Solutions for CAT 2017 QA - Afternoon (Slot-2)

Solutions for questions 67 to 100:

67. The square grid is filled by 9 numbers from 1 to 9. Their sum (1 + 2 + 3 +... 9) equals 45. Since the sum of numbers in each row and each column and each diagonal must be equal, the sum of terms in each row and in each column and in each diagonal, must be 15. For this to happen, the middle element in the 2nd row and the 2nd column must be the middle-most term of the 9 terms, i.e. 5. The corner elements in the first row are 6 and 2 (given), so the middle element in the first row must be 7. In the 2nd column, the top most element is 7 and the middle element is 5, so the bottom row middle element must be 3. Ans: (3)

68. A beats B by 1 km, means A travels 10 km in the same time that B travels 9 km. The ratio of speeds of A and B is 10 : 9. Similarly, the ratio of speeds of B and C is 10 : 9.
A : B = 10 : 9
B : C = 10 : 9
⇒ The ratio of speeds of A : B : C = 100 : 90 : 81
In the same time that A travels 100 m, C travels 81 metres
⇒ In the same time that A travels 10000 m, C would travel 8100 m or A would beat C by 1900 m. Ans: (1900)

69. Concentration of milk in the first bottle is 7/9 and that in the second bottle is 9/13. They need to be mixed in a certain ratio to get a solution which has 3/4th milk.
Applying alligation,
(Liquid taken from bottle 1)/(Liquid taken from bottle 2) = $\dfrac{\frac{9}{13}-\frac{3}{4}}{\frac{3}{4}-\frac{7}{9}} = \dfrac{27}{13}$. Choice (2)

70. Let the distance from his home to his hostel be x miles.
Time taken on his onward journey = x/60 hours
Time taken on his return journey = (x/2)/25 + (x/2 + 5)/50
Given, his return journey took 0.5 hours more than his onward journey
⇒ x/60 + 0.5 = x/50 + (x/2 + 5)/50
Upon solving, x = 30
Therefore, total distance travelled = 30 + 15 + 20 = 65 miles. Choice (3)

71. A total of 15% shirts are defective. Of the remaining 85%, 20% are sold in the domestic market. So, 20% of 85%, i.e. 17% are sold in the domestic market and the remaining 68% are exported. But, it is given that 8840 shirts were exported.
If 68% is 8840, 100% = $\dfrac{(100)(8840)}{68}$ = 13000.
Choice (2)

72. Let the average height of the 20 toddlers be x inches. When the 2 toddlers are included, the average of the group comes down by 2, i.e. it becomes x – 2. Also, given, that the average height of the 2 toddlers is one-third of the average height of the 22 toddlers, viz. x – 2.

$$\frac{20(x)+\frac{2(x-2)}{3}}{22} = x - 2$$

Upon solving, x = 32 inches. Choice (3)

73. Let the manufacturing cost of the table be ₹ m.
The wholesaler buys it for ₹ 1.1m.
The retailer buys it for ₹ (1.1)(1.3)m.
The customer buys it for ₹ (1.1)(1.3)(1.5)m, viz. ₹ 2.145m.
Given, 2.145m = 4,290
⇒ m = ₹ 2,000. Choice (2)

74. The inlet pipe can normally fill the tank in 8 hours, but it takes 10 hours when the outlet pipe is also open. In the two additional hours, the inlet pipe fills 2/8 or 25% more, and the outlet pipe took 10 hours to drain the additional 25%. So, the outlet pipe can drain 25% in 10 hours, and to drain 50%, it'll take 20 hours.
Alternately, we can assume total capacity of the tank as LCM [8, 10] or 40 litres. We'll know the filling rates and the emptying rates of both the pipes and we can answer the question based on the same. Choice (1)

75. He buys x dozen candies at ₹15 a dozen and x more dozen at ₹12 a dozen, so the average cost per dozen is ₹ (15 + 12)/2 = ₹13.5 a dozen.
By selling a dozen at ₹16.5, he'll make a profit of ₹3 per dozen.
To make an overall profit of ₹150, he needs to sell 50 dozen. Choice (1)

76. Let the initial production be p, population be x and the initial per capita consumption be c.
As p = (c)(x), we can say $\dfrac{(x_1)(c_1)}{p_1} = \dfrac{(x_2)(c_2)}{p_2}$
Finally, production became 1.4p and per capita consumption became 1.27c.

$$\frac{(x)(c)}{p} = \frac{(x_2)(1.27c)}{1.4p} \Rightarrow x_2 = \frac{(1.4x)}{1.27} = 1.102x$$

Therefore, population (x) increased by approximately 10%. Choice (3)

