 = 2b – 2	a +3 = 3b a – 1 = –2b + 2
a+3 = –3b a – 1 = 2b – 2	a + 3 = –3b a – 1 = –2b + 2

Subtracting the second equation from the first we get,

	I	II	III	IV
4	b + 2	5b – 2	–5b +2	–b –2

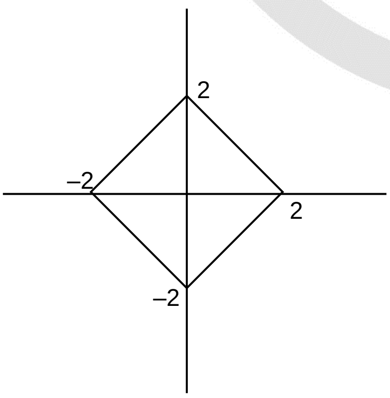
I ⇒ b = 2, a = 3 Rejected
II, III ⇒ b is not an integer. Rejected
IV ⇒ b = –6 , a = 15

$\therefore \frac{a^2}{b^2} = \left(\frac{15}{6}\right)^2 = \frac{25}{4}$. Choice (4)

80. Let the average score of the boys in the mid-semester examination be b.
Average score of the girls = b + 5
In the final exam, average score of the girls = b + 5 – 3 = b + 2.
Average score of the entire class increased by 2 and is hence $\frac{20b + 30(b+5)}{50}$ + 2 i.e. b + 5
Average score of the boys
$\frac{50(b+5) - 30(b+2)}{20} = b + 9.5$
Increases in the average of boys is 9.5.

Choice (1)

81.



The closed region bounded by |ax| + |by| = c in the two- dimensional plane has x-intercepts of $\pm\frac{c}{|a|}$ and y- intercepts of $\pm\frac{c}{|b|}$.This is in general a rhombus. In the given question, we have a square which has each of its diagonals as 4.

Area = $\frac{1}{2}(4)(4) = 8$

Choice (3)

82. The medians of a triangle divide the triangle into six parts of equal area.

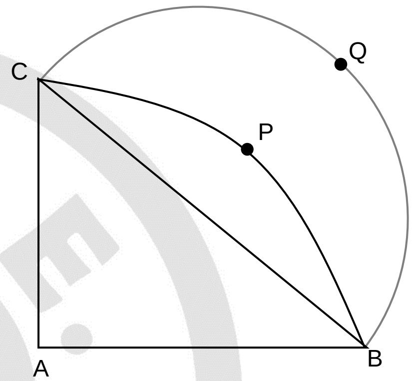
Area of GBC = $\frac{1}{3}$ (Area of the triangle)

$= \frac{1}{3}\sqrt{5(5-a)(s-b)(s-c)} = \frac{250}{\sqrt{3}}$

Area of the remaining portion = $2\left(\frac{250}{\sqrt{3}}\right) = \frac{500}{\sqrt{3}}$

Choice (2)

83.



Let AB = a (a = 6)

CQB is a semicircle of radius $\frac{a}{\sqrt{2}}$

CPB is a quarter circle (quadrant) of radius a

$\therefore$ Area of semicircle = $\frac{\pi a^2}{4}$

Area of quadrant = $\frac{\pi a^2}{4}$

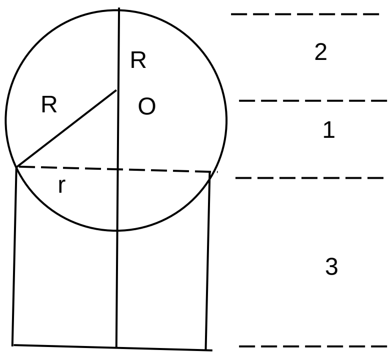
$\therefore$ Area of region enclosed by BPC, BQC = Area of ΔABC = 18.

Choice (2)

84. The volumes of the 5 smaller cubes and the original big one are in the ratio 1 : 1 : 8 : 27 : 27 : 64. Therefore, the sides are in the ratio 1 : 1 : 2 : 3 : 3 : 4 while the areas are in the ratio 1 : 1 : 4 : 9 : 9 : 16. The sum of the areas of the 5 smaller cubes is 24 parts while that of the big cube is 16 parts. The sum is 50% greater.