77. a : b = 3 : 4 and b : c = 2 : 1
Multiplying the second ratio by 2, we have
a : b = 3 : 4
b : c = 4 : 2
Therefore, a : b : c = 3 : 4 : 2.
⇒ a = 3k, b = 4k, c = 2k.
Since a, b and c are positive integers, their sum should be 9k, a positive integer.
From the options, only choice (3), which is 207, is a multiple of 9. Choice (3)

78. The motorbike that left A travelled 168 km from 1:00 p.m. to 3:40 p.m., i.e. in $2\frac{2}{3}$ hours or $\frac{8}{3}$ hours.
The car that left B, started at 2:00 p.m. and travelled till 3:40 p.m., i.e. for $1\frac{2}{3}$ hours or $\frac{5}{3}$ hours.

The car would've travelled $\frac{5}{8}$th of the distance as the bike, but since the car travelled at twice the speed, the car would've travelled $\frac{10}{8}$th of the distance, i.e. (10/8) x 168 = 210 km.
Therefore, total distance between A and B = 168 + 210 = 378 km. Choice (2)

79. The team completes the job in 4 days.
Since Amol needs 10 days to complete the job, in 4 days, he does 4/10th or 40% of the work.
Since Bimal needs 8 days to complete the job, in 4 days, he does 4/8th or 50% of the work.
Therefore, Kamal did 10% of the work and for doing 10% of the work, he gets 10% of the payment, i.e. ₹100. Choice (1)

80. First mixture has $\frac{2}{3}^{rds}$ A and the rest water.

Second mixture has $\frac{3}{4}$ths B and the rest water.

Third mixture has $\frac{4}{5}$ths C and the rest water

The 3 mixtures are mixed in the ratio 4 : 3 : 2. Let's say we get a 1 litre mixture containing the three mixtures.

There will be $\frac{4}{9}\left(\frac{2}{3}\right)$rd A, $\frac{3}{9}\left(\frac{3}{4}\right)$th B and $\frac{2}{9}\left(\frac{4}{5}\right)$th C. (the remaining part would be water)

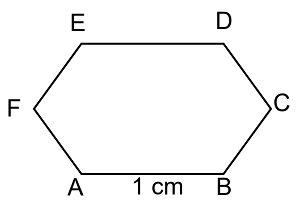
$\frac{8}{27}$ litres A, $\frac{9}{36}$ litres B, $\frac{8}{45}$ litres C

Water = $1-\left[\frac{8}{27}+\frac{9}{36}+\frac{8}{45}\right]=\frac{149}{540}$ litres

We can see that $\frac{149}{540}<\frac{8}{27}$ and $\frac{149}{540}>\frac{9}{36}$

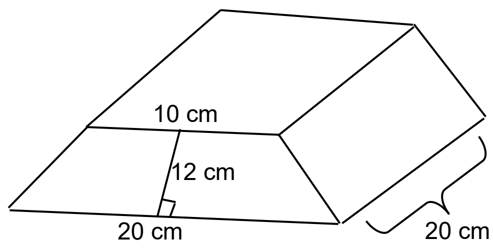
$\therefore$ There is more water than B in the resultant solution. Choice (3)

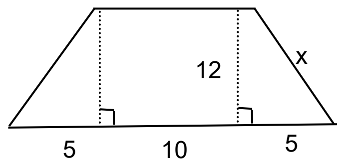
81.



In ΔABC, AB = 1 cm, BC = 1 cm
As $\angle$B = 120°,
$AC^2 = 1^2 + 1^2 - 2(1)(1)\cos(120°)$ (Cosine rule)
$= 1^2 + 1^2 + 1 = 3$
$\Rightarrow$ AC = $\sqrt{3}$ cm
The square of side $\sqrt{3}$ cm will have an area of $(\sqrt{3})^2 = 3$ cm^2 Choice (2)

82.

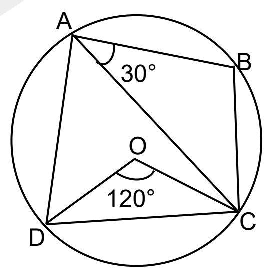




Given, the non-parallel sides are equal.
Let the non-parallel sides be x cm each
x = $\sqrt{12^2+5^2}=13$
So, we have 6 faces, two are trapezoid faces and 4 are rectangular faces.
Area of 2 trapeziums
$= 2\left[\frac{1}{2}(12)(10+20)\right]=360$ cm^2
Area of 4 rectangles
$= 2[13\text{x}20]+20(20)+10(20)=1120$ cm^2
Total area = 1120 + 360 = 1480 cm^2 Choice (3)

83. In a rectangle, diagonals bisect each other, so one diagonal should pass through the midpoint of the other.
Midpoint of the diagonal connecting (2, 5) and (6, 3) = $\left(\frac{2+6}{2},\frac{5+3}{2}\right)=(4,4)$
The other diagonal, y = 3x + c should also pass through (4, 4).
On substitution, 4 = 3 (4) + c $\Rightarrow$ –8.
Choice (4)

84.



Given, $\angle$COD = 120° and $\angle$BAC = 30°.
As $\angle$COD = 120°, $\angle$DAC = 60° (Central angle)
$\angle$DAC + $\angle$BAC = 60° + 30° = 90°
$\angle$A = 90° $\Rightarrow$ $\angle$BCD = 90° (Opposite angles are supplementary in a cyclic quadrilateral)
Ans: 90

85. Let one side be l and the other be b. (l is not necessarily greater than b)
Given, 2l + b = 400
For area to be maximum, lb should be maximum.
$\therefore$ l (400 – 2l) should be maximum
l (400 – 2l) = l (2) (200–l) = 2 (l) (200 – l)

l (200 – l) will be maximum when l = 200 – l or 2l = 200 $\Rightarrow$ l = 100
If l = 100, b = 200.
$\therefore$ The longer side must be 200 feet long.
Ans: (200)

86. As P is equidistant from the sides, P is the incenter of the triangle.
r is the inradius of the triangle, viz. $4(\sqrt{2}-1)$cm
Let the sides of the triangle be a, a, a$\sqrt{2}$

As $\Delta = \frac{1}{2}(a)(a) = r(s)$,

$$\frac{a^2}{2} = 4(\sqrt{2}-1)\frac{(a+a+a\sqrt{2})}{2}$$

$\Rightarrow$ a = $4\sqrt{2}$

Area $= \frac{1}{2}(a^2) = 16$ sq.units   Ans: (16)

87. Given, (n – 1) (n) (n + 1) = 15600
As 15600 has 2 zeroes in it, one of n – 1, n or n + 1 should be a multiple of 25. Dividing 15600 by 25, we get 624, but 624 = 24 $\times$ 26 so, the numbers are 24, 25 and 26
$24^2 + 25^2 + 26^2 = 1877$ Choice (4)

88. $\log_3 5 = \log_5(x+2)$

$\log_3 3 < \log_3 5 < \log_3 9$

$1 < \log_3 5 < 2$

So, $1 < \log_5 (x + 2) < 2$

$5^1 < x + 2 < 5^2$

$3 < x < 23$ Choice (4)

89. f [f (2^x) + g (x^2)]
as x = 1,
f [f (2) + g (1)]
= f [$2^2 + 2^1$]
= f [6] = 6^2 = 36   Choice (3)

90. $x^2 + (a + 3)x – (a+5) = 0$
$\alpha^2 + \beta^2 = (\alpha + \beta)^2 – 2\alpha\beta = (-(a + 3))^2 – 2(– (a + 5))$
$= a^2 + 9 + 6a + 2 (a + 5)$
$= a^2 + 8a + 19$
$= (a + 4)^2 + 3$
The minimum value is 3, at a = –4.   Choice (3)

91. $9^{x-\frac{1}{2}} - 2^{2x-2} = 4^x - 3^{2x-3}$

$$\frac{9^x}{9^{1/2}} - \frac{2^{2x}}{4} = 4^x - \frac{3^{2x}}{27}$$

$$\frac{9^x}{3} - \frac{2^{2x}}{4} = 4^x - \frac{9^x}{27}$$

$$\frac{9^x}{3} - \frac{4^x}{4} = 4^x - \frac{9^x}{27}$$

$$\frac{9^x}{3} + \frac{9^x}{27} = 4^x + \frac{4^x}{4}$$

$$9^x\frac{(10)}{27} = 4^x\frac{(5)}{4}$$

$$\frac{9^x}{4^x} = \frac{27}{8}$$

$$\left(\frac{3}{2}\right)^{2x} = \frac{27}{8} = \frac{3^3}{2^3} = \left(\frac{3}{2}\right)^3$$

$2x = 3 \Rightarrow x = \frac{3}{2}$

Alternately, we could've substituted the value of x from the options and it would've taken much less time.   Choice (1)

92. $\log (2^a \times 3^b \times 5^c) = \frac{1}{3}[\log (2^2 \times 3^3 \times 5) + \log (2^6 \times 3 \times 5^7) + \log (2 \times 3^2 \times 5^4)]$

$\log (2^a 3^b 5^c) = \frac{1}{3}[\log (2^2 \times 3^3 \times 5 \times 2^6 \times 3 \times 5^7 \times 2 \times 3^2 \times 5^4]$

$\log (2^a 3^b 5^c) = \frac{1}{3}[\log (2^9 \times 3^6 \times 5^{12})]$

$\log (2^a 3^b 5^c) = \log (2^3 \times 3^2 \times 5^4)$

While it is not explicitly stated that a, b and c are integers, going by the spirit of the question, we are forced to assume that they are integers. In that case, we can equate the powers of 2, 3 and 5 on the LHS and the RHS and say that a= 3, b = 2, and c = 4.   Ans: (3)

Note: If we don't assume that a, b and c are integers, we can't find the value of any of the variables.