Choice (2)

85.



The height of the cylinder (h) = 3
The volume = 9π
$\pi r^2 h = 9\pi \Rightarrow r = \sqrt{3}$
The radius of the ball (R) = 2
The height of O, the centre of the ball, above the line representing the top of the cylinder is say a. (a = 1)
$\therefore$ The height of the topmost point of the ball from the base of the cylinder is h + a +R = 3 + 1 + 2 = 6

Ans: (6)

86. In a 3, 4, 5 triangle, the length of the altitude to the hypotenuse = 3(4)/5 = 2.4. Therefore, in a 15, 20, 25 triangle, it is 12. This is the shortest distance from A to BC. At 60 km/hr, i.e., 1 km/min, it would take 24 min to cover 24 km.

Ans: (24)

87. $\log_3 x = a \Rightarrow x = 3^a$
$\log_{12} y = a \Rightarrow y = 12^a$
$\therefore xy = 36^a$ and $\sqrt{xy} = G = 6^a$
$\therefore \log_6 G = a$ Choice (4)

88. $x + 1 = x^2 \Rightarrow x^2 - x - 1 = 0 \Rightarrow x = \frac{1+\sqrt{5}}{2}$ $(\because x > 0)$

Also, $x^2 = x + 1 \Rightarrow x^4 = x^2 + 2x + 1 = 3x + 2$
$\Rightarrow 2x^4 = 6x + 4 = 3 + 3\sqrt{5} + 4 = 7 + 3\sqrt{5}$

Alternative Solution:

$x + 1 = x^2 \Rightarrow x^2 - x - 1 = 0 \Rightarrow x = \frac{1+\sqrt{5}}{2}$ $(\because x > 0)$

≈ 1.62
$\therefore 2x^4 \approx 13.8$
I ≈ 14.8, II ≈ 14, III ≈ 11.6, IV ≈ 13.6
IV matches (we have used $\sqrt{5} \approx 2.2$. Actually, it is a bit more and we would get a bit more than 13.6)

Choice (4)

89. $0.008 = \frac{8}{1000} = 5^{-3}$
$\therefore \log_{0.008}\sqrt{5} = \frac{1/2}{-3} = \frac{-1}{6}$ and $\log_{\sqrt{3}} 81 = \frac{4}{1/2} = 8$
$\therefore$ The given expression is $\frac{5}{6}$ Choice (3)

90. $9^{2x-1} - 9^{2x-2} = 9^{2x-2}(9 - 1) = 1944 = 8(243) = 8(9^{2.5})$
$\therefore 2x - 2 = 2.5 \Rightarrow x = \frac{4.5}{2} = \frac{9}{4}$

Choice (2)

91. x = 25 + y + z. The possible values of x, y, z and the corresponding number of values of y, z are tabulated below (x, y, z are positive integers). We see that $27 \le x \le 40$

x	y	z	No of values of (x, y)
27	1	1	1
28	1,2	2,1	2
-	-	-	-
38	1, …. 2	12, … 1	12
39	2, …. 12,	12, … 2	11
40	3, …. 12	12, …. 3	10

The number of solutions is 1 + 2 + …… + 12 + 11 + 10 = 78 + 21 = 99 Choice (2)

92. $(n - 5)(n - 10) - 3(n - 2) \le 0$
$\Rightarrow n^2 - 18n + 56 \le 0$
$\Rightarrow (n - 4)(n - 14) \le 0$
As n is an integer, n can be 4, 5, 6 ……14, i.e. it can have 11 values.

Ans: (11)

93. $x^2 + 11x + n = x \Rightarrow x^2 + 10x + n = 0$
$x^2 + 10x + 25 = 0$ has real and equal roots
$x^2 + 10x + n = 0$ where n > 25 has complex roots.
The maximum value of n for which the equation has two distinct real roots in 24.