93. The 5 consecutive odd numbers are
a_1, a_2, a_3, a_4, a_5
The 5 consecutive even numbers are
$2a_3 – 8, 2a_3 – 6, 2a_3 – 4, 2a_3 – 2, 2a_3$
The sum of these 5 numbers = $10a_3 – 20 = 450$ (given)
$\therefore a_3 = 47$ and $a_5 = 51$.   Ans: (51)

94. $\frac{1}{a} + \frac{1}{b} = \frac{1}{9}$
$\Rightarrow 9(a + b) = ab$
$\Rightarrow ab – 9a – 9b + 81 = 81$
$\Rightarrow (a – 9) (b – 9) = 81 = 3^4$
As a, b > 0 and a $\leq$ b, there are only 3 ordered pairs, given by a – 9 = 1, 3 or 9 and correspondingly b – 9 = 81, 27, 9.
Ans: (3)

95. The data is shown below.

A	B	K
1	2	3

The remaining 2 pens can go to different people (3 ways – 1,1,0; 0,1,1; 1,0,1) or the same person (3 ways – 2,0,0; 0,2,0; 0,0,2).

Alternately, we can distribute the last 2 identical pens among the three of them using
$x_1 + x_2 + x_3 = 2$, which has ${}^4C_2$ non-negative integral solutions, i.e. 6.
Ans: (6)

96. The sum of the digits must be a multiple of 3. We can use (A) 2,4,0,3 or (B) 2,4,0,6 or (C) 2,4,3,6

(A) _ _ _ 0 (6 numbers)
_ _ _ 2 (4 numbers)
_ _ _ 4 (4 numbers)

(B) _ _ _ 0 (6 numbers)
_ _ _ 2 (4 numbers)
_ _ _ 4 (4 numbers)
_ _ _ 6 (4 numbers)

(C) 2, 4, 3, 6 (18 numbers, with even digit in the units place)

There are a total of 50 numbers. Ans: (50)

97. f (1 × 1) = f (1) f (1) Let f (1) = x

$\therefore$ x = x^2 i.e., x = 0 or 1. The 'largest' value is1.

Ans: (1)

Alternately, the function which satisfies the condition is an exponential function of the form f(a) = a^x. When x = 1, the largest value of f (1) is 1^x, which is 1.

98. f(x) = 2x – 5, g(x) = 7 – 2x.

Given, $|f(x)+g(x)|=|f(x)|+|g(x)|$

$2 = |f(x)|+|g(x)|$

In the 3 ranges $\left(-\infty,\frac{5}{2}\right],\left[\frac{5}{2},\frac{7}{2}\right]$ and $\left[\frac{7}{2},\infty\right)$,

$|f(x)|+|g(x)|=2$ only when x $\in\left[\frac{5}{2},\frac{7}{2}\right]$.

Choice (4)

99. For any n $\geq$ 1, $a_n = 3\,(a_{n+1} + a_{n+2} + \ldots\ldots)$

$\therefore a_1 = 3\,(a_2 + a_3 + \text{-----})$ or r = ¼ and

$a_1 + a_2 + \text{----} = \frac{4a_1}{3} = 32$ (given) $\therefore a_1 = 24$

The GP is 24, 6, 1.5, $\frac{1.5}{4},\frac{1.5}{16}$ …..

$\therefore a_5 = \frac{1.5}{16} = \frac{3}{32}$ Choice (3)

100. $a_1 = \frac{1}{2(5)} = \frac{1}{3}\left(\frac{1}{2}-\frac{1}{5}\right)$

$a_2 = \frac{1}{5(8)} = \frac{1}{3}\left(\frac{1}{5}-\frac{1}{8}\right)$

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$a_{100} = \frac{1}{299(302)} = \frac{1}{3}\left(\frac{1}{299}-\frac{1}{302}\right)$

All the terms like 1/5, 1/8,……..1/299 will cancel out.

$\therefore$ The sum = $\frac{1}{3}\left(\frac{1}{2}-\frac{1}{302}\right)$

$= \frac{1}{3}\frac{(300)}{(2)\,(302)} = \frac{50}{302} = \frac{25}{151}$ Choice (1)