Ans: (24)

94. a + b + c + d = 30, a, b, c, d are integers.
$(a - b)^2 + (a - c)^2 + (a - d)^2$ would have its maximum value when each bracket has the least possible value.
Let (a, b, c, d) = (8, 8, 7, 7)
The given expression would be 2. It cannot have a smaller value.

Ans: (2)

95. There are 5 pairs of diametrically opposite points and the centre O.
If O is not selected, the number of triangles = ${}^{10}C_3 = 120$.
If O is selected, the other two points can be selected in 10(8)/2, i.e., 40 ways. The number of triangles is 160.

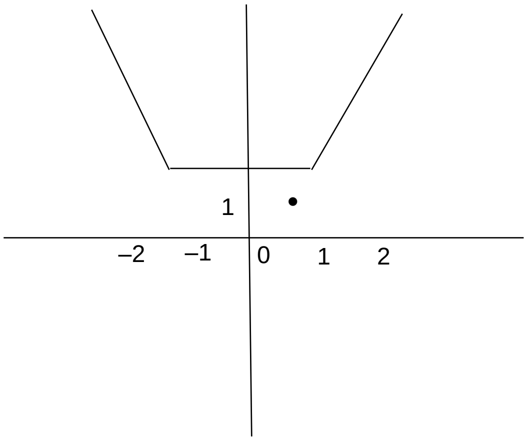
Ans: (160)

Alternative Solution:

There are 11 points. We can select 3 in $\frac{11(10)(9)}{1(2)(3)}$, ie 165 ways. These choices include the 5 diameters. Therefore, the number of triangles is 160.

Ans: (160)

96.



The graph of y = |x – 1| + |x + 1| is shown above.

The shortest distance of $\left(\frac{1}{2}, 1\right)$ from the graph is 1. Choice (1)

97. Let the first term be a and the common difference be d.

$(a + 6d)^2 = (a + 2d)(a + 16d)$

$\Rightarrow a^2 + 12ad + 36d^2 = a^2 + 18ad + 32d^2$

$\Rightarrow 4d^2 = 6ad$

$\Rightarrow \frac{a}{d} = \frac{2}{3}$ Choice (1)

98. After giving one eraser to each of the 4 kids, there are 3 left.

They can split 2, 1 or 1, 1, 1. (No kid can get 4)

There are ${}^4P_2 + {}^4C_3$, i.e., 16 ways of distributing the erasers.

Alternative Solution:

Let the number of erasers given to the 4 kids be w, x, y, z.

w + x + y + z = 7. The number of positive integral solutions is ${}^6C_3$, i.e. 20. This includes (4,1,1,1); (1,4,1,1); (1,1,4,1); (1,1,1,4).

The required number = 20 – 4 = 16

Choice (1)

99. $f(x) = \frac{5x + 2}{3x - 5}$, $g(x) = x^2 - 2x - 1$

$f(3) = \frac{5(3) + 2}{3(3) - 5} = \frac{17}{4}$

$f(17) = \frac{5\left(\frac{17}{4}\right) + 2}{3\left(\frac{17}{4}\right) - 5} = \frac{85 + 8}{51 - 20} = \frac{93}{31} = 3$

$g(3) = 3^2 - 2(3) - 1 = 2.$ Choice (1)

100. $a_1 = 3$, $a_2 = 7$, ….. $a_n = 4n - 1$, …. $a_{3n} = 4(3n) - 1$

$a_1 + a_2 + …. + a_{3n} = \frac{3n(12n + 2)}{2} = 1830$

$\Rightarrow n(6n + 1) = 610$

$\Rightarrow 6n^2 + n - 610 = 0$

$\Rightarrow (6n + 61)(n - 10) = 0$

$\Rightarrow n = 10$ ($\because$ n is an integer)

$\therefore a_1 + a_2 + …. + a_n = 3 + 7 + … + [4(10) - 1]$

$= \frac{4(10)(11)}{2} - 10 = 210.$

$210\,m > 1830 \Rightarrow n > \frac{1830}{210} = 8.7$

The minimum integral value of m is 9.

Choice (2)